Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.2-Cosine/93-4.2.4.1-a+b-cos-^m-A+B-cos+C-cos^2-

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [393]. This is test number [93].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (393)	0.00 (0)
Mathematica	98.98 (389)	1.02 (4)
Maple	60.56 (238)	39.44 (155)
Fricas	60.56 (238)	39.44 (155)
Maxima	30.28 (119)	69.72 (274)
Mupad	19.08 (75)	80.92 (318)
Giac	6.62 (26)	93.38 (367)
Sympy	3.82 (15)	96.18 (378)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

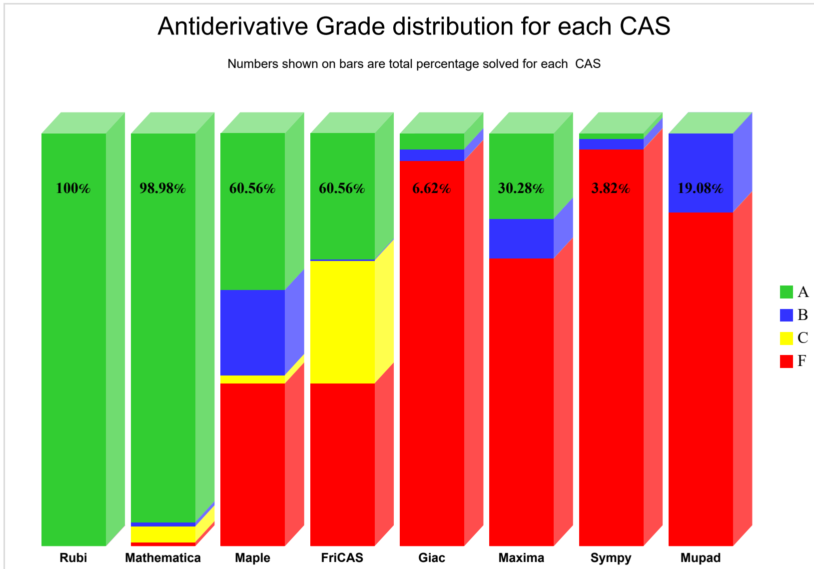
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

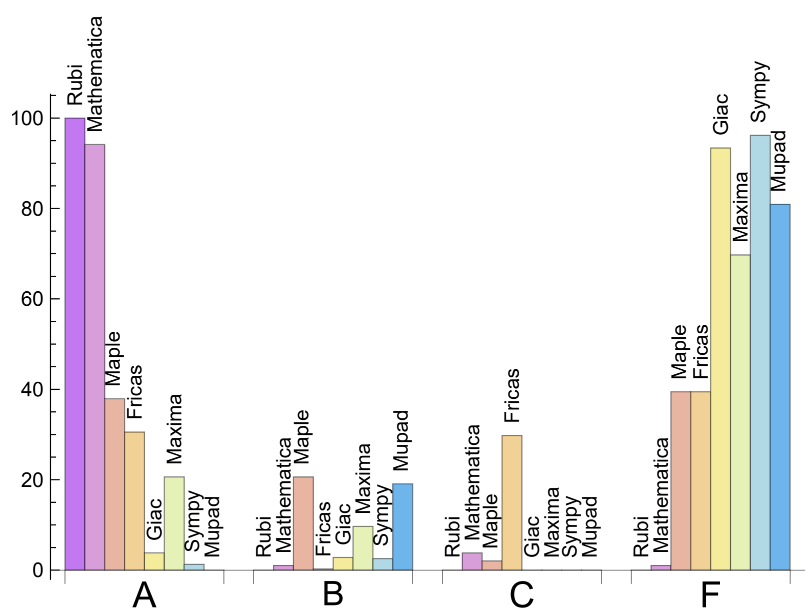
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	94.148	1.018	3.817	1.018
Maple	37.913	20.611	2.036	39.440
Fricas	30.534	0.254	29.771	39.440
Maxima	20.611	9.669	0.000	69.720
Giac	3.817	2.799	0.000	93.384
Sympy	1.272	2.545	0.000	96.183
Mupad	0.000	19.084	0.000	80.916

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	155	100.00	0.00	0.00
Maple	155	100.00	0.00	0.00
Maxima	274	100.00	0.00	0.00
Mupad	318	0.00	100.00	0.00
Giac	367	99.18	0.82	0.00
Sympy	378	21.96	78.04	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.16
Fricas	0.20
Maxima	0.45
Mathematica	0.72
Mupad	1.52
Giac	2.32
Sympy	10.46
Maple	38.56

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	94.19	1.00	84.00	0.86
Rubi	132.62	1.00	120.00	1.00
Fricas	172.05	1.52	170.00	1.35
Sympy	191.47	2.94	184.00	2.10
Mathematica	210.58	1.19	91.00	0.76
Maple	242.72	2.07	213.50	1.62
Maxima	473.18	3.74	107.00	1.18
Giac	3422.92	48.98	95.50	1.45

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

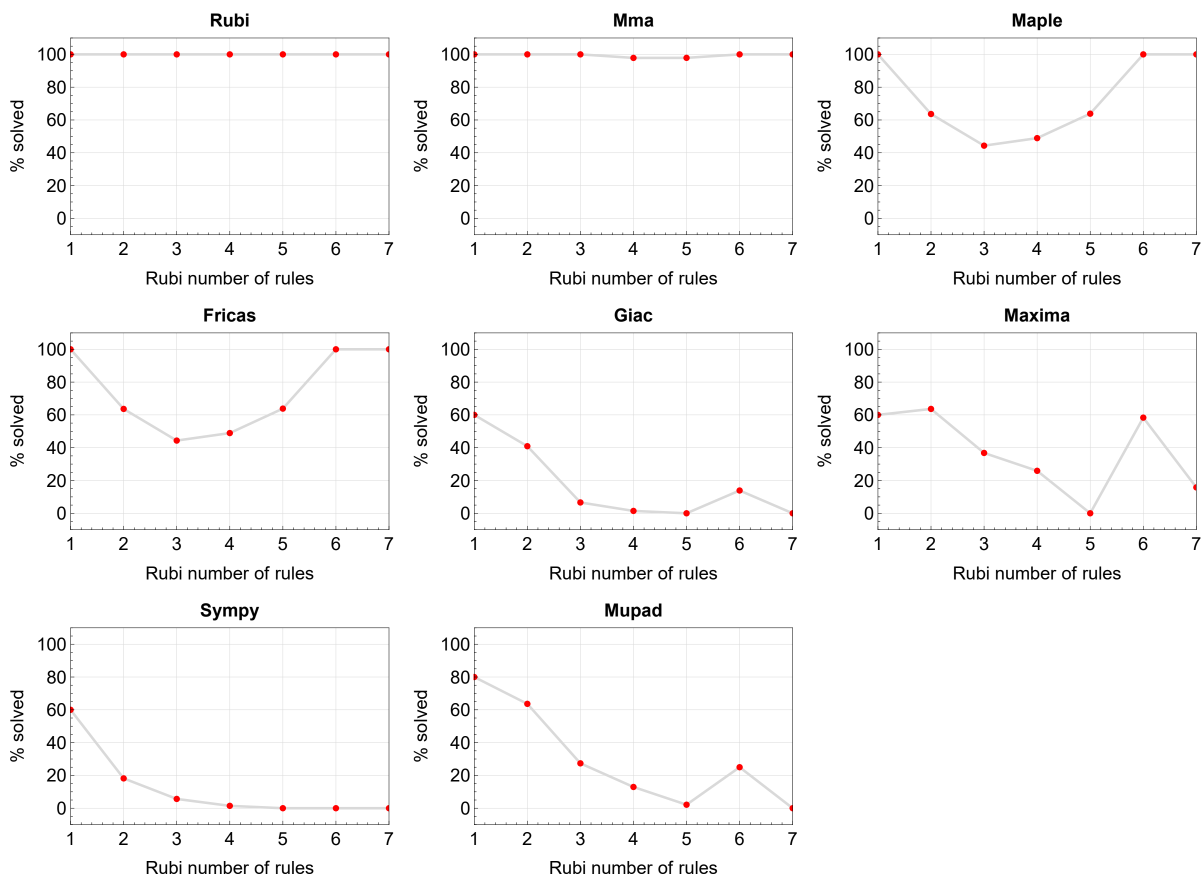


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

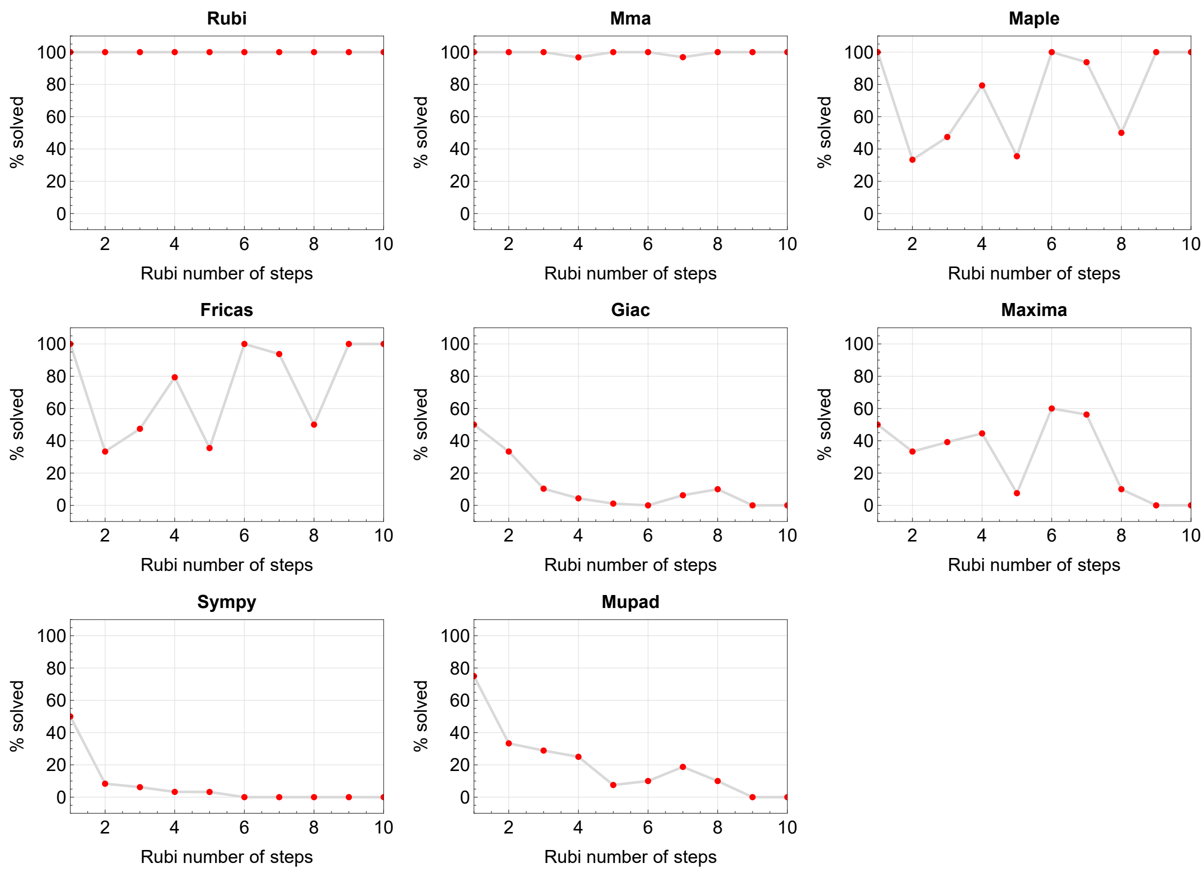


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

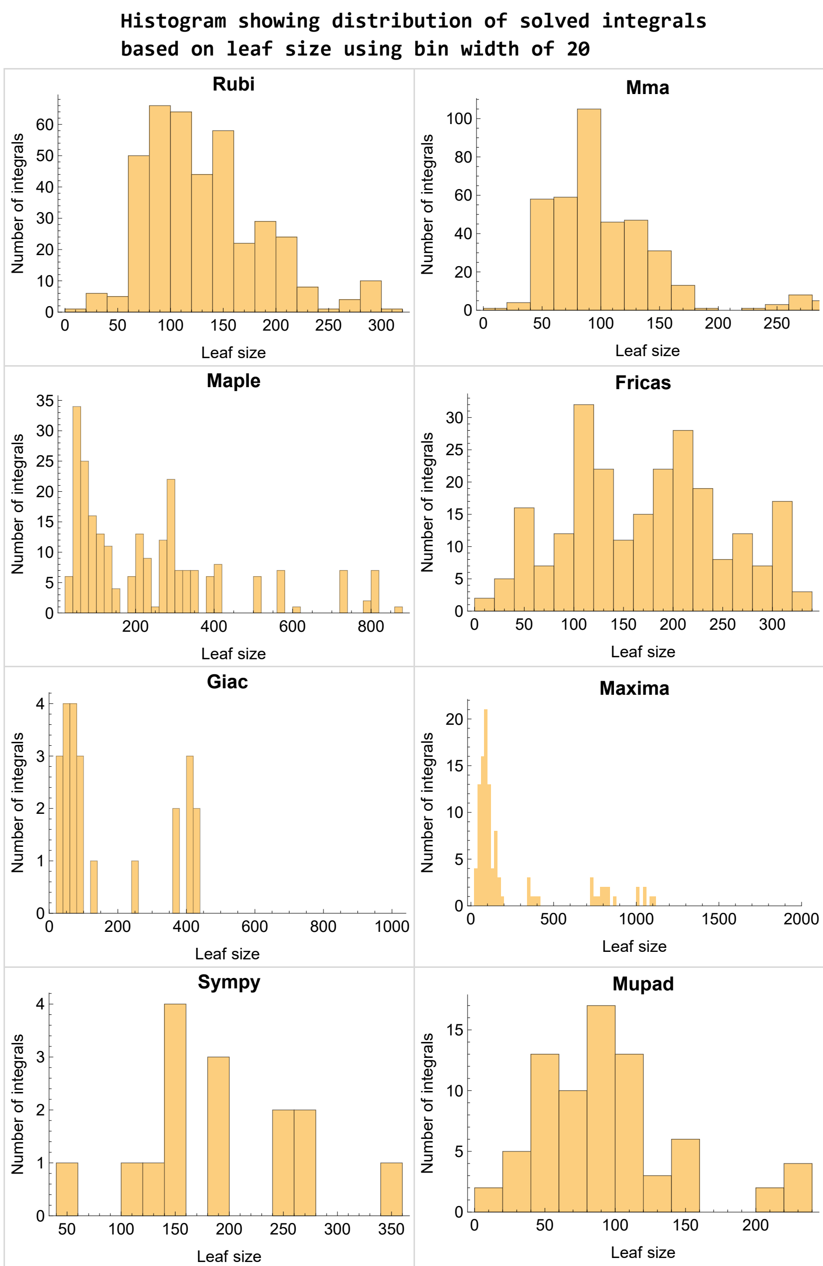


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

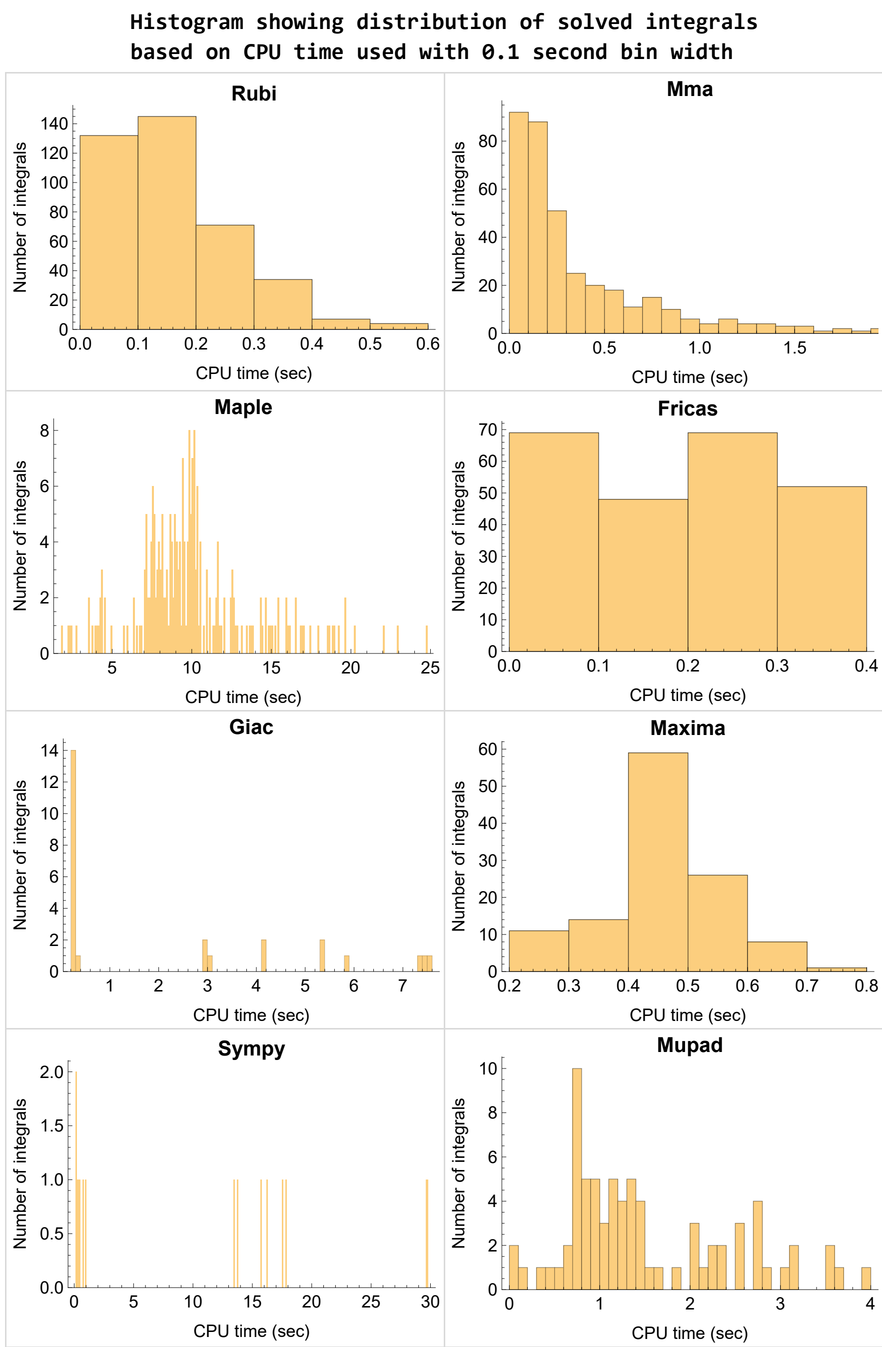


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

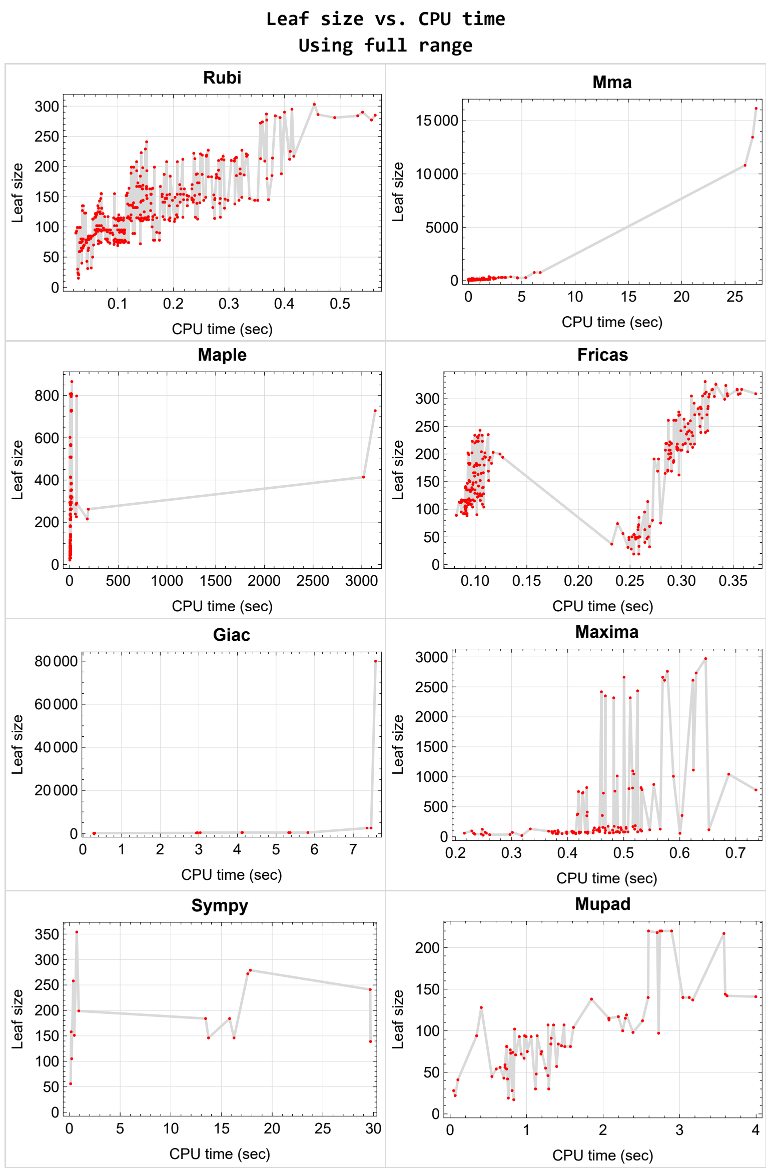


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {198, 206, 208, 233, 267, 268, 276, 283, 391, 393}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

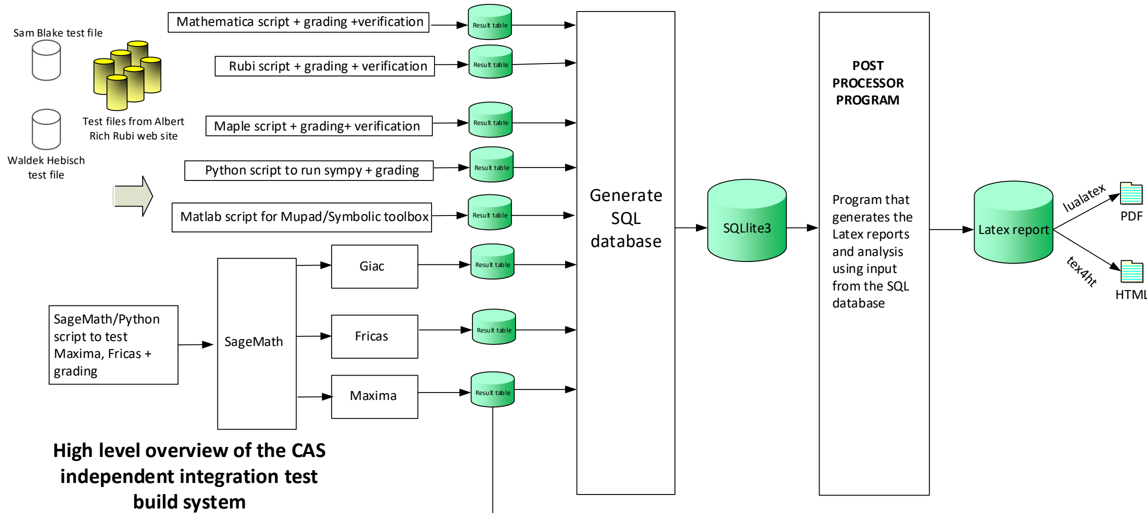
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 201, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391 }

B grade { 200, 208, 233, 393 }

C grade { 35, 36, 66, 67, 68, 76, 198, 232, 267, 268, 276, 283, 383, 384, 386 }

F normal fail { 202, 385, 387, 392 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 35, 36, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 255, 256, 257, 258, 259, 263, 264, 265, 266, 267, 271, 272, 273, 274, 275, 279, 280, 281, 282, 283, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

B grade { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 244, 245, 246, 252, 253, 254, 260, 261, 262, 268, 269, 270, 276, 277, 278, 284, 285, 286, 287 }

C grade { 26, 27, 28, 29, 30, 31, 32, 33 }

F normal fail { 34, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 24, 25, 35, 36, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

B grade { 12 }

C grade { 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287 }

F normal fail { 34, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 132, 133, 134, 135, 136, 288, 289, 290, 291, 292, 293, 297, 298, 299, 300, 301, 302, 306, 307, 308, 309, 310, 311, 315, 316, 317, 318, 319, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335 }

B grade { 35, 36, 95, 96, 97, 104, 105, 106, 113, 114, 115, 121, 122, 123, 129, 130, 131, 137, 138, 139, 294, 295, 296, 303, 304, 305, 312, 313, 314, 320, 321, 322, 328, 329, 330, 336, 337, 338 }

C grade { }

F normal fail { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 }

B grade { 35, 36, 90, 91, 99, 288, 289, 290, 297, 298, 306 }

C grade { }

F normal fail { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { 89, 98, 107 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 25, 35, 36, 65, 89, 90, 91, 92, 94, 96, 98, 99, 100, 101, 103, 105, 107, 108, 109, 110, 112, 114, 116, 117, 118, 120, 122, 124, 125, 126, 128, 130, 132, 133, 134, 136, 138, 266, 288, 289, 290, 291, 297, 298, 299, 300, 306, 307, 308, 309, 315, 316, 317, 323, 324, 325, 331, 332, 333 }

C grade { }

F normal fail { }

F(-1) timedout fail { 16, 17, 18, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 95, 97, 102, 104, 106, 111, 113, 115, 119, 121, 123, 127, 129, 131, 135, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 292, 293, 294, 295, 296, 301, 302, 303, 304, 305, 310, 311, 312, 313, 314, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-2) exception fail { }

Sympy

A grade { 92, 118, 290, 291, 317 }

B grade { 1, 2, 3, 4, 9, 10, 11, 35, 36, 91 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 30, 31, 32, 33, 34, 40, 66, 67, 76, 77, 93, 119, 120, 128, 161, 162, 167, 168, 173, 178, 179, 180, 181, 182, 185, 186, 193, 194, 198, 200, 201, 202, 204, 205, 206, 209, 210, 213, 214, 215, 216, 219, 220, 227, 228, 229, 232, 235, 236, 237, 238, 242, 267, 268, 276, 277, 292, 318, 319, 327, 354, 355, 365, 366, 367, 368, 369, 372, 373, 379, 380, 383, 385, 386, 387, 389, 390, 391 }

F(-1) timedout fail { 8, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 169, 170, 171, 172, 174, 175, 176, 177, 183, 184, 187, 188, 189, 190, 191, 192, 195, 196, 197, 199, 203, 207, 208, 211, 212, 217, 218, 221, 222, 223, 224, 225, 226, 230, 231, 233, 234, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 356, 357, 358, 359, 360, 361, 362, 363, 364, 370, 371, 374, 375, 376, 377, 378, 381, 382, 384, 388, 392, 393 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	133	82	75	80	199	93	74
N.S.	1	1.00	1.45	0.89	0.82	0.87	2.16	1.01	0.80
time (sec)	N/A	0.081	0.038	4.380	0.253	0.272	0.915	0.286	0.818

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	101	66	60	63	151	76	59
N.S.	1	1.00	1.40	0.92	0.83	0.88	2.10	1.06	0.82
time (sec)	N/A	0.065	0.030	3.784	0.255	0.264	0.479	0.281	0.717

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	71	49	43	45	105	57	43
N.S.	1	1.00	1.42	0.98	0.86	0.90	2.10	1.14	0.86
time (sec)	N/A	0.055	0.016	4.303	0.234	0.249	0.217	0.281	0.703

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	50	31	34	28	56	34	28
N.S.	1	1.00	1.67	1.03	1.13	0.93	1.87	1.13	0.93
time (sec)	N/A	0.028	0.028	2.498	0.261	0.251	0.115	0.278	0.046

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	30	38	40	0	40	22
N.S.	1	1.00	1.46	1.25	1.58	1.67	0.00	1.67	0.92
time (sec)	N/A	0.028	0.028	1.870	0.297	0.264	0.000	0.281	0.066

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	55	58	72	0	60	41
N.S.	1	1.00	1.20	1.38	1.45	1.80	0.00	1.50	1.02
time (sec)	N/A	0.035	0.024	3.579	0.232	0.258	0.000	0.299	0.101

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	93	85	97	95	0	98	77
N.S.	1	1.00	1.33	1.21	1.39	1.36	0.00	1.40	1.10
time (sec)	N/A	0.056	0.015	4.240	0.229	0.264	0.000	0.291	0.786

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	137	108	126	114	0	121	102
N.S.	1	1.00	1.40	1.10	1.29	1.16	0.00	1.23	1.04
time (sec)	N/A	0.074	0.024	4.073	0.248	0.267	0.000	0.301	0.843

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	93	79	130	85	354	87	119
N.S.	1	1.00	0.79	0.68	1.11	0.73	3.03	0.74	1.02
time (sec)	N/A	0.082	0.191	3.957	0.333	0.259	0.716	0.291	2.305

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	68	61	103	68	258	68	91
N.S.	1	1.00	0.76	0.69	1.16	0.76	2.90	0.76	1.02
time (sec)	N/A	0.061	0.118	3.546	0.382	0.258	0.375	0.281	1.321

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	44	73	49	158	43	67
N.S.	1	1.00	0.74	0.72	1.20	0.80	2.59	0.70	1.10
time (sec)	N/A	0.046	0.105	2.319	0.301	0.256	0.167	0.280	0.967

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	21	20	31	0	20	17
N.S.	1	1.00	1.00	1.40	1.33	2.07	0.00	1.33	1.13
time (sec)	N/A	0.029	0.008	2.784	0.318	0.248	0.000	0.285	0.832

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	27	37	0	34	28
N.S.	1	1.00	0.84	0.81	0.63	0.86	0.00	0.79	0.65
time (sec)	N/A	0.044	0.070	4.168	0.249	0.232	0.000	0.286	0.810

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	58	43	56	0	57	42
N.S.	1	1.00	0.94	0.89	0.66	0.86	0.00	0.88	0.65
time (sec)	N/A	0.047	0.150	4.928	0.246	0.243	0.000	0.284	0.750

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	78	60	74	0	79	56
N.S.	1	1.00	0.93	0.90	0.69	0.85	0.00	0.91	0.64
time (sec)	N/A	0.058	0.217	4.378	0.216	0.238	0.000	0.291	0.717

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	88	324	0	128	0	0	0
N.S.	1	1.00	0.78	2.87	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.097	0.542	14.492	0.000	0.102	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	86	296	0	112	0	0	0
N.S.	1	1.00	0.76	2.62	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.147	0.461	9.919	0.000	0.094	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	70	261	0	101	0	0	0
N.S.	1	1.00	0.91	3.39	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.099	0.284	8.708	0.000	0.090	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	58	236	0	92	0	0	94
N.S.	1	1.00	0.77	3.15	0.00	1.23	0.00	0.00	1.25
time (sec)	N/A	0.094	0.244	5.785	0.000	0.092	0.000	0.000	0.346

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	117	0	0	0
N.S.	1	1.00	0.77	2.92	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.104	0.287	7.553	0.000	0.087	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	116	0	0	0
N.S.	1	1.00	0.74	3.77	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.099	0.341	7.073	0.000	0.090	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	567	0	139	0	0	0
N.S.	1	1.00	0.70	4.93	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.153	0.445	11.535	0.000	0.097	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	77	414	0	135	0	0	0
N.S.	1	1.00	0.67	3.60	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.105	0.516	10.321	0.000	0.096	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	99	0	19	0	0	0
N.S.	1	1.00	1.10	4.71	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.030	0.278	6.352	0.000	0.259	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	99	0	19	0	0	19
N.S.	1	1.00	1.00	4.71	0.00	0.90	0.00	0.00	0.90
time (sec)	N/A	0.029	0.137	4.295	0.000	0.254	0.000	0.000	0.760

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	78	291	0	142	0	0	0
N.S.	1	1.00	0.68	2.53	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.137	1.825	72.991	0.000	0.103	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	79	798	0	147	0	0	0
N.S.	1	1.00	0.69	6.94	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.143	1.385	72.588	0.000	0.092	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	226	0	112	0	0	0
N.S.	1	1.00	0.74	2.90	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.112	1.113	70.412	0.000	0.086	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	55	313	0	95	0	0	0
N.S.	1	1.00	0.74	4.23	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.119	1.146	17.086	0.000	0.087	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	58	226	0	97	0	0	0
N.S.	1	1.00	0.77	3.01	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.108	0.722	12.642	0.000	0.091	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	61	795	0	108	0	0	0
N.S.	1	1.00	0.79	10.32	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.109	0.574	16.523	0.000	0.093	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	79	291	0	119	0	0	0
N.S.	1	1.00	0.69	2.53	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.148	0.933	17.412	0.000	0.094	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	866	0	129	0	0	0
N.S.	1	1.00	0.70	7.53	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.151	1.000	22.996	0.000	0.100	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	114	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	113	31	175	33	279	2494	30
N.S.	1	1.00	3.65	1.00	5.65	1.06	9.00	80.45	0.97
time (sec)	N/A	0.046	0.150	10.997	0.483	0.259	17.842	7.462	1.115

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	114	31	175	32	272	2489	30
N.S.	1	1.00	3.56	0.97	5.47	1.00	8.50	77.78	0.94
time (sec)	N/A	0.052	0.154	8.207	0.472	0.269	17.589	7.355	1.289

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	322	0	122	0	0	0
N.S.	1	1.00	0.79	2.88	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.106	0.435	12.699	0.000	0.102	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	89	294	0	106	0	0	0
N.S.	1	1.00	0.81	2.67	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.125	0.202	9.934	0.000	0.091	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	70	261	0	101	0	0	0
N.S.	1	1.00	0.91	3.39	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.065	0.068	9.481	0.000	0.091	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	59	237	0	89	0	0	0
N.S.	1	1.00	0.81	3.25	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.086	0.065	7.933	0.000	0.082	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	55	214	0	114	0	0	0
N.S.	1	1.00	0.80	3.10	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.100	0.594	8.075	0.000	0.091	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	56	292	0	113	0	0	0
N.S.	1	1.00	0.74	3.84	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.100	0.378	7.799	0.000	0.084	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	84	562	0	136	0	0	0
N.S.	1	1.00	0.76	5.11	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.141	0.531	11.664	0.000	0.097	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	83	412	0	132	0	0	0
N.S.	1	1.00	0.73	3.65	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.139	0.676	10.361	0.000	0.095	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	91	324	0	124	0	0	0
N.S.	1	1.00	0.83	2.95	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.101	0.242	12.579	0.000	0.103	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	86	296	0	112	0	0	0
N.S.	1	1.00	0.76	2.62	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.096	0.215	10.533	0.000	0.092	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	263	0	104	0	0	0
N.S.	1	1.00	0.95	3.51	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.082	0.053	9.154	0.000	0.090	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	239	0	88	0	0	0
N.S.	1	1.00	0.80	3.14	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.114	0.075	7.368	0.000	0.092	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	57	216	0	113	0	0	0
N.S.	1	1.00	0.79	3.00	0.00	1.57	0.00	0.00	0.00
time (sec)	N/A	0.141	0.438	7.287	0.000	0.087	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	112	0	0	0
N.S.	1	1.00	0.74	3.77	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.164	0.411	7.112	0.000	0.086	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	84	565	0	141	0	0	0
N.S.	1	1.00	0.74	5.00	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.203	0.537	11.513	0.000	0.091	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	83	414	0	136	0	0	0
N.S.	1	1.00	0.72	3.60	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.194	0.741	9.941	0.000	0.097	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	88	324	0	128	0	0	0
N.S.	1	1.00	0.78	2.87	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.121	0.228	14.602	0.000	0.105	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	296	0	116	0	0	0
N.S.	1	1.00	0.78	2.64	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.155	0.209	16.852	0.000	0.105	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	73	263	0	106	0	0	0
N.S.	1	1.00	0.94	3.37	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.166	0.059	18.822	0.000	0.090	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	239	0	90	0	0	0
N.S.	1	1.00	0.83	3.06	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.175	0.470	59.150	0.000	0.102	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	115	0	0	0
N.S.	1	1.00	0.77	2.92	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.111	0.353	181.955	0.000	0.099	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	114	0	0	0
N.S.	1	1.00	0.74	3.77	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.107	0.511	2.246	0.000	0.087	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	80	602	0	145	0	0	0
N.S.	1	1.00	0.70	5.23	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.134	0.873	4.553	0.000	0.098	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	83	414	0	140	0	0	0
N.S.	1	1.00	0.72	3.60	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.150	1.435	3018.439	0.000	0.108	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	94	349	0	128	0	0	0
N.S.	1	1.00	0.64	2.37	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.153	0.789	12.534	0.000	0.108	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	83	321	0	125	0	0	0
N.S.	1	1.00	0.72	2.79	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.123	0.734	11.141	0.000	0.106	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	77	293	0	109	0	0	0
N.S.	1	1.00	0.69	2.62	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.100	0.490	10.062	0.000	0.095	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	73	260	0	104	0	0	0
N.S.	1	1.00	0.91	3.25	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.081	0.078	8.460	0.000	0.098	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	58	236	0	92	0	0	94
N.S.	1	1.00	0.77	3.15	0.00	1.23	0.00	0.00	1.25
time (sec)	N/A	0.068	0.072	5.907	0.000	0.090	0.000	0.000	0.973

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	198	213	0	117	0	0	0
N.S.	1	1.00	2.79	3.00	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.093	2.540	8.102	0.000	0.094	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	141	291	0	116	0	0	0
N.S.	1	1.00	1.93	3.99	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.100	1.462	7.855	0.000	0.096	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	235	564	0	139	0	0	0
N.S.	1	1.00	2.10	5.04	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.141	4.598	12.401	0.000	0.091	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	74	413	0	135	0	0	0
N.S.	1	1.00	0.67	3.75	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.144	0.675	11.195	0.000	0.100	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	97	731	0	160	0	0	0
N.S.	1	1.00	0.66	4.97	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.185	1.013	16.574	0.000	0.100	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	86	324	0	125	0	0	0
N.S.	1	1.00	0.75	2.82	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.110	0.824	12.549	0.000	0.105	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	80	296	0	109	0	0	0
N.S.	1	1.00	0.70	2.57	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.117	0.704	10.516	0.000	0.107	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	263	0	104	0	0	0
N.S.	1	1.00	0.86	3.29	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.078	0.572	9.151	0.000	0.091	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	239	0	92	0	0	0
N.S.	1	1.00	0.78	3.06	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.080	0.075	7.214	0.000	0.091	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	117	0	0	0
N.S.	1	1.00	0.77	2.92	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.070	0.082	7.401	0.000	0.092	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	140	294	0	116	0	0	0
N.S.	1	1.00	1.87	3.92	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.098	1.000	7.469	0.000	0.088	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	81	567	0	139	0	0	0
N.S.	1	1.00	0.72	5.02	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.147	0.204	12.026	0.000	0.090	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	77	414	0	135	0	0	0
N.S.	1	1.00	0.69	3.70	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.152	0.604	11.475	0.000	0.094	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	86	324	0	125	0	0	0
N.S.	1	1.00	0.75	2.82	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.134	1.146	12.092	0.000	0.106	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	80	296	0	109	0	0	0
N.S.	1	1.00	0.70	2.57	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.170	0.874	10.383	0.000	0.096	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	263	0	104	0	0	0
N.S.	1	1.00	0.86	3.29	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.095	0.662	8.905	0.000	0.108	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	239	0	92	0	0	0
N.S.	1	1.00	0.78	3.06	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.103	0.079	7.937	0.000	0.089	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	117	0	0	0
N.S.	1	1.00	0.77	2.92	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.119	0.086	7.691	0.000	0.098	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	116	0	0	0
N.S.	1	1.00	0.74	3.77	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.093	0.130	6.575	0.000	0.087	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	81	567	0	139	0	0	0
N.S.	1	1.00	0.72	5.06	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.163	0.169	11.630	0.000	0.107	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	77	414	0	135	0	0	0
N.S.	1	1.00	0.68	3.66	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.244	0.233	10.281	0.000	0.092	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	567	0	139	0	0	0
N.S.	1	1.00	0.70	4.93	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.092	0.047	11.799	0.000	0.090	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	77	414	0	135	0	0	0
N.S.	1	1.00	0.67	3.60	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.100	0.049	10.553	0.000	0.093	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	70	70	111	63	0	0	97
N.S.	1	1.00	0.60	0.60	0.96	0.54	0.00	0.00	0.84
time (sec)	N/A	0.060	0.160	7.590	0.452	0.258	0.000	0.000	2.725

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	67	88	75	200	0	362	112
N.S.	1	1.00	0.59	0.78	0.66	1.77	0.00	3.20	0.99
time (sec)	N/A	0.055	0.498	7.062	0.431	0.287	0.000	2.965	2.515

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	47	57	46	139	79987	72
N.S.	1	1.00	0.70	0.64	0.77	0.62	1.88	1080.91	0.97
time (sec)	N/A	0.031	0.065	7.189	0.600	0.256	29.700	7.577	1.187

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	52	54	52	162	146	0	45
N.S.	1	1.00	0.58	0.60	0.58	1.80	1.62	0.00	0.50
time (sec)	N/A	0.025	0.059	6.898	0.414	0.297	13.719	0.000	0.546

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	44	53	80	201	0	0	0
N.S.	1	1.00	0.65	0.78	1.18	2.96	0.00	0.00	0.00
time (sec)	N/A	0.038	0.039	7.063	0.436	0.290	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	45	45	80	185	0	0	81
N.S.	1	1.00	0.76	0.76	1.36	3.14	0.00	0.00	1.37
time (sec)	N/A	0.031	0.051	8.606	0.411	0.292	0.000	0.000	1.499

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	59	115	728	213	0	0	0
N.S.	1	1.00	0.76	1.47	9.33	2.73	0.00	0.00	0.00
time (sec)	N/A	0.044	0.081	9.421	0.464	0.295	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	51	54	350	47	0	0	217
N.S.	1	1.00	0.65	0.68	4.43	0.59	0.00	0.00	2.75
time (sec)	N/A	0.046	0.150	8.821	0.434	0.266	0.000	0.000	3.579

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	180	2318	255	0	0	0
N.S.	1	1.00	0.66	1.48	19.00	2.09	0.00	0.00	0.00
time (sec)	N/A	0.068	0.160	8.638	0.512	0.306	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	70	71	117	69	0	0	98
N.S.	1	1.00	0.59	0.60	0.98	0.58	0.00	0.00	0.82
time (sec)	N/A	0.061	0.173	8.972	0.454	0.269	0.000	0.000	2.390

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	67	89	82	209	0	363	113
N.S.	1	1.00	0.58	0.77	0.71	1.80	0.00	3.13	0.97
time (sec)	N/A	0.059	0.380	9.022	0.525	0.295	0.000	3.036	2.077

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	53	48	60	50	0	0	54
N.S.	1	1.00	0.70	0.63	0.79	0.66	0.00	0.00	0.71
time (sec)	N/A	0.036	0.052	9.201	0.421	0.259	0.000	0.000	0.739

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	52	55	55	165	0	0	46
N.S.	1	1.00	0.56	0.59	0.59	1.77	0.00	0.00	0.49
time (sec)	N/A	0.026	0.061	9.024	0.398	0.284	0.000	0.000	1.279

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	44	54	83	204	0	0	0
N.S.	1	1.00	0.63	0.77	1.19	2.91	0.00	0.00	0.00
time (sec)	N/A	0.040	0.044	9.529	0.457	0.306	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	46	80	188	0	0	82
N.S.	1	1.00	0.74	0.75	1.31	3.08	0.00	0.00	1.34
time (sec)	N/A	0.035	0.047	8.980	0.398	0.292	0.000	0.000	1.455

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	59	116	761	216	0	0	0
N.S.	1	1.00	0.74	1.45	9.51	2.70	0.00	0.00	0.00
time (sec)	N/A	0.046	0.085	9.068	0.484	0.288	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	52	55	355	50	0	0	218
N.S.	1	1.00	0.64	0.68	4.38	0.62	0.00	0.00	2.69
time (sec)	N/A	0.051	0.023	8.637	0.462	0.268	0.000	0.000	2.707

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	81	181	2434	260	0	0	0
N.S.	1	1.00	0.65	1.45	19.47	2.08	0.00	0.00	0.00
time (sec)	N/A	0.068	0.100	8.691	0.525	0.309	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	70	73	127	75	0	0	100
N.S.	1	1.00	0.56	0.58	1.02	0.60	0.00	0.00	0.80
time (sec)	N/A	0.064	0.182	8.434	0.565	0.280	0.000	0.000	2.256

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	67	91	92	219	0	0	72
N.S.	1	1.00	0.55	0.75	0.75	1.80	0.00	0.00	0.59
time (sec)	N/A	0.062	0.718	7.868	0.408	0.286	0.000	0.000	0.928

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	52	50	64	54	0	0	56
N.S.	1	1.00	0.65	0.62	0.80	0.68	0.00	0.00	0.70
time (sec)	N/A	0.034	0.102	8.286	0.409	0.254	0.000	0.000	0.653

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	52	57	59	171	0	0	48
N.S.	1	1.00	0.53	0.58	0.60	1.73	0.00	0.00	0.48
time (sec)	N/A	0.027	0.085	8.513	0.373	0.287	0.000	0.000	1.125

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	44	56	87	210	0	0	0
N.S.	1	1.00	0.59	0.76	1.18	2.84	0.00	0.00	0.00
time (sec)	N/A	0.034	0.061	8.717	0.408	0.296	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	48	80	194	0	0	84
N.S.	1	1.00	0.69	0.74	1.23	2.98	0.00	0.00	1.29
time (sec)	N/A	0.034	0.056	8.105	0.383	0.289	0.000	0.000	1.318

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	59	118	821	222	0	0	0
N.S.	1	1.00	0.70	1.40	9.77	2.64	0.00	0.00	0.00
time (sec)	N/A	0.047	0.091	8.878	0.434	0.289	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	51	57	367	54	0	0	220
N.S.	1	1.00	0.60	0.67	4.32	0.64	0.00	0.00	2.59
time (sec)	N/A	0.047	0.170	7.655	0.417	0.254	0.000	0.000	2.766

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	80	183	2662	270	0	0	0
N.S.	1	1.00	0.61	1.40	20.32	2.06	0.00	0.00	0.00
time (sec)	N/A	0.065	0.183	7.189	0.501	0.297	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	67	88	75	207	0	0	115
N.S.	1	1.00	0.59	0.78	0.66	1.83	0.00	0.00	1.02
time (sec)	N/A	0.055	0.536	8.101	0.423	0.300	0.000	0.000	2.075

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	47	57	49	0	0	75
N.S.	1	1.00	0.70	0.64	0.77	0.66	0.00	0.00	1.01
time (sec)	N/A	0.033	0.068	8.078	0.414	0.255	0.000	0.000	1.196

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	52	54	52	169	146	0	81
N.S.	1	1.00	0.58	0.60	0.58	1.88	1.62	0.00	0.90
time (sec)	N/A	0.024	0.052	8.490	0.379	0.277	16.232	0.000	1.572

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	44	53	80	207	0	0	0
N.S.	1	1.00	0.65	0.78	1.18	3.04	0.00	0.00	0.00
time (sec)	N/A	0.038	0.035	7.514	0.420	0.289	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	45	45	85	191	0	0	84
N.S.	1	1.00	0.76	0.76	1.44	3.24	0.00	0.00	1.42
time (sec)	N/A	0.031	0.035	7.128	0.372	0.273	0.000	0.000	1.414

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	59	115	728	219	0	0	0
N.S.	1	1.00	0.76	1.47	9.33	2.81	0.00	0.00	0.00
time (sec)	N/A	0.042	0.056	7.164	0.426	0.304	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	51	54	355	50	0	0	220
N.S.	1	1.00	0.65	0.68	4.49	0.63	0.00	0.00	2.78
time (sec)	N/A	0.048	0.085	7.677	0.604	0.260	0.000	0.000	2.895

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	180	2318	261	0	0	0
N.S.	1	1.00	0.66	1.48	19.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.073	0.085	7.359	0.482	0.295	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	67	91	75	207	0	0	115
N.S.	1	1.00	0.55	0.75	0.61	1.70	0.00	0.00	0.94
time (sec)	N/A	0.061	0.647	7.569	0.423	0.288	0.000	0.000	2.290

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	52	50	57	49	0	0	75
N.S.	1	1.00	0.65	0.62	0.71	0.61	0.00	0.00	0.94
time (sec)	N/A	0.036	0.078	7.543	0.415	0.249	0.000	0.000	1.008

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	52	57	52	169	0	0	81
N.S.	1	1.00	0.53	0.58	0.53	1.71	0.00	0.00	0.82
time (sec)	N/A	0.030	0.047	7.825	0.390	0.287	0.000	0.000	0.738

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	44	56	80	207	0	0	0
N.S.	1	1.00	0.59	0.76	1.08	2.80	0.00	0.00	0.00
time (sec)	N/A	0.044	0.039	8.375	0.410	0.284	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	48	93	191	0	0	84
N.S.	1	1.00	0.69	0.74	1.43	2.94	0.00	0.00	1.29
time (sec)	N/A	0.037	0.044	8.129	0.377	0.287	0.000	0.000	1.317

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	59	118	736	219	0	0	0
N.S.	1	1.00	0.70	1.40	8.76	2.61	0.00	0.00	0.00
time (sec)	N/A	0.047	0.054	8.173	0.427	0.290	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	51	57	380	50	0	0	220
N.S.	1	1.00	0.60	0.67	4.47	0.59	0.00	0.00	2.59
time (sec)	N/A	0.055	0.094	8.037	0.418	0.259	0.000	0.000	2.743

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	80	183	2350	261	0	0	0
N.S.	1	1.00	0.61	1.40	17.94	1.99	0.00	0.00	0.00
time (sec)	N/A	0.071	0.081	7.463	0.467	0.287	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	70	91	75	207	0	0	115
N.S.	1	1.00	0.57	0.75	0.61	1.70	0.00	0.00	0.94
time (sec)	N/A	0.059	0.953	7.729	0.433	0.296	0.000	0.000	2.077

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	55	50	57	49	0	0	75
N.S.	1	1.00	0.69	0.62	0.71	0.61	0.00	0.00	0.94
time (sec)	N/A	0.038	0.065	7.638	0.436	0.258	0.000	0.000	1.007

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	55	57	52	169	0	0	81
N.S.	1	1.00	0.56	0.58	0.53	1.71	0.00	0.00	0.82
time (sec)	N/A	0.032	0.047	7.487	0.382	0.290	0.000	0.000	0.741

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	47	56	80	207	0	0	0
N.S.	1	1.00	0.64	0.76	1.08	2.80	0.00	0.00	0.00
time (sec)	N/A	0.043	0.043	7.915	0.502	0.299	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	48	93	191	0	0	117
N.S.	1	1.00	0.69	0.74	1.43	2.94	0.00	0.00	1.80
time (sec)	N/A	0.035	0.046	7.559	0.366	0.278	0.000	0.000	2.199

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	59	118	754	219	0	0	0
N.S.	1	1.00	0.70	1.40	8.98	2.61	0.00	0.00	0.00
time (sec)	N/A	0.052	0.056	7.608	0.419	0.285	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	51	57	412	50	0	0	220
N.S.	1	1.00	0.60	0.67	4.85	0.59	0.00	0.00	2.59
time (sec)	N/A	0.050	0.110	6.719	0.435	0.252	0.000	0.000	2.593

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	80	183	2418	261	0	0	0
N.S.	1	1.00	0.61	1.40	18.46	1.99	0.00	0.00	0.00
time (sec)	N/A	0.071	0.145	7.927	0.460	0.292	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.103	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	88	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.092	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	0.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	96	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	0.086	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	0.119	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	88	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	0.119	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	96	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.175	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	0.195	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.008	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.127	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	90	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	0.114	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.081	0.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	87	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	91	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.341	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	89	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.283	0.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	91	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.333	0.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.014	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	87	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.008	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	91	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.275	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	89	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.264	0.000	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	91	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	0.316	0.000	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	90	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.006	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	87	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	88	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	0.010	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	90	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	0.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	91	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	138	142	0	0	0	0	0	0
N.S.	1	0.93	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	0.207	0.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	136	142	0	0	0	0	0	0
N.S.	1	0.93	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.167	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	136	142	0	0	0	0	0	0
N.S.	1	0.93	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.223	0.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	136	142	0	0	0	0	0	0
N.S.	1	0.93	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.203	0.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	142	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	0.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	139	142	0	0	0	0	0	0
N.S.	1	0.93	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.160	0.198	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	132	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.190	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	122	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	0.147	0.000	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	120	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.163	0.128	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	114	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	0.119	0.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	111	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	0.166	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	117	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	0.143	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	114	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	122	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.124	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0	0
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.135	0.168	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0	0
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	0.148	0.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0	0
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	0.128	0.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	140	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	132	140	0	0	0	0	0	0
N.S.	1	0.94	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.132	0.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0	0
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0	0
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	0.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	242	0	0	0	0	0	0
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	1.293	0.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	175	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.801	0.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	289	0	0	0	0	0	0
N.S.	1	1.00	2.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	2.819	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	144	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.592	0.000	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	279	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.556	2.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	276	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.368	2.001	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	256	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.360	1.337	0.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	272	272	256	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.357	1.458	0.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	119	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	0.459	0.000	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	285	285	10805	0	0	0	0	0	0
N.S.	1	1.00	37.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.563	25.938	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	118	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	0.185	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.269	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.152	0.257	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	140	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	0.366	0.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	0.294	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.134	0.285	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	140	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.241	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	136	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.235	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	120	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	0.312	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	120	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.187	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	118	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.155	0.000	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	0.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	109	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.271	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	109	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	0.121	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	118	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.195	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	133	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.172	0.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.152	0.151	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.157	0.150	0.000	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	356	0	0	0	0	0	0
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	3.968	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	13441	0	0	0	0	0	0
N.S.	1	1.00	45.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	26.644	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	284	284	290	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.532	3.488	0.000	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	284	284	289	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	3.426	0.000	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	281	281	263	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.392	2.362	0.000	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	281	281	261	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	2.319	0.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	144	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.263	0.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	125	382	0	191	0	0	0
N.S.	1	1.00	0.60	1.83	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.364	1.782	15.997	0.000	0.107	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	111	351	0	177	0	0	0
N.S.	1	1.00	0.62	1.95	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.368	2.013	14.391	0.000	0.099	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	94	317	0	163	0	0	0
N.S.	1	1.00	0.65	2.19	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.233	1.603	11.845	0.000	0.103	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	83	283	0	149	0	0	0
N.S.	1	1.00	0.74	2.53	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.184	0.848	9.816	0.000	0.101	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	78	260	0	180	0	0	0
N.S.	1	1.00	0.72	2.39	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.202	1.101	9.473	0.000	0.100	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	90	506	0	199	0	0	0
N.S.	1	1.00	0.64	3.61	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.247	0.686	10.985	0.000	0.094	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	122	801	0	220	0	0	0
N.S.	1	1.00	0.67	4.43	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.281	0.851	14.958	0.000	0.100	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	143	726	0	231	0	0	0
N.S.	1	1.00	0.68	3.46	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.304	1.056	16.143	0.000	0.102	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	128	384	0	195	0	0	0
N.S.	1	1.00	0.61	1.83	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.281	1.951	15.447	0.000	0.109	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	108	353	0	183	0	0	0
N.S.	1	1.00	0.60	1.95	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.277	0.470	15.254	0.000	0.116	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	95	319	0	165	0	0	0
N.S.	1	1.00	0.65	2.18	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.293	0.205	13.843	0.000	0.106	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	85	285	0	148	0	0	0
N.S.	1	1.00	0.73	2.46	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.284	0.122	9.855	0.000	0.098	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	80	262	0	179	0	0	0
N.S.	1	1.00	0.70	2.30	0.00	1.57	0.00	0.00	0.00
time (sec)	N/A	0.274	0.711	9.713	0.000	0.101	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	92	507	0	200	0	0	0
N.S.	1	1.00	0.63	3.50	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.371	0.723	11.605	0.000	0.124	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	122	806	0	223	0	0	0
N.S.	1	1.00	0.66	4.33	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.327	0.764	15.008	0.000	0.107	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	134	728	0	235	0	0	0
N.S.	1	1.00	0.62	3.39	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.313	1.580	16.026	0.000	0.113	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	125	384	0	203	0	0	0
N.S.	1	1.00	0.59	1.81	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.217	0.548	18.694	0.000	0.117	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	109	353	0	189	0	0	0
N.S.	1	1.00	0.60	1.93	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.263	0.493	22.063	0.000	0.113	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	97	319	0	169	0	0	0
N.S.	1	1.00	0.64	2.11	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.227	0.196	24.783	0.000	0.113	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	79	285	0	150	0	0	0
N.S.	1	1.00	0.66	2.38	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.206	0.712	67.323	0.000	0.095	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	262	0	181	0	0	0
N.S.	1	1.00	0.69	2.26	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.207	0.576	192.874	0.000	0.094	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	92	509	0	204	0	0	0
N.S.	1	1.00	0.63	3.46	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.275	0.631	4.544	0.000	0.106	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	121	808	0	229	0	0	0
N.S.	1	1.00	0.64	4.30	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.394	0.919	6.372	0.000	0.103	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	134	728	0	243	0	0	0
N.S.	1	1.00	0.62	3.35	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.417	2.368	3137.824	0.000	0.105	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	127	381	0	194	0	0	0
N.S.	1	1.00	0.59	1.78	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.378	1.377	14.619	0.000	0.114	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	108	350	0	180	0	0	0
N.S.	1	1.00	0.58	1.89	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.313	1.249	14.367	0.000	0.104	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	97	316	0	166	0	0	0
N.S.	1	1.00	0.65	2.11	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.180	0.224	12.444	0.000	0.099	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	82	282	0	152	0	0	128
N.S.	1	1.00	0.70	2.41	0.00	1.30	0.00	0.00	1.09
time (sec)	N/A	0.155	0.111	9.405	0.000	0.105	0.000	0.000	0.407

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	279	259	0	183	0	0	0
N.S.	1	1.00	2.54	2.35	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.193	5.370	12.707	0.000	0.095	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	757	509	0	202	0	0	0
N.S.	1	1.00	5.45	3.66	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.242	6.734	13.678	0.000	0.093	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	116	802	0	223	0	0	0
N.S.	1	1.00	0.64	4.46	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.283	0.656	17.985	0.000	0.097	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	133	727	0	234	0	0	0
N.S.	1	1.00	0.64	3.48	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.310	0.729	20.215	0.000	0.104	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	130	384	0	194	0	0	0
N.S.	1	1.00	0.60	1.77	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.332	1.532	16.942	0.000	0.127	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	108	353	0	180	0	0	0
N.S.	1	1.00	0.57	1.88	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.320	1.356	15.411	0.000	0.102	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	94	319	0	166	0	0	0
N.S.	1	1.00	0.61	2.08	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.273	1.031	12.839	0.000	0.100	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	85	285	0	152	0	0	0
N.S.	1	1.00	0.71	2.38	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.206	0.109	10.706	0.000	0.094	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	262	0	183	0	0	0
N.S.	1	1.00	0.69	2.26	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.240	0.249	11.632	0.000	0.094	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	761	509	0	202	0	0	0
N.S.	1	1.00	5.28	3.53	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.346	6.171	13.446	0.000	0.109	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	119	808	0	223	0	0	0
N.S.	1	1.00	0.65	4.42	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.278	0.530	18.524	0.000	0.101	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	136	730	0	234	0	0	0
N.S.	1	1.00	0.64	3.44	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.287	0.805	19.623	0.000	0.100	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	130	384	0	194	0	0	0
N.S.	1	1.00	0.60	1.77	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.259	1.715	15.923	0.000	0.110	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	111	353	0	180	0	0	0
N.S.	1	1.00	0.59	1.88	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.243	1.507	14.898	0.000	0.109	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	97	319	0	166	0	0	0
N.S.	1	1.00	0.63	2.08	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.188	1.128	13.125	0.000	0.108	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	85	285	0	152	0	0	0
N.S.	1	1.00	0.71	2.38	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.166	0.113	10.992	0.000	0.113	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	279	262	0	183	0	0	0
N.S.	1	1.00	2.41	2.26	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.217	4.578	11.395	0.000	0.101	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	92	509	0	202	0	0	0
N.S.	1	1.00	0.63	3.46	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.284	0.469	13.723	0.000	0.097	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	119	808	0	223	0	0	0
N.S.	1	1.00	0.64	4.37	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.377	0.267	19.286	0.000	0.103	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	136	730	0	234	0	0	0
N.S.	1	1.00	0.64	3.44	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.409	0.757	19.694	0.000	0.107	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	119	808	0	223	0	0	0
N.S.	1	1.00	0.63	4.30	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.320	0.067	18.900	0.000	0.103	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	109	134	159	292	0	411	141
N.S.	1	1.00	0.49	0.60	0.71	1.31	0.00	1.84	0.63
time (sec)	N/A	0.142	0.835	9.250	0.504	0.312	0.000	5.365	3.995

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	92	114	116	276	0	428	137
N.S.	1	1.00	0.50	0.62	0.63	1.50	0.00	2.33	0.74
time (sec)	N/A	0.141	0.643	8.715	0.496	0.297	0.000	4.114	3.172

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	75	83	80	236	241	243	104
N.S.	1	1.00	0.52	0.58	0.56	1.65	1.69	1.70	0.73
time (sec)	N/A	0.067	0.489	9.908	0.488	0.305	29.671	2.936	1.612

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	61	63	64	212	184	0	54
N.S.	1	1.00	0.50	0.51	0.52	1.72	1.50	0.00	0.44
time (sec)	N/A	0.042	0.074	9.552	0.445	0.311	13.428	0.000	0.602

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	61	104	304	0	0	0
N.S.	1	1.00	1.00	0.66	1.12	3.27	0.00	0.00	0.00
time (sec)	N/A	0.065	0.534	10.138	0.446	0.332	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	60	70	144	312	0	0	0
N.S.	1	1.00	0.65	0.75	1.55	3.35	0.00	0.00	0.00
time (sec)	N/A	0.063	0.056	9.871	0.466	0.324	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	69	130	780	233	0	0	0
N.S.	1	1.00	0.62	1.17	7.03	2.10	0.00	0.00	0.00
time (sec)	N/A	0.110	0.107	10.234	0.736	0.311	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	87	139	1009	265	0	0	0
N.S.	1	1.00	0.57	0.91	6.64	1.74	0.00	0.00	0.00
time (sec)	N/A	0.116	0.293	10.066	0.589	0.320	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	110	212	2611	299	0	0	0
N.S.	1	1.00	0.57	1.10	13.53	1.55	0.00	0.00	0.00
time (sec)	N/A	0.152	0.233	10.327	0.623	0.342	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	109	135	169	309	0	412	142
N.S.	1	1.00	0.48	0.59	0.74	1.35	0.00	1.80	0.62
time (sec)	N/A	0.150	0.612	8.944	0.516	0.322	0.000	5.326	3.619

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	92	115	126	285	0	429	138
N.S.	1	1.00	0.49	0.61	0.67	1.51	0.00	2.27	0.73
time (sec)	N/A	0.124	0.441	8.633	0.519	0.318	0.000	4.129	1.848

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	76	84	86	249	0	0	71
N.S.	1	1.00	0.52	0.57	0.59	1.69	0.00	0.00	0.48
time (sec)	N/A	0.069	0.100	9.817	0.489	0.303	0.000	0.000	0.856

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	61	64	67	217	0	0	55
N.S.	1	1.00	0.48	0.50	0.53	1.71	0.00	0.00	0.43
time (sec)	N/A	0.036	0.070	9.996	0.482	0.301	0.000	0.000	1.248

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	93	62	107	308	0	0	0
N.S.	1	1.00	0.97	0.65	1.11	3.21	0.00	0.00	0.00
time (sec)	N/A	0.062	0.407	10.192	0.454	0.355	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	60	71	147	316	0	0	0
N.S.	1	1.00	0.62	0.74	1.53	3.29	0.00	0.00	0.00
time (sec)	N/A	0.065	0.063	10.336	0.448	0.354	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	69	131	813	240	0	0	0
N.S.	1	1.00	0.61	1.15	7.13	2.11	0.00	0.00	0.00
time (sec)	N/A	0.101	0.119	10.290	0.516	0.306	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	88	140	1044	272	0	0	0
N.S.	1	1.00	0.56	0.90	6.69	1.74	0.00	0.00	0.00
time (sec)	N/A	0.117	0.191	9.883	0.687	0.317	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	111	213	2732	308	0	0	0
N.S.	1	1.00	0.56	1.08	13.80	1.56	0.00	0.00	0.00
time (sec)	N/A	0.145	0.218	10.036	0.629	0.327	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	109	137	185	331	0	411	144
N.S.	1	1.00	0.45	0.57	0.77	1.37	0.00	1.71	0.60
time (sec)	N/A	0.152	0.642	8.918	0.521	0.323	0.000	5.823	3.596

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	92	117	140	303	0	0	94
N.S.	1	1.00	0.46	0.59	0.70	1.52	0.00	0.00	0.47
time (sec)	N/A	0.166	1.270	8.345	0.504	0.326	0.000	0.000	1.138

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	75	86	94	263	0	0	73
N.S.	1	1.00	0.48	0.55	0.61	1.70	0.00	0.00	0.47
time (sec)	N/A	0.094	1.197	9.717	0.532	0.302	0.000	0.000	0.792

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	61	66	71	227	0	0	57
N.S.	1	1.00	0.45	0.49	0.53	1.68	0.00	0.00	0.42
time (sec)	N/A	0.064	0.118	10.057	0.493	0.307	0.000	0.000	1.392

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	93	64	111	316	0	0	0
N.S.	1	1.00	0.91	0.63	1.09	3.10	0.00	0.00	0.00
time (sec)	N/A	0.094	0.505	9.067	0.480	0.329	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	60	73	151	324	0	0	0
N.S.	1	1.00	0.59	0.72	1.48	3.18	0.00	0.00	0.00
time (sec)	N/A	0.098	0.084	9.743	0.495	0.343	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	69	133	873	250	0	0	0
N.S.	1	1.00	0.58	1.11	7.28	2.08	0.00	0.00	0.00
time (sec)	N/A	0.141	0.136	10.110	0.554	0.318	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	87	142	1112	286	0	0	0
N.S.	1	1.00	0.53	0.87	6.78	1.74	0.00	0.00	0.00
time (sec)	N/A	0.151	0.322	9.240	0.624	0.326	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	110	215	2972	326	0	0	0
N.S.	1	1.00	0.53	1.03	14.29	1.57	0.00	0.00	0.00
time (sec)	N/A	0.207	0.258	10.133	0.646	0.333	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	92	114	116	282	0	0	140
N.S.	1	1.00	0.50	0.62	0.63	1.53	0.00	0.00	0.76
time (sec)	N/A	0.194	0.779	9.503	0.546	0.325	0.000	0.000	3.126

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	75	83	80	242	0	0	107
N.S.	1	1.00	0.52	0.58	0.56	1.69	0.00	0.00	0.75
time (sec)	N/A	0.142	0.598	9.458	0.487	0.325	0.000	0.000	1.489

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	61	63	64	218	184	0	93
N.S.	1	1.00	0.50	0.51	0.52	1.77	1.50	0.00	0.76
time (sec)	N/A	0.040	0.073	10.156	0.467	0.296	15.793	0.000	1.059

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	61	104	309	0	0	0
N.S.	1	1.00	1.00	0.66	1.12	3.32	0.00	0.00	0.00
time (sec)	N/A	0.061	0.293	9.769	0.467	0.344	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	60	70	149	317	0	0	0
N.S.	1	1.00	0.65	0.75	1.60	3.41	0.00	0.00	0.00
time (sec)	N/A	0.063	0.049	9.813	0.456	0.330	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	69	130	785	239	0	0	0
N.S.	1	1.00	0.62	1.17	7.07	2.15	0.00	0.00	0.00
time (sec)	N/A	0.103	0.090	10.115	0.533	0.319	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	87	139	1014	271	0	0	0
N.S.	1	1.00	0.57	0.91	6.67	1.78	0.00	0.00	0.00
time (sec)	N/A	0.115	0.233	9.822	0.488	0.313	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	110	212	2611	305	0	0	0
N.S.	1	1.00	0.57	1.10	13.53	1.58	0.00	0.00	0.00
time (sec)	N/A	0.134	0.195	10.164	0.573	0.320	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	92	117	116	282	0	0	140
N.S.	1	1.00	0.46	0.59	0.58	1.42	0.00	0.00	0.70
time (sec)	N/A	0.129	0.936	10.024	0.652	0.319	0.000	0.000	3.043

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	75	86	80	242	0	0	107
N.S.	1	1.00	0.48	0.55	0.52	1.56	0.00	0.00	0.69
time (sec)	N/A	0.070	0.724	9.321	0.508	0.307	0.000	0.000	1.280

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	61	66	64	218	0	0	93
N.S.	1	1.00	0.45	0.49	0.47	1.61	0.00	0.00	0.69
time (sec)	N/A	0.036	0.095	10.339	0.476	0.312	0.000	0.000	0.904

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	48	64	104	309	0	0	0
N.S.	1	1.00	0.47	0.63	1.02	3.03	0.00	0.00	0.00
time (sec)	N/A	0.064	0.399	10.441	0.457	0.372	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	60	73	157	317	0	0	0
N.S.	1	1.00	0.59	0.72	1.54	3.11	0.00	0.00	0.00
time (sec)	N/A	0.070	0.052	9.890	0.490	0.358	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	69	133	802	239	0	0	0
N.S.	1	1.00	0.58	1.11	6.68	1.99	0.00	0.00	0.00
time (sec)	N/A	0.102	0.096	9.659	0.509	0.310	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	87	142	1048	271	0	0	0
N.S.	1	1.00	0.53	0.87	6.39	1.65	0.00	0.00	0.00
time (sec)	N/A	0.120	0.212	8.705	0.518	0.325	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	110	215	2660	305	0	0	0
N.S.	1	1.00	0.53	1.03	12.79	1.47	0.00	0.00	0.00
time (sec)	N/A	0.134	0.171	9.451	0.569	0.309	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	95	117	116	282	0	0	140
N.S.	1	1.00	0.48	0.59	0.58	1.42	0.00	0.00	0.70
time (sec)	N/A	0.124	1.215	9.107	0.530	0.316	0.000	0.000	2.589

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	78	86	80	242	0	0	107
N.S.	1	1.00	0.50	0.55	0.52	1.56	0.00	0.00	0.69
time (sec)	N/A	0.070	0.795	9.405	0.499	0.299	0.000	0.000	1.352

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	64	66	64	218	0	0	93
N.S.	1	1.00	0.47	0.49	0.47	1.61	0.00	0.00	0.69
time (sec)	N/A	0.038	0.100	10.164	0.474	0.309	0.000	0.000	0.993

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	53	64	104	309	0	0	0
N.S.	1	1.00	0.52	0.63	1.02	3.03	0.00	0.00	0.00
time (sec)	N/A	0.062	0.424	10.013	0.457	0.357	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	60	73	157	317	0	0	0
N.S.	1	1.00	0.59	0.72	1.54	3.11	0.00	0.00	0.00
time (sec)	N/A	0.067	0.056	10.511	0.463	0.354	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	69	134	820	239	0	0	0
N.S.	1	1.00	0.58	1.12	6.83	1.99	0.00	0.00	0.00
time (sec)	N/A	0.124	0.104	10.065	0.531	0.323	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	87	142	1098	271	0	0	0
N.S.	1	1.00	0.53	0.87	6.70	1.65	0.00	0.00	0.00
time (sec)	N/A	0.163	0.242	9.528	0.516	0.298	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	110	215	2760	305	0	0	0
N.S.	1	1.00	0.53	1.03	13.27	1.47	0.00	0.00	0.00
time (sec)	N/A	0.188	0.216	9.265	0.578	0.345	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	117	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.210	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	121	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	0.446	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	122	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.376	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	116	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	0.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	123	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	0.156	0.000	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	123	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	117	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.246	0.000	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	114	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.160	0.197	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	121	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	0.420	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	122	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.569	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	117	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	124	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.137	0.000	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	133	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.873	0.000	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	124	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	121	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	0.155	0.000	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	116	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.392	0.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	116	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	0.416	0.000	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	118	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.179	0.000	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	122	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.160	0.231	0.000	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	122	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.222	0.000	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	124	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	115	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.152	0.148	0.000	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	115	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.188	0.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	118	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	222	178	0	0	0	0	0	0
N.S.	1	0.96	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.497	0.000	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	219	175	0	0	0	0	0	0
N.S.	1	0.96	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.351	0.000	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	219	175	0	0	0	0	0	0
N.S.	1	0.96	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	0.334	0.000	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	219	175	0	0	0	0	0	0
N.S.	1	0.96	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	0.342	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	227	227	173	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	235	225	175	0	0	0	0	0	0
N.S.	1	0.96	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	0.395	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	227	227	157	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	0.335	0.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	153	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	0.253	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	151	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.233	0.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	144	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	0.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	138	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.389	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	130	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	0.568	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	150	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	150	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.236	0.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	213	173	0	0	0	0	0	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.357	0.294	0.000	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	213	173	0	0	0	0	0	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.310	0.311	0.000	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	171	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	0.303	0.000	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	166	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.310	0.000	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	173	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	0.307	0.000	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	213	173	0	0	0	0	0	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	0.312	0.000	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	376	0	0	0	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	1.923	0.000	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	137	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	0.990	0.000	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	105	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.813	0.000	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	290	290	296	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	3.199	0.000	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	290	290	294	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.540	3.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	268	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	2.413	0.000	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	286	286	266	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.460	2.343	0.000	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	215	215	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	303	303	16142	0	0	0	0	0	0
N.S.	1	1.00	53.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.454	26.978	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [26] had the largest ratio of [.2000000000000000011]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	21	0.095
2	A	3	2	1.00	21	0.095
3	A	3	2	1.00	21	0.095
4	A	2	1	1.00	19	0.053
5	A	2	2	1.00	19	0.105
6	A	2	2	1.00	21	0.095
7	A	3	3	1.00	21	0.143
8	A	4	3	1.00	21	0.143
9	A	5	3	1.00	21	0.143
10	A	4	3	1.00	21	0.143
11	A	3	3	1.00	21	0.143
12	A	2	2	1.00	21	0.095
13	A	3	3	1.00	21	0.143
14	A	3	2	1.00	21	0.095
15	A	3	2	1.00	21	0.095
16	A	4	4	1.00	25	0.160
17	A	4	4	1.00	25	0.160
18	A	3	3	1.00	25	0.120
19	A	3	3	1.00	25	0.120
20	A	3	3	1.00	25	0.120
21	A	3	3	1.00	25	0.120
22	A	4	4	1.00	25	0.160
23	A	4	4	1.00	25	0.160
24	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	1	1	1.00	23	0.043
26	A	5	5	1.00	25	0.200
27	A	5	5	1.00	25	0.200
28	A	4	4	1.00	25	0.160
29	A	4	4	1.00	25	0.160
30	A	4	4	1.00	25	0.160
31	A	4	4	1.00	25	0.160
32	A	5	5	1.00	25	0.200
33	A	5	5	1.00	25	0.200
34	A	2	2	1.00	23	0.087
35	A	1	1	1.00	33	0.030
36	A	1	1	1.00	32	0.031
37	A	5	5	1.00	33	0.152
38	A	5	5	1.00	31	0.161
39	A	3	3	1.00	25	0.120
40	A	4	4	1.00	31	0.129
41	A	4	4	1.00	33	0.121
42	A	4	4	1.00	33	0.121
43	A	5	5	1.00	33	0.152
44	A	5	5	1.00	33	0.152
45	A	5	5	1.00	31	0.161
46	A	4	4	1.00	25	0.160
47	A	4	4	1.00	31	0.129
48	A	4	4	1.00	33	0.121
49	A	4	4	1.00	33	0.121
50	A	4	4	1.00	33	0.121
51	A	5	5	1.00	33	0.152
52	A	5	5	1.00	33	0.152
53	A	4	4	1.00	25	0.160
54	A	5	5	1.00	31	0.161
55	A	4	4	1.00	33	0.121
56	A	4	4	1.00	33	0.121
57	A	4	4	1.00	33	0.121
58	A	4	4	1.00	33	0.121
59	A	5	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	5	5	1.00	33	0.152
61	A	6	5	1.00	33	0.152
62	A	5	5	1.00	33	0.152
63	A	5	5	1.00	33	0.152
64	A	4	4	1.00	31	0.129
65	A	3	3	1.00	25	0.120
66	A	4	4	1.00	31	0.129
67	A	4	4	1.00	33	0.121
68	A	5	5	1.00	33	0.152
69	A	5	5	1.00	33	0.152
70	A	6	5	1.00	33	0.152
71	A	5	5	1.00	33	0.152
72	A	5	5	1.00	33	0.152
73	A	4	4	1.00	33	0.121
74	A	4	4	1.00	31	0.129
75	A	3	3	1.00	25	0.120
76	A	4	4	1.00	31	0.129
77	A	5	5	1.00	33	0.152
78	A	5	5	1.00	33	0.152
79	A	5	5	1.00	33	0.152
80	A	5	5	1.00	33	0.152
81	A	4	4	1.00	33	0.121
82	A	4	4	1.00	33	0.121
83	A	4	4	1.00	31	0.129
84	A	3	3	1.00	25	0.120
85	A	5	5	1.00	31	0.161
86	A	5	5	1.00	33	0.152
87	A	4	4	1.00	25	0.160
88	A	4	4	1.00	25	0.160
89	A	4	3	1.00	35	0.086
90	A	4	4	1.00	35	0.114
91	A	3	2	1.00	35	0.057
92	A	4	3	1.00	35	0.086
93	A	3	3	1.00	35	0.086
94	A	3	3	1.00	35	0.086

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	3	3	1.00	35	0.086
96	A	4	4	1.00	35	0.114
97	A	4	4	1.00	35	0.114
98	A	4	3	1.00	35	0.086
99	A	4	4	1.00	35	0.114
100	A	3	2	1.00	35	0.057
101	A	4	3	1.00	35	0.086
102	A	3	3	1.00	35	0.086
103	A	3	3	1.00	35	0.086
104	A	3	3	1.00	35	0.086
105	A	4	4	1.00	35	0.114
106	A	4	4	1.00	35	0.114
107	A	4	3	1.00	35	0.086
108	A	4	4	1.00	35	0.114
109	A	3	2	1.00	35	0.057
110	A	4	3	1.00	35	0.086
111	A	3	3	1.00	35	0.086
112	A	3	3	1.00	35	0.086
113	A	3	3	1.00	35	0.086
114	A	4	4	1.00	35	0.114
115	A	4	4	1.00	35	0.114
116	A	4	4	1.00	35	0.114
117	A	3	2	1.00	35	0.057
118	A	4	3	1.00	35	0.086
119	A	3	3	1.00	35	0.086
120	A	3	3	1.00	35	0.086
121	A	3	3	1.00	35	0.086
122	A	4	4	1.00	35	0.114
123	A	4	4	1.00	35	0.114
124	A	4	4	1.00	35	0.114
125	A	3	2	1.00	35	0.057
126	A	4	3	1.00	35	0.086
127	A	3	3	1.00	35	0.086
128	A	3	3	1.00	35	0.086
129	A	3	3	1.00	35	0.086

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	4	4	1.00	35	0.114
131	A	4	4	1.00	35	0.114
132	A	4	4	1.00	35	0.114
133	A	3	2	1.00	35	0.057
134	A	4	3	1.00	35	0.086
135	A	3	3	1.00	35	0.086
136	A	3	3	1.00	35	0.086
137	A	3	3	1.00	35	0.086
138	A	4	4	1.00	35	0.114
139	A	4	4	1.00	35	0.114
140	A	3	3	1.00	33	0.091
141	A	3	3	1.00	31	0.097
142	A	2	2	1.00	25	0.080
143	A	3	3	1.00	31	0.097
144	A	3	3	1.00	33	0.091
145	A	3	3	1.00	33	0.091
146	A	3	3	1.00	33	0.091
147	A	3	3	1.00	31	0.097
148	A	2	2	1.00	25	0.080
149	A	3	3	1.00	31	0.097
150	A	3	3	1.00	33	0.091
151	A	3	3	1.00	33	0.091
152	A	3	3	1.00	33	0.091
153	A	3	3	1.00	31	0.097
154	A	2	2	1.00	25	0.080
155	A	3	3	1.00	31	0.097
156	A	3	3	1.00	33	0.091
157	A	3	3	1.00	33	0.091
158	A	3	3	1.00	33	0.091
159	A	3	3	1.00	31	0.097
160	A	2	2	1.00	25	0.080
161	A	3	3	1.00	31	0.097
162	A	3	3	1.00	33	0.091
163	A	3	3	1.00	33	0.091
164	A	3	3	1.00	33	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	3	3	1.00	31	0.097
166	A	2	2	1.00	25	0.080
167	A	3	3	1.00	31	0.097
168	A	3	3	1.00	33	0.091
169	A	3	3	1.00	33	0.091
170	A	3	3	1.00	33	0.091
171	A	3	3	1.00	31	0.097
172	A	2	2	1.00	25	0.080
173	A	3	3	1.00	31	0.097
174	A	3	3	1.00	33	0.091
175	A	3	3	1.00	33	0.091
176	A	3	3	0.93	33	0.091
177	A	3	3	0.93	33	0.091
178	A	3	3	0.93	33	0.091
179	A	3	3	0.93	33	0.091
180	A	3	3	1.00	33	0.091
181	A	3	3	0.93	33	0.091
182	A	3	3	1.00	33	0.091
183	A	3	3	1.00	31	0.097
184	A	3	3	1.00	29	0.103
185	A	2	2	1.00	23	0.087
186	A	3	3	1.00	29	0.103
187	A	3	3	1.00	31	0.097
188	A	3	3	1.00	31	0.097
189	A	3	3	1.00	31	0.097
190	A	3	3	0.93	33	0.091
191	A	3	3	0.93	33	0.091
192	A	3	3	0.93	33	0.091
193	A	3	3	1.00	33	0.091
194	A	3	3	1.00	33	0.091
195	A	3	3	0.94	33	0.091
196	A	3	3	0.93	33	0.091
197	A	3	3	0.93	33	0.091
198	A	4	4	1.00	25	0.160
199	A	4	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	4	4	1.00	27	0.148
201	A	4	4	1.00	27	0.148
202	A	4	4	1.00	27	0.148
203	A	8	5	1.00	27	0.185
204	A	8	5	1.00	27	0.185
205	A	8	5	1.00	27	0.185
206	A	8	5	1.00	27	0.185
207	A	7	5	1.00	26	0.192
208	A	8	5	1.00	25	0.200
209	A	4	3	1.00	30	0.100
210	A	5	4	1.00	40	0.100
211	A	5	4	1.00	40	0.100
212	A	5	4	1.00	40	0.100
213	A	5	4	1.00	40	0.100
214	A	5	4	1.00	40	0.100
215	A	5	4	1.00	40	0.100
216	A	5	4	1.00	40	0.100
217	A	5	4	1.00	38	0.105
218	A	5	4	1.00	36	0.111
219	A	4	3	1.00	30	0.100
220	A	5	4	1.00	36	0.111
221	A	5	4	1.00	38	0.105
222	A	5	4	1.00	38	0.105
223	A	5	4	1.00	38	0.105
224	A	5	4	1.00	40	0.100
225	A	5	4	1.00	40	0.100
226	A	5	4	1.00	40	0.100
227	A	5	4	1.00	40	0.100
228	A	5	4	1.00	40	0.100
229	A	5	4	1.00	40	0.100
230	A	5	4	1.00	40	0.100
231	A	5	4	1.00	40	0.100
232	A	4	4	1.00	32	0.125
233	A	8	5	1.00	32	0.156
234	A	8	5	1.00	34	0.147

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	8	5	1.00	34	0.147
236	A	8	5	1.00	34	0.147
237	A	8	5	1.00	34	0.147
238	A	4	3	1.00	31	0.097
239	A	10	7	1.00	41	0.171
240	A	9	7	1.00	39	0.180
241	A	7	6	1.00	33	0.182
242	A	7	6	1.00	39	0.154
243	A	7	6	1.00	41	0.146
244	A	8	7	1.00	41	0.171
245	A	9	7	1.00	41	0.171
246	A	10	7	1.00	41	0.171
247	A	10	7	1.00	39	0.180
248	A	8	6	1.00	33	0.182
249	A	8	7	1.00	39	0.180
250	A	7	6	1.00	41	0.146
251	A	7	6	1.00	41	0.146
252	A	8	7	1.00	41	0.171
253	A	9	7	1.00	41	0.171
254	A	10	7	1.00	41	0.171
255	A	9	6	1.00	33	0.182
256	A	9	7	1.00	39	0.180
257	A	8	7	1.00	41	0.171
258	A	7	6	1.00	41	0.146
259	A	7	6	1.00	41	0.146
260	A	8	7	1.00	41	0.171
261	A	9	7	1.00	41	0.171
262	A	10	7	1.00	41	0.171
263	A	10	7	1.00	41	0.171
264	A	9	7	1.00	41	0.171
265	A	8	7	1.00	39	0.180
266	A	6	5	1.00	33	0.152
267	A	7	6	1.00	39	0.154
268	A	8	7	1.00	41	0.171
269	A	9	7	1.00	41	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	10	7	1.00	41	0.171
271	A	10	7	1.00	41	0.171
272	A	9	7	1.00	41	0.171
273	A	8	7	1.00	41	0.171
274	A	7	6	1.00	39	0.154
275	A	6	5	1.00	33	0.152
276	A	8	7	1.00	39	0.180
277	A	9	7	1.00	41	0.171
278	A	10	7	1.00	41	0.171
279	A	10	7	1.00	41	0.171
280	A	9	7	1.00	41	0.171
281	A	8	7	1.00	41	0.171
282	A	7	6	1.00	41	0.146
283	A	7	6	1.00	39	0.154
284	A	7	6	1.00	33	0.182
285	A	9	7	1.00	39	0.180
286	A	10	7	1.00	41	0.171
287	A	8	6	1.00	33	0.182
288	A	8	6	1.00	43	0.140
289	A	7	6	1.00	43	0.140
290	A	3	3	1.00	43	0.070
291	A	5	4	1.00	43	0.093
292	A	4	4	1.00	43	0.093
293	A	4	4	1.00	43	0.093
294	A	6	6	1.00	43	0.140
295	A	7	7	1.00	43	0.163
296	A	7	6	1.00	43	0.140
297	A	8	6	1.00	43	0.140
298	A	7	6	1.00	43	0.140
299	A	3	3	1.00	43	0.070
300	A	5	4	1.00	43	0.093
301	A	4	4	1.00	43	0.093
302	A	4	4	1.00	43	0.093
303	A	6	6	1.00	43	0.140
304	A	7	7	1.00	43	0.163

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	7	6	1.00	43	0.140
306	A	8	6	1.00	43	0.140
307	A	7	6	1.00	43	0.140
308	A	3	3	1.00	43	0.070
309	A	5	4	1.00	43	0.093
310	A	4	4	1.00	43	0.093
311	A	4	4	1.00	43	0.093
312	A	6	6	1.00	43	0.140
313	A	7	7	1.00	43	0.163
314	A	7	6	1.00	43	0.140
315	A	7	6	1.00	43	0.140
316	A	3	3	1.00	43	0.070
317	A	5	4	1.00	43	0.093
318	A	4	4	1.00	43	0.093
319	A	4	4	1.00	43	0.093
320	A	6	6	1.00	43	0.140
321	A	7	7	1.00	43	0.163
322	A	7	6	1.00	43	0.140
323	A	7	6	1.00	43	0.140
324	A	3	3	1.00	43	0.070
325	A	5	4	1.00	43	0.093
326	A	4	4	1.00	43	0.093
327	A	4	4	1.00	43	0.093
328	A	6	6	1.00	43	0.140
329	A	7	7	1.00	43	0.163
330	A	7	6	1.00	43	0.140
331	A	7	6	1.00	43	0.140
332	A	3	3	1.00	43	0.070
333	A	5	4	1.00	43	0.093
334	A	4	4	1.00	43	0.093
335	A	4	4	1.00	43	0.093
336	A	6	6	1.00	43	0.140
337	A	7	7	1.00	43	0.163
338	A	7	6	1.00	43	0.140
339	A	5	4	1.00	39	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	4	3	1.00	33	0.091
341	A	5	4	1.00	39	0.103
342	A	5	4	1.00	41	0.098
343	A	5	4	1.00	41	0.098
344	A	5	4	1.00	41	0.098
345	A	5	4	1.00	39	0.103
346	A	4	3	1.00	33	0.091
347	A	5	4	1.00	39	0.103
348	A	5	4	1.00	41	0.098
349	A	5	4	1.00	41	0.098
350	A	5	4	1.00	41	0.098
351	A	5	4	1.00	41	0.098
352	A	5	4	1.00	39	0.103
353	A	4	3	1.00	33	0.091
354	A	5	4	1.00	39	0.103
355	A	5	4	1.00	41	0.098
356	A	5	4	1.00	41	0.098
357	A	5	4	1.00	41	0.098
358	A	5	4	1.00	41	0.098
359	A	5	4	1.00	39	0.103
360	A	4	3	1.00	33	0.091
361	A	5	4	1.00	39	0.103
362	A	5	4	1.00	41	0.098
363	A	5	4	0.96	41	0.098
364	A	5	4	0.96	41	0.098
365	A	5	4	0.96	41	0.098
366	A	5	4	0.96	41	0.098
367	A	5	4	1.00	41	0.098
368	A	5	4	0.96	41	0.098
369	A	5	4	1.00	41	0.098
370	A	5	4	1.00	39	0.103
371	A	5	4	1.00	37	0.108
372	A	4	3	1.00	31	0.097
373	A	5	4	1.00	37	0.108
374	A	5	4	1.00	39	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	5	4	1.00	39	0.103
376	A	5	4	1.00	39	0.103
377	A	5	4	0.96	41	0.098
378	A	5	4	0.96	41	0.098
379	A	5	4	1.00	41	0.098
380	A	5	4	1.00	41	0.098
381	A	5	4	1.00	41	0.098
382	A	5	4	0.96	41	0.098
383	A	4	4	1.00	33	0.121
384	A	4	4	1.00	35	0.114
385	A	4	4	1.00	35	0.114
386	A	4	4	1.00	35	0.114
387	A	4	4	1.00	35	0.114
388	A	8	5	1.00	35	0.143
389	A	8	5	1.00	35	0.143
390	A	8	5	1.00	35	0.143
391	A	8	5	1.00	35	0.143
392	A	7	5	1.00	35	0.143
393	A	8	5	1.00	33	0.152

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{(3A+4C)\sin^3(c+dx)}{3d}+\frac{3(A+2C)\sin^5(c+dx)}{5d}-\frac{(A+4C)\sin^7(c+dx)}{7d}+\frac{C\sin^9(c+dx)}{9d}$$

```
[Out] (A+C)*sin(d*x+c)/d-1/3*(3*A+4*C)*sin(d*x+c)^3/d+3/5*(A+2*C)*sin(d*x+c)^5/d-
1/7*(A+4*C)*sin(d*x+c)^7/d+1/9*C*sin(d*x+c)^9/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3092, 380}

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=-\frac{(A+4C)\sin^7(c+dx)}{7d}+\frac{3(A+2C)\sin^5(c+dx)}{5d}-\frac{(3A+4C)\sin^3(c+dx)}{3d}+\frac{(A+C)\sin(c+dx)}{d}+\frac{C\sin^9(c+dx)}{9d}$$

```
[In] Int[Cos[c + d*x]^7*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - ((3*A + 4*C)*Sin[c + d*x]^3)/(3*d) + (3*(A + 2*C
)*Sin[c + d*x]^5)/(5*d) - ((A + 4*C)*Sin[c + d*x]^7)/(7*d) + (C*Sin[c + d*x
]^9)/(9*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int (1-x^2)^3 (A+C-Cx^2)\,dx, x, -\sin(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(A\left(1+\frac{C}{A}\right)-(3A+4C)x^2+3(A+2C)x^4-(A+4C)x^6+Cx^8\right)\,dx, x, -\sin(c+dx)\right)}{d}\\ &= \frac{(A+C)\sin(c+dx)}{d}-\frac{(3A+4C)\sin^3(c+dx)}{3d}\\ &\quad+\frac{3(A+2C)\sin^5(c+dx)}{5d}-\frac{(A+4C)\sin^7(c+dx)}{7d}+\frac{C\sin^9(c+dx)}{9d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.45

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{A\sin(c+dx)}{d}+\frac{C\sin(c+dx)}{d}-\frac{A\sin^3(c+dx)}{d}-\frac{4C\sin^3(c+dx)}{3d}+\frac{3A\sin^5(c+dx)}{5d}+\frac{6C\sin^5(c+dx)}{5d}-\frac{A\sin^7(c+dx)}{7d}-\frac{4C\sin^7(c+dx)}{7d}+\frac{C\sin^9(c+dx)}{9d}$$

```
[In] Integrate[Cos[c + d*x]^7*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (A*Sin[c + d*x])/d + (C*Sin[c + d*x])/d - (A*Sin[c + d*x]^3)/d - (4*C*Sin[c
 + d*x]^3)/(3*d) + (3*A*Sin[c + d*x]^5)/(5*d) + (6*C*Sin[c + d*x]^5)/(5*d)
- (A*Sin[c + d*x]^7)/(7*d) - (4*C*Sin[c + d*x]^7)/(7*d) + (C*Sin[c + d*x]^9
)/(9*d)
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

method	result
parallelrisch	$\frac{8820(A+C)\sin(3dx+3c)+252(7A+9C)\sin(5dx+5c)+45(4A+9C)\sin(7dx+7c)+35C\sin(9dx+9c)+44100\left(A+\frac{9C}{10}\right)\sin(d}{80640d}$
derivativedivides	$\frac{\frac{C\left(\frac{128}{35}+\cos^8(dx+c)+\frac{8\left(\cos^6(dx+c)\right)}{7}+\frac{48\left(\cos^4(dx+c)\right)}{35}+\frac{64\left(\cos^2(dx+c)\right)}{35}\right)\sin(dx+c)}{9}+\frac{A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8(c}{}\right.}{7}}{d}$
default	$\frac{\frac{C\left(\frac{128}{35}+\cos^8(dx+c)+\frac{8\left(\cos^6(dx+c)\right)}{7}+\frac{48\left(\cos^4(dx+c)\right)}{35}+\frac{64\left(\cos^2(dx+c)\right)}{35}\right)\sin(dx+c)}{9}+\frac{A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8(co}{}\right.}{7}}{d}$
parts	$\frac{A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7d}+\frac{C\left(\frac{128}{35}+\cos^8(dx+c)+\frac{8\left(\cos^6(dx+c)\right)}{7}+\frac{48\left(\cos^4(dx+c)\right.}{35}\right.}{9d}$
risch	$\frac{35\sin(dx+c)A}{64d}+\frac{63C\sin(dx+c)}{128d}+\frac{C\sin(9dx+9c)}{2304d}+\frac{\sin(7dx+7c)A}{448d}+\frac{9\sin(7dx+7c)C}{1792d}+\frac{7\sin(5dx+5c)A}{320d}+\frac{9\sin}{}$
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\left(\tan^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8(3A+2C)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(3A+2C)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(17A+19C)\left(\tan^5\left(\frac{d}{}\right.\right.}{5d}}{\left(1+t\right.}$

```
[In] int(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/80640*(8820*(A+C)*sin(3*d*x+3*c)+252*(7*A+9*C)*sin(5*d*x+5*c)+45*(4*A+9*C
)*sin(7*d*x+7*c)+35*C*sin(9*d*x+9*c)+44100*(A+9/10*C)*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\left(35\,C\cos\left(dx+c\right)^8+5\,(9\,A+8\,C)\cos\left(dx+c\right)^6+6\,(9\,A+8\,C)\cos\left(dx+c\right)^4+8\,(9\,A+8\,C)\cos\left(dx+c\right)^2}{315\,d}$$

```
[In] integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*C*cos(d*x + c)^8 + 5*(9*A + 8*C)*cos(d*x + c)^6 + 6*(9*A + 8*C)*c
os(d*x + c)^4 + 8*(9*A + 8*C)*cos(d*x + c)^2 + 144*A + 128*C)*sin(d*x + c)/
d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(80) = 160$.

Time = 0.92 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.16

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{16A\sin^7(c+dx)}{35d} + \frac{8A\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2A\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{A\sin(c+dx)\cos^6(c+dx)}{d} + \frac{128C\sin^9(c+dx)}{315d} + \frac{64C\,\mathrm{s}}{} \\ x(A+C\cos^2(c))\cos^7(c) \end{cases}$$

```
[In] integrate(cos(d*x+c)**7*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((16*A*sin(c + d*x)**7/(35*d) + 8*A*sin(c + d*x)**5*cos(c + d*x)**
2/(5*d) + 2*A*sin(c + d*x)**3*cos(c + d*x)**4/d + A*sin(c + d*x)*cos(c + d*
x)**6/d + 128*C*sin(c + d*x)**9/(315*d) + 64*C*sin(c + d*x)**7*cos(c + d*x)
**2/(35*d) + 16*C*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*C*sin(c + d*x)*
*3*cos(c + d*x)**6/(3*d) + C*sin(c + d*x)*cos(c + d*x)**8/d, Ne(d, 0)), (x*
(A + C*cos(c)**2)*cos(c)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{35\,C\sin(dx+c)^9 - 45\,(A+4\,C)\sin(dx+c)^7 + 189\,(A+2\,C)\sin(dx+c)^5 - 105\,(3\,A+4\,C)\sin(dx+c}{315\,d}$$

```
[In] integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*C*sin(d*x + c)^9 - 45*(A + 4*C)*sin(d*x + c)^7 + 189*(A + 2*C)*si
n(d*x + c)^5 - 105*(3*A + 4*C)*sin(d*x + c)^3 + 315*(A + C)*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\sin(9\,dx+9\,c)}{2304\,d} + \frac{(4\,A+9\,C)\sin(7\,dx+7\,c)}{1792\,d} + \frac{(7\,A+9\,C)\sin(5\,dx+5\,c)}{320\,d} + \frac{7\,(A+C)\sin(3\,dx+3\,c)}{64\,d} + \frac{7\,(10\,A+9\,C)\sin(dx+c)}{128\,d}$$

```
[In] integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2304*C*sin(9*d*x + 9*c)/d + 1/1792*(4*A + 9*C)*sin(7*d*x + 7*c)/d + 1/320
*(7*A + 9*C)*sin(5*d*x + 5*c)/d + 7/64*(A + C)*sin(3*d*x + 3*c)/d + 7/128*(
10*A + 9*C)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\frac{C\sin(c+d\,x)^9}{9}+\left(-\frac{A}{7}-\frac{4\,C}{7}\right)\,\sin\left(c+d\,x\right)^7+\left(\frac{3\,A}{5}+\frac{6\,C}{5}\right)\,\sin\left(c+d\,x\right)^5+\left(-A-\frac{4\,C}{3}\right)\,\sin\left(c+d\,x\right)^3+(A+C)}{d}$$

```
[In] int(cos(c + d*x)^7*(A + C*cos(c + d*x)^2),x)
```

```
[Out] ((C*sin(c + d*x)^9)/9 - sin(c + d*x)^3*(A + (4*C)/3) + sin(c + d*x)*(A + C)
 + sin(c + d*x)^5*((3*A)/5 + (6*C)/5) - sin(c + d*x)^7*(A/7 + (4*C)/7))/d
```

3.2 $\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{(2A+3C)\sin^3(c+dx)}{3d}+\frac{(A+3C)\sin^5(c+dx)}{5d}-\frac{C\sin^7(c+dx)}{7d}$$

```
[Out] (A+C)*sin(d*x+c)/d-1/3*(2*A+3*C)*sin(d*x+c)^3/d+1/5*(A+3*C)*sin(d*x+c)^5/d-
1/7*C*sin(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3092, 380}

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(A+3C)\sin^5(c+dx)}{5d}-\frac{(2A+3C)\sin^3(c+dx)}{3d}+\frac{(A+C)\sin(c+dx)}{d}-\frac{C\sin^7(c+dx)}{7d}$$

```
[In] Int[Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - ((2*A + 3*C)*Sin[c + d*x]^3)/(3*d) + ((A + 3*C)*
Sin[c + d*x]^5)/(5*d) - (C*Sin[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int (1-x^2)^2 (A+C-Cx^2)\, dx, x, -\sin(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(A\left(1+\frac{C}{A}\right)-(2A+3C)x^2+(A+3C)x^4-Cx^6\right)\, dx, x, -\sin(c+dx)\right)}{d}\\ &= \frac{(A+C)\sin(c+dx)}{d}-\frac{(2A+3C)\sin^3(c+dx)}{3d}+\frac{(A+3C)\sin^5(c+dx)}{5d}-\frac{C\sin^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\, dx = \frac{A\sin(c+dx)}{d}+\frac{C\sin(c+dx)}{d}-\frac{2A\sin^3(c+dx)}{3d}-\frac{C\sin^3(c+dx)}{d}+\frac{A\sin^5(c+dx)}{5d}+\frac{3C\sin^5(c+dx)}{5d}-\frac{C\sin^7(c+dx)}{7d}$$

```
[In] Integrate[Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2),x]
[Out] (A*Sin[c + d*x])/d + (C*Sin[c + d*x])/d - (2*A*Sin[c + d*x]^3)/(3*d) - (C*S
in[c + d*x]^3)/d + (A*Sin[c + d*x]^5)/(5*d) + (3*C*Sin[c + d*x]^5)/(5*d) -
(C*Sin[c + d*x]^7)/(7*d)
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{(700A+735C)\sin(3dx+3c)+(84A+147C)\sin(5dx+5c)+15\sin(7dx+7c)C+4200\left(A+\frac{7C}{8}\right)\sin(dx+c)}{6720d}$
derivativedivides	$\frac{\frac{C\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+\frac{A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{\frac{C\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+\frac{A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
parts	$\frac{A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}+\frac{C\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7d}$
risch	$\frac{5\sin(dx+c)A}{8d}+\frac{35C\sin(dx+c)}{64d}+\frac{\sin(7dx+7c)C}{448d}+\frac{\sin(5dx+5c)A}{80d}+\frac{7\sin(5dx+5c)C}{320d}+\frac{5\sin(3dx+3c)A}{48d}+\frac{7\,\mathrm{s}}{}$
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4(5A+3C)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4(5A+3C)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(91A+53C)\left(\tan^7\right.}{35d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$

```
[In] int(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6720*((700*A+735*C)*sin(3*d*x+3*c)+(84*A+147*C)*sin(5*d*x+5*c)+15*sin(7*d
*x+7*c)*C+4200*(A+7/8*C)*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\left(15\,C\cos\left(dx+c\right)^6+3\left(7\,A+6\,C\right)\cos\left(dx+c\right)^4+4\left(7\,A+6\,C\right)\cos\left(dx+c\right)^2+56\,A+48\,C\right)\sin\left(dx+c\right)}{105\,d}$$

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*C*cos(d*x + c)^6 + 3*(7*A + 6*C)*cos(d*x + c)^4 + 4*(7*A + 6*C)*c
os(d*x + c)^2 + 56*A + 48*C)*sin(d*x + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(60) = 120.

Time = 0.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.10

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{8A\sin^5(c+dx)}{15d} + \frac{4A\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{A\sin(c+dx)\cos^4(c+dx)}{d} + \frac{16C\sin^7(c+dx)}{35d} + \frac{8C\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2C\sin^3(c \\ x(A+C\cos^2(c))\cos^5(c) \end{cases}$$

```
[In] integrate(cos(d*x+c)**5*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((8*A*sin(c + d*x)**5/(15*d) + 4*A*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + A*sin(c + d*x)*cos(c + d*x)**4/d + 16*C*sin(c + d*x)**7/(35*d) + 8
*C*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*C*sin(c + d*x)**3*cos(c + d*x)
**4/d + C*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*c
os(c)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{15\,C\sin(dx+c)^7 - 21\,(A+3\,C)\sin(dx+c)^5 + 35\,(2\,A+3\,C)\sin(dx+c)^3 - 105\,(A+C)\sin(dx+c)}{105\,d}$$

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/105*(15*C*sin(d*x + c)^7 - 21*(A + 3*C)*sin(d*x + c)^5 + 35*(2*A + 3*C)*
sin(d*x + c)^3 - 105*(A + C)*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\sin(7\,dx+7\,c)}{448\,d} + \frac{(4\,A+7\,C)\sin(5\,dx+5\,c)}{320\,d} + \frac{(20\,A+21\,C)\sin(3\,dx+3\,c)}{192\,d} + \frac{5\,(8\,A+7\,C)\sin(dx+c)}{64\,d}$$

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/448*C*sin(7*d*x + 7*c)/d + 1/320*(4*A + 7*C)*sin(5*d*x + 5*c)/d + 1/192*(
20*A + 21*C)*sin(3*d*x + 3*c)/d + 5/64*(8*A + 7*C)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{\frac{C\sin(c+d\,x)^7}{7}+\left(-\frac{A}{5}-\frac{3\,C}{5}\right)\,\sin\left(c+d\,x\right)^5+\left(\frac{2\,A}{3}+C\right)\,\sin\left(c+d\,x\right)^3+\left(-A-C\right)\,\sin\left(c+d\,x\right)}{d}$$

```
[In] int(cos(c + d*x)^5*(A + C*cos(c + d*x)^2),x)
```

```
[Out] -(sin(c + d*x)^3*((2*A)/3 + C) + (C*sin(c + d*x)^7)/7 - sin(c + d*x)*(A + C
) - sin(c + d*x)^5*(A/5 + (3*C)/5))/d
```

3.3 $\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{(A+2C)\sin^3(c+dx)}{3d}+\frac{C\sin^5(c+dx)}{5d}$$

```
[Out] (A+C)*sin(d*x+c)/d-1/3*(A+2*C)*sin(d*x+c)^3/d+1/5*C*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3092, 380}

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=-\frac{(A+2C)\sin^3(c+dx)}{3d}+\frac{(A+C)\sin(c+dx)}{d}+\frac{C\sin^5(c+dx)}{5d}$$

```
[In] Int[Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - ((A + 2*C)*Sin[c + d*x]^3)/(3*d) + (C*Sin[c + d*
x]^5)/(5*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (1-x^2)\left(A+C-Cx^2\right)\,dx, x, -\sin(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(A\left(1+\frac{C}{A}\right)-(A+2C)x^2+Cx^4\right)\,dx, x, -\sin(c+dx)\right)}{d}\\
&= \frac{(A+C)\sin(c+dx)}{d}-\frac{(A+2C)\sin^3(c+dx)}{3d}+\frac{C\sin^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{A\sin(c+dx)}{d}+\frac{C\sin(c+dx)}{d}-\frac{A\sin^3(c+dx)}{3d}-\frac{2C\sin^3(c+dx)}{3d}+\frac{C\sin^5(c+dx)}{5d}$$

```
[In] Integrate[Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (A*Sin[c + d*x])/d + (C*Sin[c + d*x])/d - (A*Sin[c + d*x]^3)/(3*d) - (2*C*S
in[c + d*x]^3)/(3*d) + (C*Sin[c + d*x]^5)/(5*d)
```

Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{(20A+25C)\sin(3dx+3c)+3\sin(5dx+5c)C+180\left(A+\frac{5C}{6}\right)\sin(dx+c)}{240d}$
derivativedivides	$\frac{\frac{C\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+\frac{A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{\frac{C\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+\frac{A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
parts	$\frac{A(2+\cos^2(dx+c))\sin(dx+c)}{3d}+\frac{C\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}$
risch	$\frac{3\sin(dx+c)A}{4d}+\frac{5C\sin(dx+c)}{8d}+\frac{\sin(5dx+5c)C}{80d}+\frac{\sin(3dx+3c)A}{12d}+\frac{5\sin(3dx+3c)C}{48d}$
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8(2A+C)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(2A+C)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4(25A+29C)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*((20*A+25*C)*sin(3*d*x+3*c)+3*sin(5*d*x+5*c)*C+180*(A+5/6*C)*sin(d*x+
c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\left(3\,C\cos\left(dx+c\right)^4+\left(5\,A+4\,C\right)\cos\left(dx+c\right)^2+10\,A+8\,C\right)\sin\left(dx+c\right)}{15\,d}$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*C*cos(d*x + c)^4 + (5*A + 4*C)*cos(d*x + c)^2 + 10*A + 8*C)*sin(d*x
 + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(42) = 84.

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.10

$$\int \cos^3(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \begin{cases} \frac{2A\sin^3(c+dx)}{3d} + \frac{A\sin(c+dx)\cos^2(c+dx)}{d} + \frac{8C\sin^5(c+dx)}{15d} + \frac{4C\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{C\sin(c+dx)\cos^4(c+dx)}{d} & \text{for } d \neq 0 \\ x(A+C\cos^2(c))\cos^3(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2),x)
[Out] Piecewise((2*A*sin(c + d*x)**3/(3*d) + A*sin(c + d*x)*cos(c + d*x)**2/d + 8
*C*sin(c + d*x)**5/(15*d) + 4*C*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + C*s
in(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**3, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \frac{3\,C\sin\left(dx+c\right)^5 - 5\,(A+2\,C)\sin\left(dx+c\right)^3 + 15\,(A+C)\sin\left(dx+c\right)}{15\,d}$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] 1/15*(3*C*sin(d*x + c)^5 - 5*(A + 2*C)*sin(d*x + c)^3 + 15*(A + C)*sin(d*x
 + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \cos^3(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \frac{3\,C\sin\left(dx+c\right)^5 - 5\,A\sin\left(dx+c\right)^3 - 10\,C\sin\left(dx+c\right)^3 + 15\,A\sin\left(dx+c\right) + 15\,C\sin\left(dx+c\right)}{15\,d}$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] 1/15*(3*C*sin(d*x + c)^5 - 5*A*sin(d*x + c)^3 - 10*C*sin(d*x + c)^3 + 15*A*
sin(d*x + c) + 15*C*sin(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\frac{C\sin(c+d\,x)^5}{5}+\left(-\frac{A}{3}-\frac{2\,C}{3}\right)\,\sin\left(c+d\,x\right)^3+\left(A+C\right)\,\sin\left(c+d\,x\right)}{d}$$

```
[In] int(cos(c + d*x)^3*(A + C*cos(c + d*x)^2),x)
```

```
[Out] ((C*sin(c + d*x)^5)/5 + sin(c + d*x)*(A + C) - sin(c + d*x)^3*(A/3 + (2*C)/
3))/d
```

3.4 $\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}$$

```
[Out] (A+C)*sin(d*x+c)/d-1/3*C*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3092}

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}$$

```
[In] Int[Cos[c + d*x]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - (C*Sin[c + d*x]^3)/(3*d)
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\frac{\text{Subst}\left(\int\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\&=\frac{(A+C)\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{A\cos(dx)\sin(c)}{d}+\frac{A\cos(c)\sin(dx)}{d}+\frac{C\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}$$

[In] Integrate[Cos[c + d*x]*(A + C*Cos[c + d*x]^2),x]

[Out] (A*Cos[d*x]*Sin[c])/d + (A*Cos[c]*Sin[d*x])/d + (C*Sin[c + d*x])/d - (C*Sin[c + d*x]^3)/(3*d)

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{\sin(3dx+3c)C+12\left(A+\frac{3C}{4}\right)\sin(dx+c)}{12d}$	31
derivativedivides	$\frac{\frac{C\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+A\sin(dx+c)}{d}$	33
default	$\frac{\frac{C\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+A\sin(dx+c)}{d}$	33
parts	$\frac{\sin(dx+c)A}{d}+\frac{C\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3d}$	35
risch	$\frac{\sin(dx+c)A}{d}+\frac{3C\sin(dx+c)}{4d}+\frac{\sin(3dx+3c)C}{12d}$	40
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4(3A+C)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	75

[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/12*(sin(3*d*x+3*c)*C+12*(A+3/4*C)*sin(d*x+c))/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{\left(C\cos\left(dx+c\right)^2+3\,A+2\,C\right)\sin\left(dx+c\right)}{3\,d}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sin(d*x + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(24) = 48.

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\begin{cases}\frac{A\sin\left(c+dx\right)}{d}+\frac{2C\sin^3\left(c+dx\right)}{3d}+\frac{C\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\left(A+C\cos^2\left(c\right)\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2),x)
[Out] Piecewise((A*sin(c + d*x)/d + 2*C*sin(c + d*x)**3/(3*d) + C*sin(c + d*x)*co
s(c + d*x)**2/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=-\frac{\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)C-3\,A\sin\left(dx+c\right)}{3\,d}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] -1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*C - 3*A*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \cos(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = -\frac{\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)C - 3\,A\sin\left(dx+c\right)}{3\,d}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] -1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*C - 3*A*sin(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = -\frac{\frac{C\sin(c+d\,x)^3}{3} - \sin\left(c+d\,x\right)\,\left(A+C\right)}{d}$$

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2),x)
[Out] -((C*sin(c + d*x)^3)/3 - sin(c + d*x)*(A + C))/d
```

3.5 $\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx=\frac{A\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}$$

```
[Out] A*arctanh(sin(d*x+c))/d+C*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3093, 3855}

$$\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx=\frac{A\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
[Out] (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{C\sin(c+dx)}{d} + A\int \sec(c+dx)\,dx$$
$$= \frac{A\text{arctanh}(\sin(c+dx))}{d} + \frac{C\sin(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{A\text{arctanh}(\sin(c+dx))}{d} + \frac{C\cos(dx)\sin(c)}{d} + \frac{C\cos(c)\sin(dx)}{d}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
[Out] (A*ArcTanh[Sin[c + d*x]])/d + (C*Cos[d*x]*Sin[c])/d + (C*Cos[c]*Sin[d*x])/d
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))+\sin(dx+c)C}{d}$	30
default	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))+\sin(dx+c)C}{d}$	30
parts	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{C\sin(dx+c)}{d}$	32
parallelrisch	$\frac{-A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\sin(dx+c)C}{d}$	43
risch	$-\frac{iC\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{iC\,\mathrm{e}^{-i(dx+c)}}{2d}+\frac{A\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{A\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$	71
norman	$\frac{\frac{2C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2C\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	86

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(A*ln(sec(d*x+c)+tan(d*x+c))+sin(d*x+c)*C)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int (A + C\cos^2(c + dx))\sec(c + dx)\,dx = \frac{A\log(\sin(dx + c) + 1) - A\log(-\sin(dx + c) + 1) + 2\,C\sin(dx + c)}{2\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
[Out] 1/2*(A*log(sin(d*x + c) + 1) - A*log(-sin(d*x + c) + 1) + 2*C*sin(d*x + c))
/d
```

Sympy [F]

$$\int (A + C\cos^2(c + dx))\sec(c + dx)\,dx = \int (A + C\cos^2(c + dx))\sec(c + dx)\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c),x)
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int (A + C\cos^2(c + dx))\sec(c + dx)\,dx = \frac{A\log(\sin(dx + c) + 1) - A\log(\sin(dx + c) - 1) + 2\,C\sin(dx + c)}{2\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxima")
[Out] 1/2*(A*log(sin(d*x + c) + 1) - A*log(sin(d*x + c) - 1) + 2*C*sin(d*x + c))/
d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{A\log\left(|\sin\left(dx+c\right)+1|\right)-A\log\left(|\sin\left(dx+c\right)-1|\right)+2\,C\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")
[Out] 1/2*(A*log(abs(sin(d*x + c) + 1)) - A*log(abs(sin(d*x + c) - 1)) + 2*C*sin(
d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C\,\sin\left(c+d\,x\right)+A\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{d}$$

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x),x)
[Out] (C*sin(c + d*x) + A*atanh(sin(c + d*x)))/d
```

3.6 $\int (A + C\cos^2(c + dx))\sec^3(c + dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int (A + C\cos^2(c + dx))\sec^3(c + dx)\,dx = \frac{(A + 2C)\text{arctanh}(\sin(c + dx))}{2d} + \frac{A\sec(c + dx)\tan(c + dx)}{2d}$$

```
[Out] 1/2*(A+2*C)*arctanh(sin(d*x+c))/d+1/2*A*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3091, 3855}

$$\int (A + C\cos^2(c + dx))\sec^3(c + dx)\,dx = \frac{(A + 2C)\text{arctanh}(\sin(c + dx))}{2d} + \frac{A\tan(c + dx)\sec(c + dx)}{2d}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*
d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
```

```
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{A\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}(A+2C)\int\sec(c+dx)\,dx\\ &= \frac{(A+2C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{A\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{A\text{arctanh}(\sin(c+dx))}{2d}+\frac{C\text{arctanh}(\sin(c+dx))}{d}+\frac{A\sec(c+dx)\tan(c+dx)}{2d}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(2*d) + (C*ArcTanh[Sin[c + d*x]])/d + (A*Sec[c +
d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+C\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+C\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parts	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{C\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$\frac{-(1+\cos(2dx+2c))(A+2C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1+\cos(2dx+2c))(A+2C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2A\sin(dx+c)}{2d(1+\cos(2dx+2c))}$
risch	$-\frac{iA\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{A\ln\left(e^{i(dx+c)}-i\right)}{2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)C}{d}+\frac{A\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)C}{d}$
norman	$\frac{\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{A\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{(A+2C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{(A+2C}{}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+C*ln(sec(d
*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\frac{(A+2\,C)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(A+2\,C)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,A\sin\left(\right.}{4\,d\cos\left(dx+c\right)^2}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((A + 2*C)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A + 2*C)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*A*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(A+C\cos^2{(c+dx)}\right)\sec^3{(c+dx)}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{(A+2C)\log\left(\sin\left(dx+c\right)+1\right)-(A+2C)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2A\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((A + 2*C)*log(sin(d*x + c) + 1) - (A + 2*C)*log(sin(d*x + c) - 1) - 2*
A*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{(A+2C)\log\left(|\sin\left(dx+c\right)+1|\right)-(A+2C)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2A\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*((A + 2*C)*log(abs(sin(d*x + c) + 1)) - (A + 2*C)*log(abs(sin(d*x + c)
- 1)) - 2*A*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{\operatorname{atanh}(\sin\left(c+d\,x\right))\,\left(\frac{A}{2}+C\right)}{d} - \frac{A\,\sin\left(c+d\,x\right)}{2\,d\,\left({\sin\left(c+d\,x\right)}^2-1\right)}$$

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^3,x)
[Out] (atanh(sin(c + d*x))*(A/2 + C))/d - (A*sin(c + d*x))/(2*d*(sin(c + d*x)^2 -
 1))
```

3.7 $\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))}{8d} + \frac{(3A+4C)\sec(c+dx)\tan(c+dx)}{8d} + \frac{A\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] 1/8*(3*A+4*C)*arctanh(sin(d*x+c))/d+1/8*(3*A+4*C)*sec(d*x+c)*tan(d*x+c)/d+1
/4*A*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3091, 3853, 3855}

$$\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))}{8d} + \frac{(3A+4C)\tan(c+dx)\sec(c+dx)}{8d} + \frac{A\tan(c+dx)\sec^3(c+dx)}{4d}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((3*A + 4*C)*Sec[c + d*x]*Tan[c
 + d*x])/(8*d) + (A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{A\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}(3A+4C)\int\sec^3(c+dx)\,dx\\
&= \frac{(3A+4C)\sec(c+dx)\tan(c+dx)}{8d}\\
&\quad+\frac{A\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{8}(3A+4C)\int\sec(c+dx)\,dx\\
&= \frac{(3A+4C)\text{arctanh}(\sin(c+dx))}{8d}\\
&\quad+\frac{(3A+4C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{A\sec^3(c+dx)\tan(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\begin{aligned}
\int\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx &= \frac{3A\text{arctanh}(\sin(c+dx))}{8d}+\frac{C\text{arctanh}(\sin(c+dx))}{2d}\\
&\quad+\frac{3A\sec(c+dx)\tan(c+dx)}{8d}\\
&\quad+\frac{C\sec(c+dx)\tan(c+dx)}{2d}\\
&\quad+\frac{A\sec^3(c+dx)\tan(c+dx)}{4d}
\end{aligned}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (3*A*ArcTanh[Sin[c + d*x]])/(8*d) + (C*ArcTanh[Sin[c + d*x]])/(2*d) + (3*A*
Sec[c + d*x]*Tan[c + d*x])/(8*d) + (C*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (A
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(d}{2}\right.}{d}$
default	$\frac{A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(d}{2}\right.}{d}$
parts	$\frac{A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\mathrm{ta}}{2}\right.}{d}$
parallelrisch	$\frac{-6\left(A+\frac{4C}{3}\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+6\left(A+\frac{4C}{3}\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\left(\right.\right.}{4d(\cos(4dx+4c)+4\cos(2dx+2c)+3)}$
risch	$-\frac{i e^{i(dx+c)}\left(3A e^{6i(dx+c)}+4C e^{6i(dx+c)}+11A e^{4i(dx+c)}+4C e^{4i(dx+c)}-11A e^{2i(dx+c)}-4C e^{2i(dx+c)}-3A-4C\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}-\frac{3A\ln}{}$
norman	$\frac{\frac{(5A+4C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(5A+4C)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(7A-4C)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{(7A-4C)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{(13A+4C)\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{4d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))+C*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\frac{(3A+4C)\cos(dx+c)^4\log(\sin(dx+c)+1)-(3A+4C)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left((3A\right.}{16\,d\cos(dx+c)^4}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/16*((3*A + 4*C)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*A + 4*C)*cos(d*
x + c)^4*log(-sin(d*x + c) + 1) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sin(
d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \left(A+C\cos^2\left(c+dx\right)\right)\sec^5\left(c+dx\right)dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39

$$\int \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\frac{(3A+4C)\log\left(\sin\left(dx+c\right)+1\right)-(3A+4C)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left((3A+4C)\sin(dx+c)^3-(5A+4C)\sin(dx+\cdots\right.}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/16*((3*A + 4*C)*log(sin(d*x + c) + 1) - (3*A + 4*C)*log(sin(d*x + c) - 1)
 - 2*((3*A + 4*C)*sin(d*x + c)^3 - (5*A + 4*C)*sin(d*x + c))/(sin(d*x + c)^
4 - 2*sin(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\frac{(3A+4C)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-(3A+4C)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3A\sin(dx+c)^3+4C\sin(dx+c)^3-5A\sin\cdots\right.}{\left(\sin(dx+c)^2-1\right)\cdots}}{16\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/16*((3*A + 4*C)*log(abs(sin(d*x + c) + 1)) - (3*A + 4*C)*log(abs(sin(d*x
+ c) - 1)) - 2*(3*A*sin(d*x + c)^3 + 4*C*sin(d*x + c)^3 - 5*A*sin(d*x + c)
- 4*C*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int (A + C\cos^2(c+dx))\sec^5(c+dx)\,dx = \frac{\sin(c+d\,x)\,\left(\frac{5\,A}{8}+\frac{C}{2}\right)-\sin(c+d\,x)^3\,\left(\frac{3\,A}{8}+\frac{C}{2}\right)}{d\,\left(\sin(c+d\,x)^4-2\sin(c+d\,x)^2+1\right)} + \frac{\operatorname{atanh}(\sin(c+d\,x))\,\left(\frac{3\,A}{8}+\frac{C}{2}\right)}{d}$$

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^5,x)
[Out] (sin(c + d*x)*((5*A)/8 + C/2) - sin(c + d*x)^3*((3*A)/8 + C/2))/(d*(sin(c +
 d*x)^4 - 2*sin(c + d*x)^2 + 1)) + (atanh(sin(c + d*x))*((3*A)/8 + C/2))/d
```

3.8 $\int \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\frac{(5A+6C)\text{arctanh}(\sin(c+dx))}{16d}+\frac{(5A+6C)\sec(c+dx)\tan(c+dx)}{16d}+\frac{(5A+6C)\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{A\sec^5(c+dx)\tan(c+dx)}{6d}$$

```
[Out] 1/16*(5*A+6*C)*arctanh(sin(d*x+c))/d+1/16*(5*A+6*C)*sec(d*x+c)*tan(d*x+c)/d
+1/24*(5*A+6*C)*sec(d*x+c)^3*tan(d*x+c)/d+1/6*A*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3091, 3853, 3855}

$$\int \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\frac{(5A+6C)\text{arctanh}(\sin(c+dx))}{16d}+\frac{(5A+6C)\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{(5A+6C)\tan(c+dx)\sec(c+dx)}{16d}+\frac{A\tan(c+dx)\sec^5(c+dx)}{6d}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] ((5*A + 6*C)*ArcTanh[Sin[c + d*x]])/(16*d) + ((5*A + 6*C)*Sec[c + d*x]*Tan[
c + d*x])/(16*d) + ((5*A + 6*C)*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (A*Se
c[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{A\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{6}(5A+6C)\int\sec^5(c+dx)\,dx\\
&= \frac{(5A+6C)\sec^3(c+dx)\tan(c+dx)}{24d}\\
&\quad+\frac{A\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{8}(5A+6C)\int\sec^3(c+dx)\,dx\\
&= \frac{(5A+6C)\sec(c+dx)\tan(c+dx)}{16d}+\frac{(5A+6C)\sec^3(c+dx)\tan(c+dx)}{24d}\\
&\quad+\frac{A\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{16}(5A+6C)\int\sec(c+dx)\,dx\\
&= \frac{(5A+6C)\text{arctanh}(\sin(c+dx))}{16d}+\frac{(5A+6C)\sec(c+dx)\tan(c+dx)}{16d}\\
&\quad+\frac{(5A+6C)\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{A\sec^5(c+dx)\tan(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{5A\operatorname{arctanh}(\sin(c+dx))}{16d}+\frac{3C\operatorname{arctanh}(\sin(c+dx))}{8d} +\frac{5A\sec(c+dx)\tan(c+dx)}{16d} +\frac{3C\sec(c+dx)\tan(c+dx)}{8d} +\frac{5A\sec^3(c+dx)\tan(c+dx)}{24d} +\frac{C\sec^3(c+dx)\tan(c+dx)}{4d} +\frac{A\sec^5(c+dx)\tan(c+dx)}{6d}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] (5*A*ArcTanh[Sin[c + d*x]])/(16*d) + (3*C*ArcTanh[Sin[c + d*x]])/(8*d) + (5
*A*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (3*C*Sec[c + d*x]*Tan[c + d*x])/(8*d
) + (5*A*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (C*Sec[c + d*x]^3*Tan[c + d*
x])/(4*d) + (A*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+C\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\cdots}{d}$
default	$\frac{A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+C\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\cdots}{d}$
parts	$\frac{A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}+\frac{C\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}\cdots}{\cdots}$
parallelrisch	$\frac{-225\left(A+\frac{6C}{5}\right)\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}+\cos(2dx+2c)+\frac{2}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+225\left(A+\frac{6C}{5}\right)\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(\cdots}{48d(\cos(6dx+6c)+6\cos\cdots}$
risch	$-\frac{i\,e^{i(dx+c)}\left(15A\,e^{10i(dx+c)}+18C\,e^{10i(dx+c)}+85A\,e^{8i(dx+c)}+102C\,e^{8i(dx+c)}+198A\,e^{6i(dx+c)}+84C\,e^{6i(dx+c)}-198A\,e^{4i\cdots}{24d\left(e^{2i(dx+c)}+1\right)^6}$
norman	$\frac{\frac{(11A+10C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{(11A+10C)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{7(19A-6C)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{7(19A-6C)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{(71A+18C\cdots}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}\cdots\right.\right.}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5
/16*ln(sec(d*x+c)+tan(d*x+c)))+C*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d
*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int (A + C\cos^2(c+dx))\sec^7(c+dx)\,dx$$
$$= \frac{3\,(5\,A+6\,C)\cos{(dx+c)}^6\log{(\sin{(dx+c)}+1)} - 3\,(5\,A+6\,C)\cos{(dx+c)}^6\log{(-\sin{(dx+c)}+1)} + 2\,(}{96\,d\cos{(dx+c)}^6}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(5*A + 6*C)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(5*A + 6*C)*co
s(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(3*(5*A + 6*C)*cos(d*x + c)^4 + 2*(
5*A + 6*C)*cos(d*x + c)^2 + 8*A)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)]

Timed out.

$$\int (A + C\cos^2(c+dx))\sec^7(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int (A + C\cos^2(c+dx))\sec^7(c+dx)\,dx$$
$$= \frac{3\,(5\,A+6\,C)\log{(\sin{(dx+c)}+1)} - 3\,(5\,A+6\,C)\log{(\sin{(dx+c)}-1)} - \frac{2\left(3\,(5\,A+6\,C)\sin(dx+c)^5 - 8\,(5\,A+6\,C)\sin\right.}{\sin(dx+c)^6 - 3\,\sin(dx+c)}}{96\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(5*A + 6*C)*log(sin(d*x + c) + 1) - 3*(5*A + 6*C)*log(sin(d*x + c)
- 1) - 2*(3*(5*A + 6*C)*sin(d*x + c)^5 - 8*(5*A + 6*C)*sin(d*x + c)^3 + 3*(
11*A + 10*C)*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x +
 c)^2 - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$$
$$=\frac{3\,(5\,A+6\,C)\log\left(|\sin\left(dx+c\right)+1|\right)-3\,(5\,A+6\,C)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(15\,A\sin(dx+c)^5+18\,C\sin(dx+c)^5\right.}{}}{96\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/96*(3*(5*A + 6*C)*log(abs(sin(d*x + c) + 1)) - 3*(5*A + 6*C)*log(abs(sin(
d*x + c) - 1)) - 2*(15*A*sin(d*x + c)^5 + 18*C*sin(d*x + c)^5 - 40*A*sin(d*
x + c)^3 - 48*C*sin(d*x + c)^3 + 33*A*sin(d*x + c) + 30*C*sin(d*x + c))/(si
n(d*x + c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$$
$$=\frac{\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(\frac{5\,A}{16}+\frac{3\,C}{8}\right)}{d}$$
$$-\frac{\left(\frac{5\,A}{16}+\frac{3\,C}{8}\right)\,\sin\left(c+d\,x\right)^5+\left(-\frac{5\,A}{6}-C\right)\,\sin\left(c+d\,x\right)^3+\left(\frac{11\,A}{16}+\frac{5\,C}{8}\right)\,\sin\left(c+d\,x\right)}{d\,\left(\sin\left(c+d\,x\right)^6-3\sin\left(c+d\,x\right)^4+3\sin\left(c+d\,x\right)^2-1\right)}$$

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^7,x)
```

```
[Out] (atanh(sin(c + d*x))*((5*A)/16 + (3*C)/8))/d - (sin(c + d*x)*((11*A)/16 + (
5*C)/8) - sin(c + d*x)^3*((5*A)/6 + C) + sin(c + d*x)^5*((5*A)/16 + (3*C)/8
))/(d*(3*sin(c + d*x)^2 - 3*sin(c + d*x)^4 + sin(c + d*x)^6 - 1))
```

3.9 $\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{5}{128}(8A+7C)x + \frac{5(8A+7C)\cos(c+dx)\sin(c+dx)}{128d} + \frac{5(8A+7C)\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{(8A+7C)\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{C\cos^7(c+dx)\sin(c+dx)}{8d}$$

```
[Out] 5/128*(8*A+7*C)*x+5/128*(8*A+7*C)*cos(d*x+c)*sin(d*x+c)/d+5/192*(8*A+7*C)*c
os(d*x+c)^3*sin(d*x+c)/d+1/48*(8*A+7*C)*cos(d*x+c)^5*sin(d*x+c)/d+1/8*C*cos
(d*x+c)^7*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3093, 2715, 8}

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(8A+7C)\sin(c+dx)\cos^5(c+dx)}{48d} + \frac{5(8A+7C)\sin(c+dx)\cos^3(c+dx)}{192d} + \frac{5(8A+7C)\sin(c+dx)\cos(c+dx)}{128d} + \frac{5}{128}x(8A+7C) + \frac{C\sin(c+dx)\cos^7(c+dx)}{8d}$$

```
[In] Int[Cos[c + d*x]^6*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (5*(8*A + 7*C)*x)/128 + (5*(8*A + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(128*d) +
 (5*(8*A + 7*C)*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + ((8*A + 7*C)*Cos[c +
 d*x]^5*Sin[c + d*x])/(48*d) + (C*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{C\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}(8A+7C)\int \cos^6(c+dx)\,dx \\
&= \frac{(8A+7C)\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{C\cos^7(c+dx)\sin(c+dx)}{8d} \\
&\quad + \frac{1}{48}(5(8A+7C))\int \cos^4(c+dx)\,dx \\
&= \frac{5(8A+7C)\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{(8A+7C)\cos^5(c+dx)\sin(c+dx)}{48d} \\
&\quad + \frac{C\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{64}(5(8A+7C))\int \cos^2(c+dx)\,dx \\
&= \frac{5(8A+7C)\cos(c+dx)\sin(c+dx)}{128d} + \frac{5(8A+7C)\cos^3(c+dx)\sin(c+dx)}{192d} \\
&\quad + \frac{(8A+7C)\cos^5(c+dx)\sin(c+dx)}{48d} \\
&\quad + \frac{C\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{128}(5(8A+7C))\int 1\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{5}{128}(8A+7C)x + \frac{5(8A+7C)\cos(c+dx)\sin(c+dx)}{128d} \\
&\quad + \frac{5(8A+7C)\cos^3(c+dx)\sin(c+dx)}{192d} \\
&\quad + \frac{(8A+7C)\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{C\cos^7(c+dx)\sin(c+dx)}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$
\begin{aligned}
&\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx \\
&= \frac{960Ac+840cC+960Adx+840Cdx+48(15A+14C)\sin(2(c+dx))+24(6A+7C)\sin(4(c+dx))+16}{3072d}
\end{aligned}
$$

```
[In] Integrate[Cos[c + d*x]^6*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (960*A*c + 840*c*C + 960*A*d*x + 840*C*d*x + 48*(15*A + 14*C)*Sin[2*(c + d*
x)] + 24*(6*A + 7*C)*Sin[4*(c + d*x)] + 16*A*Sin[6*(c + d*x)] + 32*C*Sin[6*
(c + d*x)] + 3*C*Sin[8*(c + d*x)])/(3072*d)
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{(720A+672C)\sin(2dx+2c)+(144A+168C)\sin(4dx+4c)+(16A+32C)\sin(6dx+6c)+3C\sin(8dx+8c)+960d\left(A+\frac{7C}{8}\right)x}{3072d}$
derivativedivides	$\frac{C\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)+A\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c\right.}{4}\right.}{}\right.}{d}$
default	$\frac{C\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)+A\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c\right.}{4}\right.}{}\right.}{d}$
parts	$\frac{A\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{C\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+\right.}{24}\right.}{8}\right.}{d}$
risch	$\frac{5xA}{16}+\frac{35Cx}{128}+\frac{C\sin(8dx+8c)}{1024d}+\frac{\sin(6dx+6c)A}{192d}+\frac{\sin(6dx+6c)C}{96d}+\frac{3\sin(4dx+4c)A}{64d}+\frac{7\sin(4dx+4c)C}{128d}+\frac{15}{}$
norman	$\left(\frac{5A}{16}+\frac{35C}{128}\right)x+\left(\frac{5A}{2}+\frac{35C}{16}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{5A}{2}+\frac{35C}{16}\right)x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{5A}{16}+\frac{35C}{128}\right)x\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{35A}{2}\right.$

```
[In] int(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3072*((720*A+672*C)*sin(2*d*x+2*c)+(144*A+168*C)*sin(4*d*x+4*c)+(16*A+32*
C)*sin(6*d*x+6*c)+3*C*sin(8*d*x+8*c)+960*d*(A+7/8*C)*x)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{15\,(8\,A+7\,C)dx+\left(48\,C\cos\left(dx+c\right)^7+8\,(8\,A+7\,C)\cos\left(dx+c\right)^5+10\,(8\,A+7\,C)\cos\left(dx+c\right)^3+15\,(\right.}{384\,d}$$

```
[In] integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/384*(15*(8*A + 7*C)*d*x + (48*C*cos(d*x + c)^7 + 8*(8*A + 7*C)*cos(d*x +
c)^5 + 10*(8*A + 7*C)*cos(d*x + c)^3 + 15*(8*A + 7*C)*cos(d*x + c))*sin(d*x
 + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(109) = 218$.

Time = 0.72 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.03

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \begin{cases} \frac{5Ax\sin^6(c+dx)}{16} + \frac{15Ax\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15Ax\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5Ax\cos^6(c+dx)}{16} + \frac{5A\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5A}{} \\ x(A+C\cos^2(c))\cos^6(c) \end{cases}$$

```
[In] integrate(cos(d*x+c)**6*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((5*A*x*sin(c + d*x)**6/16 + 15*A*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*A*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*A*x*cos(c + d*x)**6/16
 + 5*A*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*A*sin(c + d*x)**3*cos(c + d*
x)**3/(6*d) + 11*A*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 35*C*x*sin(c + d*x
)**8/128 + 35*C*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 105*C*x*sin(c + d*x)
**4*cos(c + d*x)**4/64 + 35*C*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 35*C*x
*cos(c + d*x)**8/128 + 35*C*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*C*si
n(c + d*x)**5*cos(c + d*x)**3/(384*d) + 511*C*sin(c + d*x)**3*cos(c + d*x)*
*5/(384*d) + 93*C*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0)), (x*(A +
C*cos(c)**2)*cos(c)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{15\,(dx+c)(8\,A+7\,C) + \frac{15\,(8\,A+7\,C)\tan(dx+c)^7+55\,(8\,A+7\,C)\tan(dx+c)^5+73\,(8\,A+7\,C)\tan(dx+c)^3+3\,(88\,A+93\,C)\tan(dx+c)}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{384\,d}$$

```
[In] integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/384*(15*(d*x + c)*(8*A + 7*C) + (15*(8*A + 7*C)*tan(d*x + c)^7 + 55*(8*A
+ 7*C)*tan(d*x + c)^5 + 73*(8*A + 7*C)*tan(d*x + c)^3 + 3*(88*A + 93*C)*tan
(d*x + c))/(tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*
x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{5}{128}(8\,A+7\,C)x+\frac{C\sin(8\,dx+8\,c)}{1024\,d}+\frac{(A+2\,C)\sin(6\,dx+6\,c)}{192\,d}+\frac{(6\,A+7\,C)\sin(4\,dx+4\,c)}{128\,d}+\frac{(15\,A+14\,C)\sin(2\,dx+2\,c)}{64\,d}$$

```
[In] integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 5/128*(8*A + 7*C)*x + 1/1024*C*sin(8*d*x + 8*c)/d + 1/192*(A + 2*C)*sin(6*d
*x + 6*c)/d + 1/128*(6*A + 7*C)*sin(4*d*x + 4*c)/d + 1/64*(15*A + 14*C)*sin
(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = x\left(\frac{5\,A}{16}+\frac{35\,C}{128}\right)+\frac{\left(\frac{5\,A}{16}+\frac{35\,C}{128}\right)\tan(c+d\,x)^7+\left(\frac{55\,A}{48}+\frac{385\,C}{384}\right)\tan(c+d\,x)^5+\left(\frac{73\,A}{48}+\frac{511\,C}{384}\right)\tan(c+d\,x)^3+\left(\frac{11\,A}{16}+\frac{93\,C}{128}\right)}{d\,\left(\tan(c+d\,x)^8+4\tan(c+d\,x)^6+6\tan(c+d\,x)^4+4\tan(c+d\,x)^2+1\right)}$$

```
[In] int(cos(c + d*x)^6*(A + C*cos(c + d*x)^2),x)
```

```
[Out] x*((5*A)/16 + (35*C)/128) + (tan(c + d*x)*((11*A)/16 + (93*C)/128) + tan(c
+ d*x)^7*((5*A)/16 + (35*C)/128) + tan(c + d*x)^5*((55*A)/48 + (385*C)/384)
 + tan(c + d*x)^3*((73*A)/48 + (511*C)/384))/(d*(4*tan(c + d*x)^2 + 6*tan(c
 + d*x)^4 + 4*tan(c + d*x)^6 + tan(c + d*x)^8 + 1))
```

3.10 $\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{1}{16}(6A+5C)x+\frac{(6A+5C)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(6A+5C)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{C\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 1/16*(6*A+5*C)*x+1/16*(6*A+5*C)*cos(d*x+c)*sin(d*x+c)/d+1/24*(6*A+5*C)*cos(
d*x+c)^3*sin(d*x+c)/d+1/6*C*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3093, 2715, 8}

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(6A+5C)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6A+5C)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6A+5C)+\frac{C\sin(c+dx)\cos^5(c+dx)}{6d}$$

```
[In] Int[Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((6*A + 5*C)*x)/16 + ((6*A + 5*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((6*A
 + 5*C)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (C*Cos[c + d*x]^5*Sin[c + d*x
])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(6A+5C)\int \cos^4(c+dx)\,dx \\
&= \frac{(6A+5C)\cos^3(c+dx)\sin(c+dx)}{24d} \\
&\quad + \frac{C\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{8}(6A+5C)\int \cos^2(c+dx)\,dx \\
&= \frac{(6A+5C)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(6A+5C)\cos^3(c+dx)\sin(c+dx)}{24d} \\
&\quad + \frac{C\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{16}(6A+5C)\int 1\,dx \\
&= \frac{1}{16}(6A+5C)x + \frac{(6A+5C)\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad + \frac{(6A+5C)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{C\cos^5(c+dx)\sin(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{72Ac+60cC+72Adx+60Cdx+(48A+45C)\sin(2(c+dx))+(6A+9C)\sin(4(c+dx))+C\sin(6(c+}{192d}$$

```
[In] Integrate[Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2),x]
[Out] (72*A*c + 60*c*C + 72*A*d*x + 60*C*d*x + (48*A + 45*C)*Sin[2*(c + d*x)] + (
6*A + 9*C)*Sin[4*(c + d*x)] + C*Sin[6*(c + d*x)])/(192*d)
```

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{(48A+45C)\sin(2dx+2c)+(6A+9C)\sin(4dx+4c)+\sin(6dx+6c)C+72d\left(A+\frac{5C}{6}\right)x}{192d}$
risch	$\frac{3xA}{8}+\frac{5Cx}{16}+\frac{\sin(6dx+6c)C}{192d}+\frac{\sin(4dx+4c)A}{32d}+\frac{3\sin(4dx+4c)C}{64d}+\frac{\sin(2dx+2c)A}{4d}+\frac{15\sin(2dx+2c)C}{64d}$
derivativedivides	$\frac{C\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right.}{d}$
default	$\frac{C\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right.}{d}$
parts	$\frac{A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{C\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\right.}{d}$
norman	$\left(\frac{3A}{8}+\frac{5C}{16}\right)x+\left(\frac{3A}{8}+\frac{5C}{16}\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9A}{4}+\frac{15C}{8}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9A}{4}+\frac{15C}{8}\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15A}{2}+\frac{25}{4}\right.$

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/192*((48*A+45*C)*sin(2*d*x+2*c)+(6*A+9*C)*sin(4*d*x+4*c)+sin(6*d*x+6*c)*C
+72*d*(A+5/6*C)*x)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^4(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx$$
$$= \frac{3\,(6\,A+5\,C)dx + \left(8\,C\cos\left(dx+c\right)^5 + 2\,(6\,A+5\,C)\cos\left(dx+c\right)^3 + 3\,(6\,A+5\,C)\cos\left(dx+c\right)\right)\sin\left(dx+\right.}{48\,d}$$

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] 1/48*(3*(6*A + 5*C)*d*x + (8*C*cos(d*x + c)^5 + 2*(6*A + 5*C)*cos(d*x + c)^
3 + 3*(6*A + 5*C)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(82) = 164.

Time = 0.38 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.90

$$\int \cos^4(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx$$
$$= \begin{cases} \frac{3Ax\sin^4(c+dx)}{8} + \frac{3Ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Ax\cos^4(c+dx)}{8} + \frac{3A\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5A\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{5Cx}{} \\ x(A+C\cos^2(c))\cos^4(c) \end{cases}$$

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2),x)
[Out] Piecewise((3*A*x*sin(c + d*x)**4/8 + 3*A*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*A*x*cos(c + d*x)**4/8 + 3*A*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*A*
sin(c + d*x)*cos(c + d*x)**3/(8*d) + 5*C*x*sin(c + d*x)**6/16 + 15*C*x*sin(
c + d*x)**4*cos(c + d*x)**2/16 + 15*C*x*sin(c + d*x)**2*cos(c + d*x)**4/16
+ 5*C*x*cos(c + d*x)**6/16 + 5*C*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*C*
sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*C*sin(c + d*x)*cos(c + d*x)**5/(
16*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3\,(dx+c)(6\,A+5\,C)+\frac{3\,(6\,A+5\,C)\tan(dx+c)^5+8\,(6\,A+5\,C)\tan(dx+c)^3+3\,(10\,A+11\,C)\tan(dx+c)}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] 1/48*(3*(d*x + c)*(6*A + 5*C) + (3*(6*A + 5*C)*tan(d*x + c)^5 + 8*(6*A + 5*
C)*tan(d*x + c)^3 + 3*(10*A + 11*C)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d
*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{1}{16}\,(6\,A+5\,C)x+\frac{C\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{(2\,A+3\,C)\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{(16\,A+15\,C)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] 1/16*(6*A + 5*C)*x + 1/192*C*sin(6*d*x + 6*c)/d + 1/64*(2*A + 3*C)*sin(4*d*
x + 4*c)/d + 1/64*(16*A + 15*C)*sin(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = x\left(\frac{3\,A}{8}+\frac{5\,C}{16}\right)+\frac{\left(\frac{3\,A}{8}+\frac{5\,C}{16}\right)\,\tan(c+d\,x)^5+\left(A+\frac{5\,C}{6}\right)\,\tan(c+d\,x)^3+\left(\frac{5\,A}{8}+\frac{11\,C}{16}\right)\,\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^6+3\tan\left(c+d\,x\right)^4+3\tan\left(c+d\,x\right)^2+1\right)}$$

```
[In] int(cos(c + d*x)^4*(A + C*cos(c + d*x)^2),x)
```

```
[Out] x*((3*A)/8 + (5*C)/16) + (tan(c + d*x)*((5*A)/8 + (11*C)/16) + tan(c + d*x)
^3*(A + (5*C)/6) + tan(c + d*x)^5*((3*A)/8 + (5*C)/16))/(d*(3*tan(c + d*x)^
2 + 3*tan(c + d*x)^4 + tan(c + d*x)^6 + 1))
```

3.11 $\int \cos^2(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^2(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \frac{1}{8}(4A+3C)x + \frac{(4A+3C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{C\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 1/8*(4*A+3*C)*x+1/8*(4*A+3*C)*cos(d*x+c)*sin(d*x+c)/d+1/4*C*cos(d*x+c)^3*si
n(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3093, 2715, 8}

$$\int \cos^2(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \frac{(4A+3C)\sin(c+dx)\cos(c+dx)}{8d} + \frac{1}{8}x(4A+3C) + \frac{C\sin(c+dx)\cos^3(c+dx)}{4d}$$

```
[In] Int[Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((4*A + 3*C)*x)/8 + ((4*A + 3*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (C*Cos[
c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(4A+3C)\int\cos^2(c+dx)\,dx\\
&= \frac{(4A+3C)\cos(c+dx)\sin(c+dx)}{8d}+\frac{C\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{8}(4A+3C)\int 1\,dx\\
&= \frac{1}{8}(4A+3C)x+\frac{(4A+3C)\cos(c+dx)\sin(c+dx)}{8d}+\frac{C\cos^3(c+dx)\sin(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx))}{32d}$$

```
[In] Integrate[Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2),x]
[Out] (4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*Sin[4*(c + d*x)])
/(32*d)
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(8A+8C)\sin(2dx+2c)+\sin(4dx+4c)C+16d\left(A+\frac{3C}{4}\right)x}{32d}$
risch	$\frac{xA}{2}+\frac{3Cx}{8}+\frac{\sin(4dx+4c)C}{32d}+\frac{\sin(2dx+2c)A}{4d}+\frac{\sin(2dx+2c)C}{4d}$
derivativedivides	$\frac{C\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{C\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{C\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
norman	$\frac{\left(\frac{A}{2}+\frac{3C}{8}\right)x+\left(2A+\frac{3C}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2A+\frac{3C}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3A+\frac{9C}{4}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{A}{2}+\frac{3C}{8}\right)x\left(\tan\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/32*((8*A+8*C)*sin(2*d*x+2*c)+sin(4*d*x+4*c)*C+16*d*(A+3/4*C)*x)/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(4\,A+3\,C)dx+\left(2\,C\cos\left(dx+c\right)^3+(4\,A+3\,C)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] 1/8*((4*A + 3*C)*d*x + (2*C*cos(d*x + c)^3 + (4*A + 3*C)*cos(d*x + c))*sin(
d*x + c))/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(53) = 106$.

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.59

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \begin{cases} \frac{Ax\sin^2(c+dx)}{2} + \frac{Ax\cos^2(c+dx)}{2} + \frac{A\sin(c+dx)\cos(c+dx)}{2d} + \frac{3Cx\sin^4(c+dx)}{8} + \frac{3Cx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Cx\cos^4(c+dx)}{8} + \\ x(A+C\cos^2(c))\cos^2(c) \end{cases}$$

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*x*sin(c + d*x)**2/2 + A*x*cos(c + d*x)**2/2 + A*sin(c + d*x)*c
os(c + d*x)/(2*d) + 3*C*x*sin(c + d*x)**4/8 + 3*C*x*sin(c + d*x)**2*cos(c +
 d*x)**2/4 + 3*C*x*cos(c + d*x)**4/8 + 3*C*sin(c + d*x)**3*cos(c + d*x)/(8*
d) + 5*C*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + C*cos(c)**2
)*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(dx+c)(4A+3C) + \frac{(4A+3C)\tan(dx+c)^3+(4A+5C)\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((d*x + c)*(4*A + 3*C) + ((4*A + 3*C)*tan(d*x + c)^3 + (4*A + 5*C)*tan(
d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{1}{8}(4A+3C)x + \frac{C\sin(4dx+4c)}{32d} + \frac{(A+C)\sin(2dx+2c)}{4d}$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*A + 3*C)*x + 1/32*C*sin(4*d*x + 4*c)/d + 1/4*(A + C)*sin(2*d*x + 2*c
)/d
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=x\left(\frac{A}{2}+\frac{3\,C}{8}\right)+\frac{\left(\frac{A}{2}+\frac{3\,C}{8}\right)\,\tan(c+d\,x)^3+\left(\frac{A}{2}+\frac{5\,C}{8}\right)\,\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2),x)
[Out] x*(A/2 + (3*C)/8) + (tan(c + d*x)*(A/2 + (5*C)/8) + tan(c + d*x)^3*(A/2 + (
3*C)/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.12 $\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx=Cx+\frac{A\tan(c+dx)}{d}$$

```
[Out] C*x+A*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3091, 8}

$$\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx=\frac{A\tan(c+dx)}{d}+Cx$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] C*x + (A*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{A\tan(c+dx)}{d} + C\int 1\,dx \\
&= Cx + \frac{A\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = Cx + \frac{A\tan(c+dx)}{d}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] C*x + (A*Tan[c + d*x])/d
```

Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{A\tan(dx+c)+C(dx+c)}{d}$
default	$\frac{A\tan(dx+c)+C(dx+c)}{d}$
parts	$\frac{A\tan(dx+c)}{d} + \frac{C(dx+c)}{d}$
risch	$Cx + \frac{2iA}{d\left(e^{2i(dx+c)}+1\right)}$
parallelrisch	$\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)xdC-dxC-2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$
norman	$\frac{Cx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+Cx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-Cx-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-Cx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*tan(d*x+c)+C*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{Cdx\cos\left(dx+c\right)+A\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")
[Out] (C*d*x*cos(d*x + c) + A*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(A+C\cos^2\left(c+dx\right)\right)\sec^2\left(c+dx\right)dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{(dx+c)C+A\tan\left(dx+c\right)}{d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")
[Out] ((d*x + c)*C + A*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{(dx+c)C+A\tan\left(dx+c\right)}{d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
[Out] ((d*x + c)*C + A*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{A\tan(c+d\,x)+C\,d\,x}{d}$$

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^2,x)
[Out] (A*tan(c + d*x) + C*d*x)/d
```

3.13 $\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx=\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\sec^2(c+dx)\tan(c+dx)}{3d}$$

```
[Out] 1/3*(2*A+3*C)*tan(d*x+c)/d+1/3*A*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3091, 3852, 8}

$$\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx=\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] ((2*A + 3*C)*Tan[c + d*x])/(3*d) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{A\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}(2A+3C)\int \sec^2(c+dx)\,dx \\
&= \frac{A\sec^2(c+dx)\tan(c+dx)}{3d} - \frac{(2A+3C)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{3d} \\
&= \frac{(2A+3C)\tan(c+dx)}{3d} + \frac{A\sec^2(c+dx)\tan(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$
\int \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{C\tan(c+dx)}{d} + \frac{A\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}
$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
[Out] (C*Tan[c + d*x])/d + (A*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	siz
derivativedivides	$\frac{-A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+C\tan(dx+c)}{d}$	35
default	$\frac{-A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+C\tan(dx+c)}{d}$	35
parts	$-\frac{A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}+\frac{C\tan(dx+c)}{d}$	37
parallelrisch	$\frac{(2A+3C)\sin(3dx+3c)+6\sin(dx+c)\left(A+\frac{C}{2}\right)}{3d(\cos(3dx+3c)+3\cos(dx+c))}$	57
risch	$\frac{2i\left(3C\,e^{4i(dx+c)}+6A\,e^{2i(dx+c)}+6C\,e^{2i(dx+c)}+2A+3C\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$	63
norman	$\frac{-\frac{8A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{8A\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4(A-3C)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2(A+C)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	12

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
[Out] 1/d*(-A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+C*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^2+A\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \left(A + C\cos^2(c + dx)\right)\sec^4(c + dx)\,dx = \int \left(A + C\cos^2\left(c + dx\right)\right)\sec^4\left(c + dx\right)dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \left(A + C\cos^2(c + dx)\right)\sec^4(c + dx)\,dx = \frac{A\tan\left(dx + c\right)^3 + 3\left(A + C\right)\tan\left(dx + c\right)}{3\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")
[Out] 1/3*(A*tan(d*x + c)^3 + 3*(A + C)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{A\tan\left(dx + c\right)^3 + 3\,A\tan\left(dx + c\right) + 3\,C\tan\left(dx + c\right)}{3\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")
[Out] 1/3*(A*tan(d*x + c)^3 + 3*A*tan(d*x + c) + 3*C*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \left(A + C\cos^2(c + dx)\right)\sec^4(c + dx)\,dx = \frac{A\tan(c + d\,x)^3}{3\,d} + \frac{\tan(c + d\,x)\,\left(A + C\right)}{d}$$

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^4,x)
[Out] (A*tan(c + d*x)^3)/(3*d) + (tan(c + d*x)*(A + C))/d
```

3.14 $\int (A+C\cos^2(c+dx))\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int (A+C\cos^2(c+dx))\sec^6(c+dx)\,dx = \frac{(4A+5C)\tan(c+dx)}{5d} + \frac{A\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{(4A+5C)\tan^3(c+dx)}{15d}$$

```
[Out] 1/5*(4*A+5*C)*tan(d*x+c)/d+1/5*A*sec(d*x+c)^4*tan(d*x+c)/d+1/15*(4*A+5*C)*t
an(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3091, 3852}

$$\int (A+C\cos^2(c+dx))\sec^6(c+dx)\,dx = \frac{(4A+5C)\tan^3(c+dx)}{15d} + \frac{(4A+5C)\tan(c+dx)}{5d} + \frac{A\tan(c+dx)\sec^4(c+dx)}{5d}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] ((4*A + 5*C)*Tan[c + d*x])/(5*d) + (A*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) +
((4*A + 5*C)*Tan[c + d*x]^3)/(15*d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{A\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{5}(4A+5C)\int \sec^4(c+dx)\,dx \\
&= \frac{A\sec^4(c+dx)\tan(c+dx)}{5d} - \frac{(4A+5C)\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{5d} \\
&= \frac{(4A+5C)\tan(c+dx)}{5d} + \frac{A\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{(4A+5C)\tan^3(c+dx)}{15d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{C\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d} + \frac{A\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
[Out] (C*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d + (A*(Tan[c + d*x] + (2*Tan[c + d*x
]^3)/3 + Tan[c + d*x]^5/5))/d
```

Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-A\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)-C\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{-A\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)-C\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
parts	$-\frac{A\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)}{d}-\frac{C\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
parallelrisch	$\frac{(40A+50C)\sin(3dx+3c)+(8A+10C)\sin(5dx+5c)+80\sin(dx+c)\left(A+\frac{C}{2}\right)}{15d(\cos(5dx+5c)+5\cos(3dx+3c)+10\cos(dx+c))}$
risch	$\frac{4i\left(15C\,\mathrm{e}^{6i(dx+c)}+40A\,\mathrm{e}^{4i(dx+c)}+35C\,\mathrm{e}^{4i(dx+c)}+20A\,\mathrm{e}^{2i(dx+c)}+25C\,\mathrm{e}^{2i(dx+c)}+4A+5C\right)}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}$
norman	$\frac{-\frac{4(A-C)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4(A-C)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2(A+C)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(11A-5C)\left(\tan^{5}\left(\frac{dx}{2}\right.\right.}{5d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}$

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,method=_RETURNVERBOSE)

[Out] 1/d*(-A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-C*(-2/3-1/3*s
ec(d*x+c)^2)*tan(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{\left(2\,(4\,A+5\,C)\cos\left(dx+c\right)^4+(4\,A+5\,C)\cos\left(dx+c\right)^2+3\,A\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas")

[Out] 1/15*(2*(4*A + 5*C)*cos(d*x + c)^4 + (4*A + 5*C)*cos(d*x + c)^2 + 3*A)*sin(
d*x + c)/(d*cos(d*x + c)^5)

Sympy [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{3\,A\tan\left(dx+c\right)^5+5\,(2\,A+C)\tan\left(dx+c\right)^3+15\,(A+C)\tan\left(dx+c\right)}{15\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="maxima")
[Out] 1/15*(3*A*tan(d*x + c)^5 + 5*(2*A + C)*tan(d*x + c)^3 + 15*(A + C)*tan(d*x
+ c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{3\,A\tan\left(dx+c\right)^5+10\,A\tan\left(dx+c\right)^3+5\,C\tan\left(dx+c\right)^3+15\,A\tan\left(dx+c\right)+15\,C\tan\left(dx+c\right)}{15\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="giac")
[Out] 1/15*(3*A*tan(d*x + c)^5 + 10*A*tan(d*x + c)^3 + 5*C*tan(d*x + c)^3 + 15*A*
tan(d*x + c) + 15*C*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int (A + C\cos^2(c + dx))\sec^6(c + dx)\,dx$$
$$= \frac{\frac{A\tan(c+d\,x)^5}{5} + \left(\frac{2\,A}{3} + \frac{C}{3}\right)\tan(c + d\,x)^3 + (A + C)\tan(c + d\,x)}{d}$$

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^6,x)
[Out] ((A*tan(c + d*x)^5)/5 + tan(c + d*x)*(A + C) + tan(c + d*x)^3*((2*A)/3 + C/
3))/d
```

3.15 $\int (A+C\cos^2(c+dx))\sec^8(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int (A+C\cos^2(c+dx))\sec^8(c+dx)\,dx=\frac{(6A+7C)\tan(c+dx)}{7d}+\frac{A\sec^6(c+dx)\tan(c+dx)}{7d}+\frac{2(6A+7C)\tan^3(c+dx)}{21d}+\frac{(6A+7C)\tan^5(c+dx)}{35d}$$

```
[Out] 1/7*(6*A+7*C)*tan(d*x+c)/d+1/7*A*sec(d*x+c)^6*tan(d*x+c)/d+2/21*(6*A+7*C)*t
an(d*x+c)^3/d+1/35*(6*A+7*C)*tan(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3091, 3852}

$$\int (A+C\cos^2(c+dx))\sec^8(c+dx)\,dx=\frac{(6A+7C)\tan^5(c+dx)}{35d}+\frac{2(6A+7C)\tan^3(c+dx)}{21d}+\frac{(6A+7C)\tan(c+dx)}{7d}+\frac{A\tan(c+dx)\sec^6(c+dx)}{7d}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]
```

```
[Out] ((6*A + 7*C)*Tan[c + d*x])/(7*d) + (A*Sec[c + d*x]^6*Tan[c + d*x])/(7*d) +
(2*(6*A + 7*C)*Tan[c + d*x]^3)/(21*d) + ((6*A + 7*C)*Tan[c + d*x]^5)/(35*d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \frac{A\sec^6(c+dx)\tan(c+dx)}{7d} + \frac{1}{7}(6A+7C)\int \sec^6(c+dx)\,dx$$

$$= \frac{A\sec^6(c+dx)\tan(c+dx)}{7d} - \frac{(6A+7C)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(c+dx)\right)}{7d}$$

$$= \frac{(6A+7C)\tan(c+dx)}{7d} + \frac{A\sec^6(c+dx)\tan(c+dx)}{7d} + \frac{2(6A+7C)\tan^3(c+dx)}{21d} + \frac{(6A+7C)\tan^5(c+dx)}{35d}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx = \frac{C\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d} + \frac{A\left(\tan(c+dx)+\tan^3(c+dx)+\frac{3}{5}\tan^5(c+dx)+\frac{1}{7}\tan^7(c+dx)\right)}{d}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]
```

```
[Out] (C*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d + (A*(Tan[c
+ d*x] + Tan[c + d*x]^3 + (3*Tan[c + d*x]^5)/5 + Tan[c + d*x]^7/7))/d
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-A\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)-C\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+}{d}$
default	$\frac{-A\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)-C\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+}{d}$
parts	$-\frac{A\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)}{d}-\frac{C\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(}{d}$
risch	$\frac{16i\left(70C\,e^{8i(dx+c)}+210A\,e^{6i(dx+c)}+175C\,e^{6i(dx+c)}+126A\,e^{4i(dx+c)}+147C\,e^{4i(dx+c)}+42A\,e^{2i(dx+c)}+49C\,e^{2i(dx+c)}+6A}{105d\left(e^{2i(dx+c)}+1\right)^7}$
parallelrisch	$\frac{(1008A+1176C)\sin(3dx+3c)+(336A+392C)\sin(5dx+5c)+(48A+56C)\sin(7dx+7c)+1680\sin(dx+c)\left(A+\frac{C}{2}\right)}{105d(\cos(7dx+7c)+7\cos(5dx+5c)+21\cos(3dx+3c)+35\cos(dx+c))}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d
*x+c)-C*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx$$
$$=\frac{\left(8\,(6\,A+7\,C)\cos{(dx+c)}^6+4\,(6\,A+7\,C)\cos{(dx+c)}^4+3\,(6\,A+7\,C)\cos{(dx+c)}^2+15\,A\right)\sin{(dx+c)}}{105\,d\cos{(dx+c)}^7}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(8*(6*A + 7*C)*cos(d*x + c)^6 + 4*(6*A + 7*C)*cos(d*x + c)^4 + 3*(6*A
 + 7*C)*cos(d*x + c)^2 + 15*A)*sin(d*x + c)/(d*cos(d*x + c)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \left(A+C\cos ^2(c+dx)\right)\sec ^8(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**8,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \left(A+C\cos ^2(c+dx)\right)\sec ^8(c+dx)\,dx$$
$$=\frac{15\,A\tan \left(dx+c\right)^7+21\,(3\,A+C)\tan \left(dx+c\right)^5+35\,(3\,A+2\,C)\tan \left(dx+c\right)^3+105\,(A+C)\tan \left(dx+c\right)}{105\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="maxima")
[Out] 1/105*(15*A*tan(d*x + c)^7 + 21*(3*A + C)*tan(d*x + c)^5 + 35*(3*A + 2*C)*t
an(d*x + c)^3 + 105*(A + C)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \left(A+C\cos ^2(c+dx)\right)\sec ^8(c+dx)\,dx$$
$$=\frac{15\,A\tan \left(dx+c\right)^7+63\,A\tan \left(dx+c\right)^5+21\,C\tan \left(dx+c\right)^5+105\,A\tan \left(dx+c\right)^3+70\,C\tan \left(dx+c\right)^3+\cdots}{105\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="giac")
[Out] 1/105*(15*A*tan(d*x + c)^7 + 63*A*tan(d*x + c)^5 + 21*C*tan(d*x + c)^5 + 10
5*A*tan(d*x + c)^3 + 70*C*tan(d*x + c)^3 + 105*A*tan(d*x + c) + 105*C*tan(d
*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx$$
$$=\frac{\frac{A\tan(c+d\,x)^7}{7}+\left(\frac{3\,A}{5}+\frac{C}{5}\right)\,\tan(c+d\,x)^5+\left(A+\frac{2\,C}{3}\right)\,\tan(c+d\,x)^3+(A+C)\,\tan(c+d\,x)}{d}$$

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^8,x)
```

```
[Out] ((A*tan(c + d*x)^7)/7 + tan(c + d*x)^3*(A + (2*C)/3) + tan(c + d*x)*(A + C)
 + tan(c + d*x)^5*((3*A)/5 + C/5))/d
```

3.16 $\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) dx = \frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}$$

```
[Out] 2/45*b*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*C*(b*cos(d*x+c))^(7/
2)*sin(d*x+c)/b/d+2/15*b^2*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3093, 2715, 2721, 2719}

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) dx = \frac{2b^2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{2b(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sq
rt[Cos[c + d*x]]) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
45*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{1}{9}(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx \\
&= \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} \\
&\quad + \frac{1}{15}\left(b^2(9A+7C)\right)\int \sqrt{b\cos(c+dx)}\,dx \\
&= \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} \\
&\quad + \frac{\left(b^2(9A+7C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}} \\
&= \frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{5/2}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}(18A+19C+5C}{180d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[
Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*
d*Cos[c + d*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(125) = 250.

Time = 14.49 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-72A}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos\right.\right.}}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-160*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+13
6*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d
*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) dx = \frac{3i\sqrt{2}(9A+7C)b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/45*(3*I*sqrt(2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(9*A + 7*C)*b^(5
/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(5*C*b^2*cos(d*x + c)^3 + (9*A + 7*C)*b^2*cos(d*x + c))*sqrt
(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{5}{2}} dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2), x)
```

3.17 $\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\,dx = \frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/21*b^2*(7*A+5*C)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/21*b*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x
+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3093, 2715, 2721, 2720}

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\,dx = \frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21
*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{1}{7}(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx \\
&= \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} \\
&\quad + \frac{1}{21}\left(b^2(7A+5C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} \\
&\quad + \frac{\left(b^2(7A+5C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}} \\
&= \frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{3/2}\left(4(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sqrt{\cos(c+dx)}(14A+13C\right.}{42d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[C
os[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*Cos[
c + d*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(125) = 250$.

Time = 9.92 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.62

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(28A+56C)\left(\right.\right.}{21\sqrt{-b}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+
1/2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int (b\cos(c+dx))^{3/2} (A +C\cos^2(c+dx))\ dx = \frac{-i\sqrt{2}(7A+5C)b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(}{}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] 1/21*(-I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*b*cos(d*x + c)^2 + (7*A + 5*C)*
b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\ dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Maxima [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\ dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{3}{2}}\ dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2), x)
```

3.18 $\int \sqrt{b \cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3093, 2721, 2719}

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx = \frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Co
s[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{1}{5}(5A+3C)\int\sqrt{b\cos(c+dx)}\,dx\\ &= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}\\ &= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos
[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(93) = 186$.

Time = 8.71 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.39

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c}{2}}}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}}{}$

```
[In] int((cos(d*x+c)*b)^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(8*cos(1/2*
d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{2\sqrt{b\cos(dx+c)}C\cos(dx+c)\sin(dx+c)+\sqrt{2}(5iA+3iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInv}}{}$$

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(5*I*A +
3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

3.19 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(d*
x+c)*(b*cos(d*x+c))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3093, 2721, 2720}

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{1}{3}(3A+C)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int\frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*
x)])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(91) = 182$.

Time = 5.78 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.15

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\right.\right.\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\right.}$
parts	$\frac{2A\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\|\sqrt{2}\right)}{d\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\right.\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}}$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x
+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(3i\,A+i\,C)\sqrt{b}\text{weierstr}}{3\,bd}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\, dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{\sqrt{b\cos(dx + c)}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{\sqrt{b\cos(dx + c)}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\, dx = \frac{2C\sin(c + dx)\sqrt{b\cos(c + dx)}}{3bd} + \frac{2A\sqrt{\cos(c + dx)}\,\mathrm{F}\left(\frac{c}{2} + \frac{dx}{2}\middle|2\right)}{d\sqrt{b\cos(c + dx)}} + \frac{2C\sqrt{\cos(c + dx)}\,\mathrm{F}\left(\frac{c}{2} + \frac{dx}{2}\middle|2\right)}{3d\sqrt{b\cos(c + dx)}}$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/2),x)
[Out] (2*C*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2)
*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*C*cos(c + d*x
)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))
```

3.20 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3091, 2721, 2719}

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos
[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{(A-C)\int\sqrt{b\cos(c+dx)}\,dx}{b^2} \\
&= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}} \\
&= -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int\frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x]
)/(b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(94) = 188.

Time = 7.55 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.92

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$-\frac{2A\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\right.\right.}}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}}$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2
*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-i\,A+i\,C)\sqrt{b}\cos{(dx+c)}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,\ldots}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A - I*C)*sqr
t(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^2*d*co
s(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\left(b\,\cos\left(c + d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2), x)
```

3.21 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3091, 2721, 2720}

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}\\ &= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left((A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
[Out] (2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x]
))/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(94) = 188$.

Time = 7.07 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.77

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+3C)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+A\sqrt{\cdots}\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\cdots\right.\right.\right.}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx\cdots}{2}}\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\cdots}$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/b^2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}(-i\,A-3i\,C)\sqrt{b}\cos\left(dx+c\right)^2 \text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\text{si}\cdots}{\cdots}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*
cos(d*x + c))*A*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos(c+d\,x)^2 + A}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2), x)
```

3.22 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^3/d/(b
*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^4/d/cos(
d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3091, 2716, 2721, 2719}

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = -\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sq
rt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (2*
(3*A + 5*C)*Sin[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{(3A+5C)\int\frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b^2}\\
&= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}-\frac{(3A+5C)\int\sqrt{b\cos(c+dx)}\,dx}{5b^4}\\
&= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}\\
&\quad-\frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b^4\sqrt{\cos(c+dx)}}\\
&= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(3A+5C)\sin(c+dx)+A\,s}{5b^3d\sqrt{b\cos(c+dx)}}$$

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]

[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(127) = 254.

Time = 11.54 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.93

method	result
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}E\left(\right.\right.}{}$
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\right.}{}$

[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(7/2),x,method=_RETURNVERBOSE)

[Out] -2/5*A*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d-2*C/b^3*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\,dx = \frac{\sqrt{2}(-3i\,A - 5i\,C)\sqrt{b}\cos\left(dx + c\right)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^4*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\, dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{7}{2}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\, dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\left(b\,\cos\left(c + d\,x\right)\right)^{7/2}}\, dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2), x)
```

3.23 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx=\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^4d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(7/2)+2/21*(5*A+7*C)*sin(d*x+c)/b^3/d/(
b*cos(d*x+c))^(3/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(b*cos
(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3091, 2716, 2721, 2720}

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx=\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^4d\sqrt{b\cos(c+dx)}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^4*d*Sqrt
[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (2*
(5*A + 7*C)*Sin[c + d*x])/(21*b^3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{(5A+7C)\int\frac{1}{(b\cos(c+dx))^{5/2}}\,dx}{7b^2} \\
&= \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{(5A+7C)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b^4} \\
&= \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{\left((5A+7C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^4\sqrt{b\cos(c+dx)}} \\
&= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^4d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.67

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\,dx = \frac{2\Big((5A + 7C)\sqrt{\cos(c + dx)}\,\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + (5A + 7C + 3A\sec^2(c + d}{21b^4d\sqrt{b\cos(c + dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C +
 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^4*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(127) = 254$.

Time = 10.32 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.60

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{6b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2(\cos}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{d}{2}\right.\right.\right.}}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(9/2),x,method=_RETURNVERBOSE)
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4*(C*(-1/6
*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b
*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x
+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{\sqrt{2}(-5i\,A-7i\,C)\sqrt{b}\cos{(dx+c)}^4\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\mathrm{s}}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^5*d*
cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \int \frac{C\cos{(dx+c)}^2+A}{(b\cos{(dx+c)})^{\frac{9}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\,dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\,dx = \int \frac{C\cos(c + d\,x)^2 + A}{(b\,\cos(c + d\,x))^{9/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2), x)
```

3.24 $\int \sqrt{\cos(c+dx)}(3-5\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \sqrt{\cos(c+dx)}(3-5\cos^2(c+dx))\,dx = -\frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}$$

```
[Out] -2*cos(d*x+c)^(3/2)*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3090}

$$\int \sqrt{\cos(c+dx)}(3-5\cos^2(c+dx))\,dx = -\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d}$$

```
[In] Int[Sqrt[Cos[c + d*x]]*(3 - 5*Cos[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/d
```

Rule 3090

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\text{integral} = -\frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx=-\frac{\sqrt{\cos(c+dx)}\sin(2(c+dx))}{d}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(3 - 5*Cos[c + d*x]^2),x]
[Out] -((Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)])/d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(19) = 38.

Time = 6.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.71

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$
parts	$\frac{6\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}+\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}}{}$

```
[In] int((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -4*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/sin(1/2*d*x+1/2*c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx=-\frac{2\,\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)}{d}$$

```
[In] integrate((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*cos(d*x + c)^(3/2)*sin(d*x + c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((3-5*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx = \int -\left(5\,\cos\left(dx+c\right)^2-3\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
[Out] -integrate((5*cos(d*x + c)^2 - 3)*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx = \int -\left(5\,\cos\left(dx+c\right)^2-3\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(-(5*cos(d*x + c)^2 - 3)*sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx = -\int \sqrt{\cos\left(c+d\,x\right)}\,\left(5\cos\left(c+d\,x\right)^2-3\right)\,dx$$

```
[In] int(-cos(c + d*x)^(1/2)*(5*cos(c + d*x)^2 - 3),x)
[Out] -int(cos(c + d*x)^(1/2)*(5*cos(c + d*x)^2 - 3), x)
```

3.25 $\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{d}$$

```
[Out] -2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3090}

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

```
[In] Int[(1 - 3*Cos[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
[Out] (-2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d
```

Rule 3090

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{d}$$

```
[In] Integrate[(1 - 3*Cos[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
[Out] (-2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(19) = 38.

Time = 4.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.71

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
parts	$\frac{2\,\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert\sqrt{2}\right)}{d}+\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(\frac{dx}{2}+\frac{c}{2}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2(\cos^2}}$

```
[In] int((-3*cos(d*x+c)^2+1)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -4*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=-\frac{2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d}$$

```
[In] integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((1-3*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int -\frac{3\cos(dx+c)^2-1}{\sqrt{\cos(dx+c)}}\,dx$$

```
[In] integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
[Out] -integrate((3*cos(d*x + c)^2 - 1)/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int -\frac{3\cos(dx+c)^2-1}{\sqrt{\cos(dx+c)}}\,dx$$

```
[In] integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(-(3*cos(d*x + c)^2 - 1)/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{d}$$

```
[In] int(-(3*cos(c + d*x)^2 - 1)/cos(c + d*x)^(1/2),x)
[Out] -(2*cos(c + d*x)^(1/2)*sin(c + d*x))/d
```

3.26 $\int \left(A+C\cos^2(c+dx)\right) (b\sec(c+dx))^{9/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \left(A+C\cos^2(c+dx)\right) (b\sec(c+dx))^{9/2}\,dx=\frac{2b^4(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^3(5A+7C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2Ab^2(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}$$

```
[Out] 2/21*b^3*(5*A+7*C)*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/21*b^4*(5*A+7*C)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d+2/7*A*b^2*(b*sec(d*x+c))^(5
/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3317, 4131, 3853, 3856, 2720}

$$\int \left(A+C\cos^2(c+dx)\right) (b\sec(c+dx))^{9/2}\,dx=\frac{2b^4(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^3(5A+7C)\sin(c+dx)(b\sec(c+dx))^{3/2}}{21d}+\frac{2Ab^2\tan(c+dx)(b\sec(c+dx))^{5/2}}{7d}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(9/2),x]
```

```
[Out] (2*b^4*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[
c + d*x]])/(21*d) + (2*b^3*(5*A + 7*C)*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])
/(21*d) + (2*A*b^2*(b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int (b\sec(c+dx))^{5/2} \left(C + A\sec^2(c+dx)\right) \, dx \\
&= \frac{2Ab^2(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{1}{7}\left(b^2(5A+7C)\right) \int (b\sec(c+dx))^{5/2} \, dx \\
&= \frac{2b^3(5A+7C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} \\
&\quad + \frac{2Ab^2(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{1}{21}\left(b^4(5A+7C)\right) \int \sqrt{b\sec(c+dx)} \, dx
\end{aligned}$$

$$= \frac{2b^3(5A+7C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2Ab^2(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{1}{21}\left(b^4(5A+7C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$= \frac{2b^4(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2b^3(5A+7C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2Ab^2(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}$$

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{9/2}\,dx = \frac{b^2(b\sec(c+dx))^{5/2}\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C)\sin(2(c+ \right.}{21d}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(9/2),x]
```

```
[Out] (b^2*(b*Sec[c + d*x])^(5/2)*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c
+ d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))/(21*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 72.99 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.53

method	result
default	$-\frac{2b^4\sqrt{b\sec(dx+c)}\left(5i\cos(dx+c)AF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7i\cos(dx+c)CF(i(\csc(dx+c)-\cot\right.}{}$
parts	$-\frac{2iA\sqrt{b\sec(dx+c)}\,b^4\left(5\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+5F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\right.}{21d}$

```
[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*b^4/d*(b*sec(d*x+c))^(1/2)*(5*I*cos(d*x+c)*A*EllipticF(I*(csc(d*x+c)-
cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7
*I*cos(d*x+c)*C*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*I*A*EllipticF(I*(csc(d*x+c)-cot(d*
x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*I*C*E
llipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)-5*A*tan(d*x+c)-7*tan(d*x+c)*C-3*tan(d*x+c)*sec(d*x+c)
^2*A)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.23

$$\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{9/2} dx = \frac{-i\sqrt{2}(5A + 7C)b^{\frac{9}{2}} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + i\cdots}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-I*sqrt(2)*(5*A + 7*C)*b^(9/2)*cos(d*x + c)^3*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(5*A + 7*C)*b^(9/2)*cos(d*x
+ c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((5*A
+ 7*C)*b^4*cos(d*x + c)^2 + 3*A*b^4)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{9/2} dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{9/2} dx = \int (C\cos(dx + c)^2 + A)(b\sec(dx + c))^{\frac{9}{2}} dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(9/2), x)
```

Giac [F]

$$\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{9/2} dx = \int (C\cos(dx + c)^2 + A)(b\sec(dx + c))^{\frac{9}{2}} dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{9/2} dx = \int (C\cos(c + d x)^2+A) \left(\frac{b}{\cos(c + d x)}\right)^{9/2} dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(9/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(9/2), x)
```

3.27 $\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{7/2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{7/2} \, dx = -\frac{2b^4(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^3(3A+5C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2Ab^2(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

```
[Out] -2/5*b^4*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/5*b
^3*(3*A+5*C)*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+2/5*A*b^2*(b*sec(d*x+c))^(3/
2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3317, 4131, 3853, 3856, 2719}

$$\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{7/2} \, dx = -\frac{2b^4(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^3(3A+5C)\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d} + \frac{2Ab^2\tan(c+dx)(b\sec(c+dx))^{3/2}}{5d}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*b^4*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt
[b*Sec[c + d*x]]) + (2*b^3*(3*A + 5*C)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(
5*d) + (2*A*b^2*(b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int (b\sec(c+dx))^{3/2} \left(C + A\sec^2(c+dx)\right) \, dx \\
&= \frac{2Ab^2(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{1}{5}\left(b^2(3A+5C)\right) \int (b\sec(c+dx))^{3/2} \, dx \\
&= \frac{2b^3(3A+5C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2Ab^2(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} \\
&\quad - \frac{1}{5}\left(b^4(3A+5C)\right) \int \frac{1}{\sqrt{b\sec(c+dx)}} \, dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2b^3(3A+5C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad + \frac{2Ab^2(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} - \frac{\left(b^4(3A+5C)\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= -\frac{2b^4(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^3(3A+5C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad + \frac{2Ab^2(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$
\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{7/2}\,dx =
$$

$$
-\frac{b^2(b\sec(c+dx))^{3/2}\left(2(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-(3A+5C)\sin(2(c+dx))-2A\tan(c+d\right.}{5d}
$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] -1/5*(b^2*(b*Sec[c + d*x])^(3/2)*(2*(3*A + 5*C)*Cos[c + d*x]^(3/2)*Elliptic
E[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d*x]))/d
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 72.59 (sec) , antiderivative size = 798, normalized size of antiderivative = 6.94

method	result	size
default	Expression too large to display	798
parts	Expression too large to display	811

```
[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*b^3/d*(b*sec(d*x+c))^(1/2)/(1+cos(d*x+c))*(-5*I*C*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I
)*cos(d*x+c)^2-6*I*cos(d*x+c)*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+3*I*A*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)+5*I*C*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c))
,I)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+6*I*cos(
```

```
d*x+c)*A*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-10*I*cos(d*x+c)*C*(1/(1+cos(d*x+c)))^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+3*
I*A*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+10*I*cos(d*x+c)*C*EllipticF(I*(csc(
d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)-3*I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elli
pticE(I*(csc(d*x+c)-cot(d*x+c)),I)*cos(d*x+c)^2-5*I*C*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I
)+5*I*C*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+3*A*sin(d*x+c)+5*sin(d*x+c)*C+A
*tan(d*x+c)+tan(d*x+c)*sec(d*x+c)*A)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.28

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{7/2}\,dx = \frac{-i\sqrt{2}(3\,A+5\,C)b^{\frac{7}{2}}\cos{(dx+c)}^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+}}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-I*sqrt(2)*(3*A + 5*C)*b^(7/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(3*A
+ 5*C)*b^(7/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*b^3*cos(d*x + c)^2 +
A*b^3)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{7/2}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{7/2}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{7/2}\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{7/2}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(7/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(7/2), x)
```

3.28 $\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{5/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{5/2} dx = \frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2Ab^2\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d}$$

```
[Out] 2/3*b^2*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d+2/3*A*b^
2*(b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3317, 4131, 3856, 2720}

$$\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{5/2} dx = \frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2Ab^2\tan(c+dx)\sqrt{b\sec(c+dx)}}{3d}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c
+ d*x]])/(3*d) + (2*A*b^2*Sqrt[b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \sqrt{b \sec(c+dx)}\left(C + A\sec^2(c+dx)\right)\, dx \\
&= \frac{2Ab^2\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{1}{3}\left(b^2(A+3C)\right)\int \sqrt{b\sec(c+dx)}\,dx \\
&= \frac{2Ab^2\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d} \\
&\quad + \frac{1}{3}\left(b^2(A+3C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} \\
&\quad + \frac{2Ab^2\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{5/2} dx = \frac{2b^2\sqrt{b\sec(c + dx)}\left((A + 3C)\sqrt{\cos(c + dx)}\,\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + A\tan(c + dx)\right)}{3d}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[b*Sec[c + d*x]]*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] + A*Tan[c + d*x]))/(3*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 70.41 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.90

method	result
parts	$-\frac{2A\sqrt{b\sec(dx+c)}\,b^2\left(iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)+iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{1}}\right.}{3d}$
default	$\frac{2b^2\sqrt{b\sec(dx+c)}\left(iA\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}F(i(\cot(dx+c)-\csc(dx+c)),i)\cos(dx+c)+3iC\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}F(i\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*A/d*(b*sec(d*x+c))^(1/2)*b^2*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)
*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+I*El
lipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)-tan(d*x+c))-2*I*C*b^2/d*(1+cos(d*x+c))*EllipticF(I*(cs
c(d*x+c)-cot(d*x+c)),I)*(b*sec(d*x+c))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{5/2} dx = \frac{-i\sqrt{2}(A + 3C)b^{\frac{5}{2}}\cos(dx + c)\,\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + i\sqrt{}}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*A*b^2*sqrt(b/co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{5/2}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{5/2}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{5/2}\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(5/2), x)
```

3.29 $\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{3/2}\,dx = -\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

```
[Out] -2*b^2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*A*b^2*tan
(d*x+c)/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3317, 4131, 3856, 2719}

$$\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{3/2}\,dx = \frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}-\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*b^2*(A - C)*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec
[c + d*x]]) + (2*A*b^2*Tan[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \frac{C + A\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}} - \left(b^2(A-C)\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}} - \frac{\left(b^2(A-C)\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= -\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{3/2}\,dx=\frac{2b\sqrt{b\sec(c+dx)}\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d}$$

[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(3/2),x]

[Out] (2*b*Sqrt[b*Sec[c + d*x]]*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c + d*x]))/d

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 17.09 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.23

method	result
default	$\frac{2b\sqrt{b\sec(dx+c)}\left(i(-\cos(dx+c)-1)A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,E(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{1+\cos(dx+c)}}+i(1+\cos(dx+c))C\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,E\right.}{}$
parts	$\frac{2A\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,F(i(\csc(dx+c)-\cot(dx+c)),i)\left(\cos^2(dx+c)\right)-i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,E(i(\csc(dx+c)-\cot(d\right.}{}$

[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2*b/d*(b*sec(d*x+c))^(1/2)*(I*(-cos(d*x+c)-1)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)+I*(1+cos(d*x+c))*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)+I*(1+cos(d*x+c))*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)+I*(-cos(d*x+c)-1)*C*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)+A*(csc(d*x+c)-cot(d*x+c))+cot(d*x+c)*(1-cos(d*x+c))*C)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{3/2}\,dx = \frac{-i\sqrt{2}(A-C)b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] (-I*sqrt(2)*(A - C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(A - C)*b^(3/2)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*A
*b*sqrt(b/cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{3/2}\,dx = \int (C\cos(dx+c)^2+A)(b\sec(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \left(A + C\cos^2(c + dx)\right)\left(b\sec(c + dx)\right)^{3/2}\,dx = \int \left(C\cos\left(dx + c\right)^2 + A\right)\left(b\sec\left(dx + c\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)\left(b\sec(c+dx)\right)^{3/2}\,dx = \int \left(C\cos\left(c + d\,x\right)^2+A\right)\left(\frac{b}{\cos\left(c + d\,x\right)}\right)^{3/2}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(3/2), x)
```

3.30 $\int (A+C\cos^2(c+dx)) \sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int (A+C\cos^2(c+dx)) \sqrt{b\sec(c+dx)}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d+2/3*b^2*C*ta
n(d*x+c)/d/(b*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3317, 4130, 3856, 2720}

$$\int (A+C\cos^2(c+dx)) \sqrt{b\sec(c+dx)}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*
x]])/(3*d) + (2*b^2*C*Tan[c + d*x])/(3*d*(b*Sec[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \frac{C + A\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx \\
&= \frac{2b^2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}} + \frac{1}{3}(3A+C)\int \sqrt{b\sec(c+dx)}\,dx \\
&= \frac{2b^2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}} + \frac{1}{3}\left((3A+C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2b^2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int (A+C\cos^2(c+dx))\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))\right)}{3d}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2] + C*Sin[2*(c + d*x)]))/(3*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.64 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.01

method	result
parts	$-\frac{2iA(1+\cos(dx+c))F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{b\sec(dx+c)}\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{d}-\frac{2C\left(iF(i(\csc(dx+c)-\cot(dx+c\right.}{}$
default	$\frac{2\left(3iA\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}F(i(\cot(dx+c)-\csc(dx+c)),i)\cos(dx+c)+iC\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}F(i(\cot(dx+c)-\csc\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*A/d*(1+cos(d*x+c))*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(b*sec(d*x+c
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2/3*C/d
*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+I*EllipticF(I*(csc(d*x+c)-cot(d*x+c))
,I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-cos(d*x+c)*s
in(d*x+c))*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int (A+C\cos^2(c+dx))\sqrt{b\sec(c+dx)}\,dx = \frac{2\,C\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i}{3\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*C*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(-3*I*A -
 I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + s
qrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)))/d
```

Sympy [F]

$$\int (A + C\cos^2(c + dx))\,\sqrt{b\sec(c + dx)}\,dx = \int \sqrt{b\sec(c + dx)}\left(A + C\cos^2(c + dx)\right)\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*(A + C*cos(c + d*x)**2), x)
```

Maxima [F]

$$\int (A + C\cos^2(c + dx))\,\sqrt{b\sec(c + dx)}\,dx = \int \left(C\cos(dx + c)^2 + A\right)\sqrt{b\sec(dx + c)}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int (A + C\cos^2(c + dx))\,\sqrt{b\sec(c + dx)}\,dx = \int \left(C\cos(dx + c)^2 + A\right)\sqrt{b\sec(dx + c)}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)\sqrt{b\sec(c+dx)}\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(1/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(1/2), x)
```

3.31 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}$$

```
[Out] 2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/5*b^2*C*
tan(d*x+c)/d/(b*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3317, 4130, 3856, 2719}

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Se
c[c + d*x]]) + (2*b^2*C*Tan[c + d*x])/(5*d*(b*Sec[c + d*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \frac{C + A\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx \\
&= \frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}} + \frac{1}{5}(5A+3C)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}} + \frac{(5A+3C)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$
\int \frac{A + C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\frac{4(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 2C\sin(2(c+dx))}{10d\sqrt{b\sec(c+dx)}}
$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
[Out] ((4*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*C*Sin[2*(
c + d*x)])/(10*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 16.52 (sec) , antiderivative size = 795, normalized size of antiderivative = 10.32

method	result	size
default	Expression too large to display	795
parts	Expression too large to display	806

```
[In] int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d/(1+cos(d*x+c))/(b*sec(d*x+c))^(1/2)*(3*I*sec(d*x+c)*C*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)+3*I*cos(d*x+c)*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-3*I*cos(d*x+c)*C*(1/(1+cos(
d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-co
t(d*x+c)),I)+5*I*sec(d*x+c)*A*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x
+c)-cot(d*x+c)),I)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-10*I*A*(1/(1+cos(d*x+c
)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)+10*I*A*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c))
,I)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-6*I*C*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*I*sec(
d*x+c)*C*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)+6*I*C*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(I*
(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*I*cos(d*x+c)
*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(
csc(d*x+c)-cot(d*x+c)),I)-5*I*sec(d*x+c)*A*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*I*cos(d
*x+c)*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elliptic
F(I*(csc(d*x+c)-cot(d*x+c)),I)+C*cos(d*x+c)^2*sin(d*x+c)+C*cos(d*x+c)*sin(d
*x+c)+5*A*sin(d*x+c)+3*sin(d*x+c)*C)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx$$
$$= \frac{2\,C\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^2\sin\left(dx+c\right) + \sqrt{2}(5i\,A + 3i\,C)\sqrt{b}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*C*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + sqrt(2)*(5*I*A
+ 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{A + C\cos^2{(c + dx)}}{\sqrt{b\sec{(c + dx)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(1/2),x)
[Out] Integral((A + C*cos(c + d*x)**2)/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{C\cos{(dx + c)}^2 + A}{\sqrt{b\sec{(dx + c)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{C\cos{(dx + c)}^2 + A}{\sqrt{b\sec{(dx + c)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{C\cos{(c + d\,x)}^2 + A}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2), x)
```

3.32 $\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}$$

```
[Out] 2/21*(7*A+5*C)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)+2/21*(7*A+5*C)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^2/d+2/7*b^2*C*tan(d*x+c)/d/(b*sec
(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3317, 4130, 3854, 3856, 2720}

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c +
d*x]])/(21*b^2*d) + (2*(7*A + 5*C)*Sin[c + d*x])/(21*b*d*Sqrt[b*Sec[c + d*x
]]) + (2*b^2*C*Tan[c + d*x])/(7*d*(b*Sec[c + d*x])^(7/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \frac{C+A\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx \\
&= \frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}} + \frac{1}{7}(7A+5C)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx \\
&= \frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}} + \frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}} + \frac{(7A+5C)\int\sqrt{b\sec(c+dx)}\,dx}{21b^2} \\
&= \frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}} + \frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}} \\
&\quad + \frac{\left((7A+5C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^2}
\end{aligned}
$$

$$
= \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^2d} + \frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}} + \frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}
$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$
\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\frac{4(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{\sqrt{\cos(c+dx)}} + 2(14A+13C+3C\cos(2(c+dx)))\sin(c+dx)}{42bd\sqrt{b\sec(c+dx)}}
$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(14*A + 1
3*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(42*b*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 17.41 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.53

method	result
default	$\frac{\frac{2iA\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}F(i(\cot(dx+c)-\csc(dx+c)),i)}{3}+\frac{10iC\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}F(i(\cot(dx+c)-\csc(dx+c)),i)}{21}+\frac{2i\sec(dx+c}{}}{}$
parts	$-\frac{2A\left(iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+i\sec(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(}{1+\cos}}\right.}{3d\sqrt{b\sec(dx+c)}\,b}$

```
[In] int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/b/d/(b*sec(d*x+c))^(1/2)*(7*I*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elli
pticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)+5*I*C*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(1+cos(d
*x+c)))^(1/2)+7*I*sec(d*x+c)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(
I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)+5*I*sec(d*x+c)*C*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(1+
cos(d*x+c)))^(1/2)+3*C*cos(d*x+c)^2*sin(d*x+c)+7*A*sin(d*x+c)+5*sin(d*x+c)*
C)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{A + C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-7i\,A - 5i\,C)\sqrt{b}\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\,\sin(dx+c)) +$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^3 + (7*A + 5*C
)*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{A + C\cos^2(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{C\cos(dx+c)^2 + A}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\sec(c + dx))^{3/2}}\,dx = \int \frac{C\cos{(dx + c)}^2 + A}{(b\sec{(dx + c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\sec(c + dx))^{3/2}}\,dx = \int \frac{C\cos{(c + d\,x)}^2 + A}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2), x)
```

3.33 $\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}$$

```
[Out] 2/45*(9*A+7*C)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(3/2)+2/15*(9*A+7*C)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/9*b^2*C*tan(d*x+c)/d/(b*sec
(d*x+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3317, 4130, 3854, 3856, 2719}

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt
[b*Sec[c + d*x]]) + (2*(9*A + 7*C)*Sin[c + d*x])/(45*b*d*(b*Sec[c + d*x])^(
3/2)) + (2*b^2*C*Tan[c + d*x])/(9*d*(b*Sec[c + d*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \frac{C + A\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx \\
&= \frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}} + \frac{1}{9}(9A+7C)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx \\
&= \frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}} + \frac{(9A+7C)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{15b^2} \\
&= \frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}} + \frac{(9A+7C)\int \sqrt{\cos(c+dx)}\,dx}{15b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{\frac{48(9A+7C)E\left(\frac{1}{2}(c+dx)|2\right)}{\sqrt{\cos(c+dx)}}+4(18A+19C+5C\cos(2(c+dx)))\sin(2(c+dx))}{360b^2d\sqrt{b\sec(c+dx)}}$$

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]

[Out] ((48*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(360*b^2*d*Sqrt[b*Sec[c + d*x]])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 23.00 (sec) , antiderivative size = 866, normalized size of antiderivative = 7.53

method	result	size
default	Expression too large to display	866
parts	Expression too large to display	876

[In] int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/45/b^2/d/(1+cos(d*x+c))/(b*sec(d*x+c))^(1/2)*(21*I*cos(d*x+c)*C*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-21*I*sec(d*x+c)*C*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-27*I*sec(d*x+c)*A*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+54*I*A*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*C*sin(d*x+c)*cos(d*x+c)^4+27*I*sec(d*x+c)*A*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+42*I*C*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+21*I*sec(d*x+c)*C*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-21*I*cos(d*x+c)*C*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*C*cos(d*x+c)^3*sin(d*x+c)-42*I*C*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+27*I*cos(d*x+c)*A*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-9*A*sin(d*x+c)*cos(d*x+c)^2-27*I*cos(d*x+c)*A*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-54*I*A*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-7*C*cos(d*x+c)^2*si

```
n(d*x+c)-9*A*sin(d*x+c)*cos(d*x+c)-7*C*cos(d*x+c)*sin(d*x+c)-27*A*sin(d*x+c
)-21*sin(d*x+c)*C)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = -\frac{3\sqrt{2}(-9i\,A-7i\,C)\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I*C
)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^4 + (9*A + 7*C)*cos(d*x + c)^2)*sqrt
(b/cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{A+C\cos^2\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(5/2),x)
[Out] Integral((A + C*cos(c + d*x)**2)/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\sec(c + dx))^{5/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\sec\left(dx + c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\sec(c + dx))^{5/2}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2), x)
```

3.34 $\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx = \frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)} - \frac{(C(1+m)+A(2+m))(b\cos(c+dx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+m)(2+m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(1+m)*sin(d*x+c)/b/d/(2+m)-(C*(1+m)+A*(2+m))*(b*cos(d*x+c)
)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/b/d
/(1+m)/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3093, 2722}

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+2)} - \frac{(A(m+2)+C(m+1))\sin(c+dx)(b\cos(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(c+dx)\right)}{bd(m+1)(m+2)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^m*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(2 + m)) - ((C*(1 + m) + A*(
2 + m))*(b*Cos[c + d*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)} + \left(A+\frac{C(1+m)}{2+m}\right)\int (b\cos(c+dx))^m\,dx$$

$$= \frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)} - \frac{\left(A+\frac{C(1+m)}{2+m}\right)(b\cos(c+dx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{(b\cos(c+dx))^m\cot(c+dx)\left(A(3+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)+C(1+m)\,\text{c}\right.}{d(1+m)(3+m)}$$

```
[In] Integrate[(b*Cos[c + d*x])^m*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] -(((b*Cos[c + d*x])^m*Cot[c + d*x]*(A*(3 + m)*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, Cos[c + d*x]^2] + C*(1 + m)*Cos[c + d*x]^2*Hypergeometric2F
1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 +
 m)*(3 + m)))
```

Maple [F]

$$\int (\cos (dx+c) b)^m \left(A+C\left(\cos^2 (dx+c)\right)\right) dx$$

[In] int((cos(d*x+c)*b)^m*(A+C*cos(d*x+c)^2),x)

[Out] int((cos(d*x+c)*b)^m*(A+C*cos(d*x+c)^2),x)

Fricas [F]

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right) dx = \int \left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)})^m dx$$

[In] integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)

Sympy [F]

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right) dx = \int (b\cos{(c+dx)})^m \left(A+C\cos^2{(c+dx)}\right) dx$$

[In] integrate((b*cos(d*x+c))**m*(A+C*cos(d*x+c)**2),x)

[Out] Integral((b*cos(c + d*x))**m*(A + C*cos(c + d*x)**2), x)

Maxima [F]

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right) dx = \int \left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)})^m dx$$

[In] integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)

Giac [F]

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^m\,dx$$

```
[In] integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^m\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^m,x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^m, x)
```

3.35 $\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 31

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right) dx = \frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)}$$

```
[Out] C*(b*cos(d*x+c))^(1+m)*sin(d*x+c)/b/d/(2+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {3090}

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right) dx = \frac{C\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+2)}$$

```
[In] Int[(b*Cos[c + d*x])^m*(-((C*(1 + m))/(2 + m)) + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(2 + m))
```

Rule 3090

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\text{integral} = \frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.65

$$\int (b\cos(c+dx))^m\left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{C(b\cos(c+dx))^m\cot(c+dx)\left((3+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)-(2+m)\cos^2(c\right.}{d(2+m)(3+m)}$$

```
[In] Integrate[(b*Cos[c + d*x])^m*(-((C*(1 + m))/(2 + m)) + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^m*Cot[c + d*x]*((3 + m)*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, Cos[c + d*x]^2] - (2 + m)*Cos[c + d*x]^2*Hypergeometric2F1[1/
2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + m)*
(3 + m))
```

Maple [A] (verified)

Time = 11.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{C(\cos(dx+c)b)^m\sin(2dx+2c)}{2(2+m)d}$	31

```
[In] int((cos(d*x+c)*b)^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2*C/(2+m)/d*(cos(d*x+c)*b)^m*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int (b\cos(c+dx))^m\left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{(b\cos(dx+c))^m\,C\cos(dx+c)\sin(dx+c)}{dm+2\,d}$$

```
[In] integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] (b*cos(d*x + c))^m*C*cos(d*x + c)*sin(d*x + c)/(d*m + 2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(26) = 52$.

Time = 17.84 (sec) , antiderivative size = 279, normalized size of antiderivative = 9.00

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx$$
$$= \begin{cases} -\frac{2C\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^m\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2d} + \frac{2C\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right.}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+2d\tan} \\ x(b\cos(c))^m\left(-\frac{C(m+1)}{m+2}+C\cos^2(c)\right) \end{cases}$$

```
[In] integrate((b*cos(d*x+c))**m*(-C*(1+m)/(2+m)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((-2*C*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(tan(
c/2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)**3/(d*m*tan(c/2 + d*x/2)**4 + 2*d
*m*tan(c/2 + d*x/2)**2 + d*m + 2*d*tan(c/2 + d*x/2)**4 + 4*d*tan(c/2 + d*x/
2)**2 + 2*d) + 2*C*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(t
an(c/2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)/(d*m*tan(c/2 + d*x/2)**4 + 2*d
*m*tan(c/2 + d*x/2)**2 + d*m + 2*d*tan(c/2 + d*x/2)**4 + 4*d*tan(c/2 + d*x/
2)**2 + 2*d), Ne(d, 0)), (x*(b*cos(c))**m*(-C*(m + 1)/(m + 2) + C*cos(c)**2
), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(31) = 62$.

Time = 0.48 (sec) , antiderivative size = 175, normalized size of antiderivative = 5.65

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{2}\,m}Cb^m\sin\left(-(dx+c)(m+2)+m\arctan\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/4*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/2*m)*C*b^m*sin(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/2*m)*C*b^m*sin(-(d*x + c)*(m - 2) + m*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)))/(2^m*d*(m + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2494 vs. $2(31) = 62$.

Time = 7.46 (sec) , antiderivative size = 2494, normalized size of antiderivative = 80.45

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m} + C\cos^2(c+dx)\right) dx = \text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 2*(C*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1))^
m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2
 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) +
1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) +
 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*
floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b
)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sg
n(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x +
 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c
)^3 - C*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1
))^m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c
)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c))
 + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)
) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi
*m*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 +
2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4
*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*
x + 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/
2*c) - C*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 +
1))^m*tan(1/2*d*x + 1/2*c)^3 + C*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(t
an(1/2*d*x + 1/2*c)^2 + 1))^m*tan(1/2*d*x + 1/2*c))/(d*m*tan(-1/4*pi*m*sgn(
2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sgn(2*b*ta
n(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*
c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/
4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^4 + 2*d*tan(-1/4*p
i*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn
```

```
(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sg
n(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sg
n(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*
x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1
/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^4 + 2*d*m*
tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 +
 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/
4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1
/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*fl
oor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(
1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1
/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^
2 + d*m*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/
2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*
sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4
 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)
*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn
(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*
x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d
*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^4 + d*m*t
an(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 +
2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4
*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/
4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*flo
or(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/
2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2 + 2*d*m*tan(1/2*d*x +
1/2*c)^2 + 2*d*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d
*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x
 + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*
x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/
2*c)) + pi*m*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1
/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2
*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c))
 - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(
tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2 + 4*d*
tan(1/2*d*x + 1/2*c)^2 + d*m + 2*d)
```

Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx=\frac{C\,\sin\left(2\,c+2\,d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^m}{2\,d\,\left(m+2\right)}$$

```
[In] int((b*cos(c + d*x))^m*(C*cos(c + d*x)^2 - (C*(m + 1))/(m + 2)),x)
[Out] (C*sin(2*c + 2*d*x)*(b*cos(c + d*x))^m)/(2*d*(m + 2))
```

3.36 $\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)\,dx = -\frac{A(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(1+m)}$$

```
[Out] -A*(b*cos(d*x+c))^(1+m)*sin(d*x+c)/b/d/(1+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3090}

$$\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)\,dx = -\frac{A\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+1)}$$

```
[In] Int[(b*Cos[c + d*x])^m*(A - (A*(2 + m)*Cos[c + d*x]^2)/(1 + m)),x]
```

```
[Out] -((A*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(1 + m)))
```

Rule 3090

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\text{integral} = -\frac{A(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(1+m)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.56

$$\int (b\cos(c+dx))^m\left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)dx =$$
$$-\frac{A(b\cos(c+dx))^m\cot(c+dx)\left((3+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)-(2+m)\cos\right.}{d(1+m)(3+m)}$$

```
[In] Integrate[(b*Cos[c + d*x])^m*(A - (A*(2 + m)*Cos[c + d*x]^2)/(1 + m)),x]
```

```
[Out] -((A*(b*Cos[c + d*x])^m*Cot[c + d*x]*((3 + m)*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, Cos[c + d*x]^2] - (2 + m)*Cos[c + d*x]^2*Hypergeometric2F1[
1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m
)*(3 + m)))
```

Maple [A] (verified)

Time = 8.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$-\frac{A(\cos(dx+c)b)^m\sin(2dx+2c)}{2(1+m)d}$	31

```
[In] int((cos(d*x+c)*b)^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*A/(1+m)/d*(cos(d*x+c)*b)^m*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^m\left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)dx$$
$$=-\frac{(b\cos(dx+c))^m A\cos(dx+c)\sin(dx+c)}{dm+d}$$

```
[In] integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="fri
cas")
```

```
[Out] -(b*cos(d*x + c))^m*A*cos(d*x + c)*sin(d*x + c)/(d*m + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(27) = 54$.

Time = 17.59 (sec) , antiderivative size = 272, normalized size of antiderivative = 8.50

$$\int (b\cos(c+dx))^m \left(A - \frac{A(2+m)\cos^2(c+dx)}{1+m}\right) dx$$
$$= \begin{cases} \frac{2A\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^m \tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+d} - \frac{2A\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^{\cdots}}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+d\tan^4\left(\frac{c}{2}\cdots\right.} \\ x(b\cos(c))^m\left(A - \frac{A(m+2)\cos^2(c)}{m+1}\right) \end{cases}$$

```
[In] integrate((b*cos(d*x+c))**m*(A-A*(2+m)*cos(d*x+c)**2/(1+m)),x)
```

```
[Out] Piecewise((2*A*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(tan(c
/2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)**3/(d*m*tan(c/2 + d*x/2)**4 + 2*d*
m*tan(c/2 + d*x/2)**2 + d*m + d*tan(c/2 + d*x/2)**4 + 2*d*tan(c/2 + d*x/2)*
*2 + d) - 2*A*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(tan(c/
2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)/(d*m*tan(c/2 + d*x/2)**4 + 2*d*m*ta
n(c/2 + d*x/2)**2 + d*m + d*tan(c/2 + d*x/2)**4 + 2*d*tan(c/2 + d*x/2)**2 +
 d), Ne(d, 0)), (x*(b*cos(c))**m*(A - A*(m + 2)*cos(c)**2/(m + 1)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(32) = 64$.

Time = 0.47 (sec) , antiderivative size = 175, normalized size of antiderivative = 5.47

$$\int (b\cos(c+dx))^m \left(A - \frac{A(2+m)\cos^2(c+dx)}{1+m}\right) dx$$
$$= \frac{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{2}m} Ab^m \sin\left(-(dx+c)(m+2)+m\arctan(\mathrm{si}\cdots\right.}{\cdots}$$

```
[In] integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="max
ima")
```

```
[Out] 1/4*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
2*m)*A*b^m*sin(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/2*m)*A*b^m*sin(-(d*x + c)*(m - 2) + m*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))/(2^m*d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2489 vs. $2(32) = 64$.

Time = 7.36 (sec) , antiderivative size = 2489, normalized size of antiderivative = 77.78

$$\int (b\cos(c+dx))^m \left(A - \frac{A(2+m)\cos^2(c+dx)}{1+m}\right) dx = \text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="gia
c")
```

```
[Out] -2*(A*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1))
^m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^
2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) +
 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c))
+ 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m
*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*
b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x
+ 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*
c)^3 - A*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 +
1))^m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*
c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)
) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c
)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + p
i*m*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 +
 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/
4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sg
n(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d
*x + 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1
/2*c) - A*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 +
 1))^m*tan(1/2*d*x + 1/2*c)^3 + A*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(
tan(1/2*d*x + 1/2*c)^2 + 1))^m*tan(1/2*d*x + 1/2*c))/(d*m*tan(-1/4*pi*m*sgn
(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sgn(2*b*t
an(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x +
 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1
/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^4 + d*tan(-1/4*pi
*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(
```

```
tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sgn
(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/
2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^4 + 2*d*m*t
an(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 +
2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4
*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/
4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*flo
or(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/
2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2
 + d*m*tan(1/2*d*x + 1/2*c)^4 + 2*d*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2
*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*s
gn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*
sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4
- 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*
sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(
tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x
 + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*
x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2 + d*tan(1/2*d*x + 1/2*c)^4 + d*m*tan(
-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b
)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi
*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*p
i*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(
1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(
1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c
)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2 + 2*d*m*tan(1/2*d*x + 1/2
*c)^2 + d*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x +
1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/
2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1
/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))
 + pi*m*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)
^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c))
- 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/
4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1
/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2 + 2*d*tan(1
/2*d*x + 1/2*c)^2 + d*m + d)
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)\,dx = -\frac{A\,\sin\left(2\,c+2\,d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^m}{2\,d\,\left(m+1\right)}$$

```
[In] int((b*cos(c + d*x))^m*(A - (A*cos(c + d*x)^2*(m + 2))/(m + 1)),x)
[Out] -(A*sin(2*c + 2*d*x)*(b*cos(c + d*x))^m)/(2*d*(m + 1))
```

3.37 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 112

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/9*C*(b*cos(d*x+c))^(7/
2)*sin(d*x+c)/b^3/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*
x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3093, 2715, 2721, 2719}

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45bd} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^3d}$$

```
[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[C
os[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d
) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}+\frac{(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx}{9b^2}\\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd}\\
&\quad+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}+\frac{1}{15}(9A+7C)\int \sqrt{b\cos(c+dx)}\,dx
\end{aligned}$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d} + \frac{\left((9A+7C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}(18A+19C+5C\cos(2(c+dx)))\sin(2(c+\cdots}{180d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[Co
s[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*d*
Sqrt[Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(124) = 248.

Time = 12.70 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.88

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-72A-2\cdots}{\cdots}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\cdots\right.\right.\right.}}$

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
[Out] -2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-160*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*
```

```
x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*
C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x
+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{3\sqrt{2}(-9i\,A-7i\,C)\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I*C
)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*sqrt(b
*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\big(A+C\cos^2(c+dx)\big)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\big(A+C\cos^2(c+dx)\big)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\big(A+C\cos^2(c+dx)\big)\,dx$$
$$=\int \cos\left(c+d\,x\right)^2\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

3.38 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 110

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/21*b*(7*A+5*C)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c
))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 3093, 2715, 2721, 2720}

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d} + \frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}$$

```
[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b
*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d)
+ (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b}\\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}+\frac{(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx}{7b}\\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}\\
&\quad+\frac{1}{21}(b(7A+5C))\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx
\end{aligned}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{\left(b(7A+5C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}$$

$$= \frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{(b\cos(c+dx))^{3/2}\left(4(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sqrt{\cos(c+dx)}(14A+13C+3C\cos(2(c+dx)\right.}{42bd\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
[Out] ((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[C
os[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*b*d*Co
s[c + d*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(122) = 244$.

Time = 9.93 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.67

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(28A+56C)\right.}{21\sqrt{-}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}$

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(48*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/
2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{\sqrt{2}(-7i\,A-5i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}(7i\,A+5i\,C)\sqrt{b}\text{weierst}}{}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^2 + 7*A + 5*C)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

3.39 $\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3093, 2721, 2719}

$$\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx = \frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Co
s[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{1}{5}(5A+3C)\int\sqrt{b\cos(c+dx)}\,dx\\ &= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}\\ &= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\begin{aligned}&\int\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx\\ &=\frac{\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}\end{aligned}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos
[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(93) = 186.

Time = 9.48 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.39

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}{}$

```
[In] int((cos(d*x+c)*b)^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(8*cos(1/2*
d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{2\sqrt{b\cos(dx+c)}C\cos(dx+c)\sin(dx+c)+\sqrt{2}(5iA+3iC)\sqrt{b}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInver}}{}$$

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(5*I*A +
3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

3.40 $\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 73

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec(c+dx)\,dx = \frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 2/3*b*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3093, 2721, 2720}

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec(c+dx)\,dx = \frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Co
s[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}(b(3A+C))\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{\left(b(3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\ &= \frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx\\ &=\frac{b\left(2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))\right)}{3d\sqrt{b\cos(c+dx)}}\end{aligned}$$

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]

[Out] (b*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)]))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(89) = 178$.

Time = 7.93 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.25

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\right.\right.\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}\right.}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}}{}$

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(3i\,A+i\,C)\sqrt{b}\text{weierstra}}{3\,d}$$

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/d

Sympy [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \int \sqrt{b\cos\left(c+dx\right)}\left(A+C\cos^2\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)*(b*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(b*cos(c + d*x))*(A + C*cos(c + d*x)**2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x), x)
```

3.41 $\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 69

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 3091, 2721, 2719}

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c +
 d*x]]) + (2*A*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + (-A+C)\int \sqrt{b\cos(c+dx)}\,dx \\
&= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left((-A+C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \sqrt{b\cos(c+dx)}\left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx \\
= \frac{2b\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\right) + A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c + d
*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(89) = 178$.

Time = 8.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.10

method	result
default	$\frac{2b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$-\frac{2Ab\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2*b*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2
*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.65

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \frac{\sqrt{2}(-i\,A+i\,C)\sqrt{b}\cos\left(dx+c\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx\right.}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] (sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A - I*C)*sqr
t(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(d*cos(d*
x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2*(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx = \int (C\cos{(dx+c)}^2+A)\sqrt{b\cos{(dx+c)}}\sec{(dx+c)}^2\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx = \int (C\cos{(dx+c)}^2+A)\sqrt{b\cos{(dx+c)}}\sec{(dx+c)}^2\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)
```

3.42 $\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 76

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx = \frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b*(A+3*C)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 3091, 2721, 2720}

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx = \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*b*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^3 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}(b(A+3C))\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left(b(A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\begin{aligned}
&\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx \\
&= \frac{2b\left((A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*b*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*
x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(92) = 184$.

Time = 7.80 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.84

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+3C)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+A\sqrt{\frac{1}{2}}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{d}{2}\right.}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*b*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.49

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\frac{\sqrt{2}(-i\,A-3i\,C)\sqrt{b}\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(i\,A+3i\,C}{3\,d\cos}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
```

```
c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*
cos(d*x + c))*A*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3*(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\begin{aligned}&\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx\\&=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^3\,dx\end{aligned}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Giac [F]

$$\begin{aligned}&\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx\\&=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^3\,dx\end{aligned}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^3}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)
```

3.43 $\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 110

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^4(c+dx)\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*b*(3*A+5*C)*sin(d*x+c)/d/(b
*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2719}

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^4(c+dx)\,dx = \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[C
os[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*
(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^4 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(b^2(3A+5C)\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} + \frac{1}{5}(-3A-5C)\int \sqrt{b\cos(c+dx)}\,dx
\end{aligned}$$

$$= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} + \frac{\left((-3A-5C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = -\frac{\sqrt{b\cos(c+dx)}\sec^2(c+dx)\left(2(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-(3A+5C)\sin(2(c+dx))-\right.}{5d}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] -1/5*(Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(2*(3*A + 5*C)*Cos[c + d*x]^(3/2)
*EllipticE[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d*x
]))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(122) = 244$.

Time = 11.66 (sec) , antiderivative size = 562, normalized size of antiderivative = 5.11

method	result
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}E\left(\right.\right.}{}$
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5*A*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(
1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d-2*C*b*(-2*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.24

$$\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{\sqrt{2}(-3i\,A-5i\,C)\sqrt{b}\cos{(dx+c)}^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(c}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^4}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^4, x)
```

3.44 $\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 113

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b^2*(5*A+7*C)*sin(d*x+c)/d
/(b*cos(d*x+c))^(3/2)+2/21*b*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos
(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3091, 2716, 2721, 2720}

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (2*b*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*
b^2*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^5 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(b^3(5A+7C)\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{1}{21}(b(5A+7C))\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&\quad + \frac{\left(b(5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}} \\
&= \frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$
\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx
$$

$$
= \frac{\sqrt{b\cos(c+dx)}\sec^3(c+dx)\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C)\sin(2(c+dx)\right.}{21d}
$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*Elli
pticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))/(
21*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(125) = 250$.

Time = 10.36 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.65

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{6b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2(\cdots}}{3\sqrt{-b\left(2\left(\sin^4(\cdots\right.\right.}}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-\frac{\cos(\cdots}{}}\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(C*(-1/6*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)
^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1
/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$= \frac{\sqrt{2}(-5i\,A-7i\,C)\sqrt{b}\cos{(dx+c)}^4\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(5i\,A+$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^5\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^5\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$= \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^5}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^5,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^5, x)
```

3.45 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 110

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2b(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*C*(b*cos(d*x+c))^(7/2)
*sin(d*x+c)/b^2/d+2/15*b*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*
x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {16, 3093, 2715, 2721, 2719}

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d} + \frac{2b(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^2d}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt
[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d
) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}+\frac{(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx}{9b}\\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}\\
&\quad+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}+\frac{1}{15}(b(9A+7C))\int \sqrt{b\cos(c+dx)}\,dx
\end{aligned}
$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d} + \frac{\left(b(9A+7C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}}$$

$$= \frac{2b(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{5/2}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}(18A+19C+5C\right.}{180bd\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[
Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*
b*d*Cos[c + d*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(122) = 244.

Time = 12.58 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.95

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-72A\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos\right.\right.}}$

```
[In] int(cos(d*x+c)*(cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-160*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*
```

```
d*x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+13
6*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d
*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) dx = \frac{3i\sqrt{2}(9A+7C)b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))}{}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/45*(3*I*sqrt(2)*(9*A + 7*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(9*A + 7*C)*b^(3
/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(5*C*b*cos(d*x + c)^3 + (9*A + 7*C)*b*cos(d*x + c))*sqrt(b*c
os(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\,dx = \int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2), x)
```

3.46 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/21*b^2*(7*A+5*C)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/21*b*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x
+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3093, 2715, 2721, 2720}

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21
*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{1}{7}(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx \\
&= \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} \\
&\quad + \frac{1}{21}\left(b^2(7A+5C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} \\
&\quad + \frac{\left(b^2(7A+5C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}} \\
&= \frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) dx = \frac{(b\cos(c+dx))^{3/2} \left(4(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sqrt{\cos(c+dx)}(14A+13C\right.}{42d\cos^{\frac{3}{2}}(c+dx)}$$

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]

[Out] ((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*Cos[c + d*x]^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(125) = 250.

Time = 10.53 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.62

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(28A+56C)\right.}{21\sqrt{-b}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$

[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] -2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{-i\sqrt{2}(7A+5C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] 1/21*(-I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*b*cos(d*x + c)^2 + (7*A + 5*C)*
b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2), x)
```

3.47 $\int (b\cos(c+dx))^{3/2} (A + C\cos^2(c+dx)) \sec(c+dx)\, dx$

Optimal result

Integrand size = 31, antiderivative size = 75

$$\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\, dx = \frac{2b(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/5*b*(5*A+3*C)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3093, 2721, 2719}

$$\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\, dx = \frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (2*b*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[
Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx\\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}(b(5A+3C))\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{\left(b(5A+3C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}\\
&= \frac{2b(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{b\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[C
os[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(91) = 182$.

Time = 9.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.51

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{d}{2}\right.\right.\right.}}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{d}{2}\right.\right.\right.}}{}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(8*cos(1/
2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2\sqrt{b\cos(dx+c)}Cb\cos(dx+c)\sin(dx+c)+i\sqrt{2}(5A+3C)b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstras}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*b*cos(d*x + c)*sin(d*x + c) + I*sqrt(2)*(5*A
+ 3*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - I*sqrt(2)*(5*A + 3*C)*b^(3/2)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x), x)
```

3.48 $\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 76

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 2/3*b^2*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b*C*
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 3093, 2721, 2720}

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bC\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = b^2 \int \frac{A + C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(b^2(3A+C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left(b^2(3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$

$$= \frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{b^2\left(2(3A+C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (b^2*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c
 + d*x)]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(92) = 184.

Time = 7.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.14

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\right.\right.\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}\right.}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\right.\right.\right.}}{}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2
*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{-i\sqrt{2}(3A+C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+C)b^{\frac{3}{2}}\mathrm{we}}{3d}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*b*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)
```

3.49 $\int (b\cos(c+dx))^{3/2} (A+C\cos^2(c+dx)) \sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 72

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx = -\frac{2b(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*b*(A-C)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 3091, 2721, 2719}

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx = \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{2b(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (-2*b*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c
 + d*x]]) + (2*A*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^3 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - (b(A-C)) \int \sqrt{b\cos(c+dx)}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(b(A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2b(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{2b^2\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
[Out] (2*b^2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c +
 d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(92) = 184.

Time = 7.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.00

method	result
default	$\frac{2b^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$-\frac{2A\,b^2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(s\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,method=_RETURNVE
RBOSE)
[Out] 2*b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.57

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{-i\sqrt{2}(A-C)b^{\frac{3}{2}}\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] (-I*sqrt(2)*(A - C)*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(A - C)*b^(3/2)
*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*b*sin(d*x + c))/(d*cos(d*x
+ c))
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)
```

3.50 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 78

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b^2*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 3091, 2721, 2720}

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^4 \int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{1}{3}\left(b^2(A+3C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{\left(b^2(A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{2b^2\left((A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c +
d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(94) = 188.

Time = 7.11 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.77

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+3C)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+A\sqrt{\frac{1}{2}}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\right.}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*b^2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{-i\sqrt{2}(A+3\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^2\,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}(A}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*C)*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(3/2)*cos(d*x + c)^
```

```
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(
d*x + c))*A*b*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^4(c + dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^4(c + dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^4,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^4, x)
```

3.51 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 113

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = -\frac{2b(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*b^2*(3*A+5*C)*sin(d*x+c)/d/
(b*cos(d*x+c))^(1/2)-2/5*b*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3091, 2716, 2721, 2719}

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{2b(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (-2*b*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt
[Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*
b^2*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^5 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(b^3(3A+5C)\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{1}{5}(b(3A+5C))\int \sqrt{b\cos(c+dx)}\,dx
\end{aligned}
$$

$$= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(b(3A+5C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}$$

$$= -\frac{2b(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = -\frac{(b\cos(c+dx))^{3/2}\sec^3(c+dx)\left(2(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-(3A+5C)\sin(2(c+dx))-2\right.}{5d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
[Out] -1/5*((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(2*(3*A + 5*C)*Cos[c + d*x]^(3/
2)*EllipticE[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d
*x]))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(125) = 250.

Time = 11.51 (sec) , antiderivative size = 565, normalized size of antiderivative = 5.00

method	result
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\right.\right.}{}$
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right.}{}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5*A*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*si
n(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d-2*C*b^2*(-
2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.25

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{-i\sqrt{2}(3A+5C)b^{\frac{3}{2}}\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] 1/5*(-I*sqrt(2)*(3*A + 5*C)*b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(3*A
+ 5*C)*b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*b*cos(d*x + c)^2 + A*
b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^5\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^5\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^5}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^5,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^5, x)
```

3.52 $\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^6(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^6(c+dx)\,dx = \frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b^3*(5*A+7*C)*sin(d*x+c)/d
/(b*cos(d*x+c))^(3/2)+2/21*b^2*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*c
os(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3091, 2716, 2721, 2720}

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^6(c+dx)\,dx = \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (2*b^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (
2*b^3*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^6 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(b^4(5A+7C)\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{1}{21}\left(b^2(5A+7C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx
\end{aligned}$$

$$= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{\left(b^2(5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}$$

$$= \frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{(b\cos(c+dx))^{3/2}\sec^4(c+dx)\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C)\sin\right.}{21d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*El
lipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))
/(21*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(127) = 254.

Time = 9.94 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.60

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{6b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2(}}{3\sqrt{-b\left(2\left(\sin^4\right.\right.}}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(C*(-1/6
*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b
*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x
+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.18

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{-i\sqrt{2}(5A+7C)b^{\frac{3}{2}}\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="fricas")
```

```
[Out] 1/21*(-I*sqrt(2)*(5*A + 7*C)*b^(3/2)*cos(d*x + c)^4*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(5*A + 7*C)*b^(3/2)*cos(d*x
+ c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((5*A
+ 7*C)*b*cos(d*x + c)^2 + 3*A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^6\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^6\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^6}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^6,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^6, x)
```

3.53 $\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) dx = \frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}$$

```
[Out] 2/45*b*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*C*(b*cos(d*x+c))^(7/
2)*sin(d*x+c)/b/d+2/15*b^2*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3093, 2715, 2721, 2719}

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) dx = \frac{2b^2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{2b(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sq
rt[Cos[c + d*x]]) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
45*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}+\frac{1}{9}(9A+7C)\int(b\cos(c+dx))^{5/2}\,dx\\
&= \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}\\
&\quad+\frac{1}{15}\left(b^2(9A+7C)\right)\int\sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}\\
&\quad+\frac{\left(b^2(9A+7C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}}\\
&= \frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{5/2}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}(18A+19C+5C}{180d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[
Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*
d*Cos[c + d*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(125) = 250.

Time = 14.60 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^3\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-72A\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^3\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos\right.\right.}}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-160*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+13
6*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d
*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3i\sqrt{2}(9A+7C)b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/45*(3*I*sqrt(2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(9*A + 7*C)*b^(5
/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(5*C*b^2*cos(d*x + c)^3 + (9*A + 7*C)*b^2*cos(d*x + c))*sqrt
(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2), x)
```

3.54 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/d+2/21*b^3*(7*A+5*C)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/21*b^2*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x
+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {16, 3093, 2715, 2721, 2720}

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (2*b^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*b^2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(
21*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx \\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}+\frac{1}{7}(b(7A+5C))\int (b\cos(c+dx))^{3/2}\,dx \\
&= \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} \\
&\quad +\frac{1}{21}\left(b^3(7A+5C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx
\end{aligned}
$$

$$= \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{\left(b^3(7A+5C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}$$

$$= \frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{b(b\cos(c+dx))^{3/2}\left(4(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sqrt{\cos(c+dx)}(14A+13C+3C\cos(2\right.}{42d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
[Out] (b*(b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt
[Cos[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*Co
s[c + d*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(124) = 248.

Time = 16.85 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.64

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(28A+56C)\right(}{21\sqrt{-b}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\right.}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(48*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+
1/2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{-i\sqrt{2}(7A+5C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(7A+5C)b^{\frac{5}{2}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/21*(-I*sqrt(2)*(7*A + 5*C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*(7*A + 5*C)*b^(5/2)*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*b^2*cos(d*x + c)^2 + (7*A + 5*C
)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x), x)
```

3.55 $\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) \sec^2(c+dx)\, dx$

Optimal result

Integrand size = 33, antiderivative size = 78

$$\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) \sec^2(c+dx)\, dx = \frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*b*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/5*b^2*(5*A+3*C)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*c
os(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3093, 2721, 2719}

$$\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) \sec^2(c+dx)\, dx = \frac{2b^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{2bC\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqr
t[Cos[c + d*x]]) + (2*b*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2\int\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx\\
&= \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}\left(b^2(5A+3C)\right)\int\sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{\left(b^2(5A+3C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}\\
&= \frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{b^2\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt
[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(94) = 188$.

Time = 18.82 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.37

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{d}{2}\right.\right.\right.}}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d} - \frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{d}{2}\right.\right.\right.}}{}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(8*cos(1/
2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.36

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{2\sqrt{b\cos(dx+c)}Cb^2\cos(dx+c)\sin(dx+c)+i\sqrt{2}(5A+3C)b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstra}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*b^2*cos(d*x + c)*sin(d*x + c) + I*sqrt(2)*(5*
A + 3*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) - I*sqrt(2)*(5*A + 3*C)*b^(5/2)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)
```

3.56 $\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 78

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 2/3*b^3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*
C*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 3093, 2721, 2720}

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^3*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = b^3 \int \frac{A + C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(b^3(3A+C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left(b^3(3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$

$$= \frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{2(b\cos(c+dx))^{5/2}\left((3A+C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{3d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
[Out] (2*(b*Cos[c + d*x])^(5/2)*((3*A + C)*EllipticF[(c + d*x)/2, 2] + C*Sqrt[Cos
[c + d*x]]*Sin[c + d*x]))/(3*d*Cos[c + d*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(94) = 188.

Time = 59.15 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.06

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\cdots\right.\right.\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}\cdots\right.}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\cdots\right.\right.\right.}}{\cdots}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,method=_RETURNVE
RBOSE)
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2
*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.15

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{-i\sqrt{2}(3A+C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+C)b^{\frac{5}{2}}\text{we}}{3d}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*b^2*sin(d*x + c))/
d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)
```

3.57 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 74

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = -\frac{2b^2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*b^2*(A-C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x
+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3091, 2721, 2719}

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (-2*b^2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos
[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^4 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \left(b^2(A-C)\right)\int \sqrt{b\cos(c+dx)}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(b^2(A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2b^2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{2b^3\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
[Out] (2*b^3*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c +
 d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(94) = 188.

Time = 181.96 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.92

method	result
default	$\frac{2b^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$-\frac{2A\,b^3\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(s\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,method=_RETURNVE
RBOSE)
[Out] 2*b^3*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.55

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{-i\sqrt{2}(A-C)b^{\frac{5}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] (-I*sqrt(2)*(A - C)*b^(5/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(A - C)*b^(5/2)
*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*b^2*sin(d*x + c))/(d*cos(d*
x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \frac{\left(C\cos(c+d\,x)^2+A\right)\,(b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^4}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^4,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^4, x)
```

3.58 $\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) \sec^5(c+dx)\, dx$

Optimal result

Integrand size = 33, antiderivative size = 78

$$\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) \sec^5(c+dx)\, dx = \frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b^3*(A+3*C)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 3091, 2721, 2720}

$$\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) \sec^5(c+dx)\, dx = \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (2*b^3*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^5 \int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}\left(b^3(A+3C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left(b^3(A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{2b^3\left((A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (2*b^3*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c +
d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(94) = 188$.

Time = 2.25 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.77

$$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+3}{3\sqrt{-b\,(2\,(}}$$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*b^3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.46

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\frac{-i\sqrt{2}(A+3\,C)b^{\frac{5}{2}}\cos\left(dx+c\right)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\sqrt{2}(A}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(
d*x + c))*A*b^2*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^5\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^5\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^5}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^5,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^5, x)
```

3.59 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = -\frac{2b^2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*b^3*(3*A+5*C)*sin(d*x+c)/d/
(b*cos(d*x+c))^(1/2)-2/5*b^2*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/co
s(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2719}

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{2b^2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (-2*b^2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sq
rt[Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (
2*b^3*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^6 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(b^4(3A+5C)\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{1}{5}\left(b^2(3A+5C)\right)\int \sqrt{b\cos(c+dx)}\,dx
\end{aligned}
$$

$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(b^2(3A+5C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}$$

$$= -\frac{2b^2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = -\frac{2b^4\left((3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) - \frac{1}{2}(3A+5C)\sin(2(c+dx)) - A\tan(c+dx)\right)}{5d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
[Out] (-2*b^4*((3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - ((3*A +
 5*C)*Sin[2*(c + d*x)])/2 - A*Tan[c + d*x]))/(5*d*(b*Cos[c + d*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(127) = 254.

Time = 4.55 (sec) , antiderivative size = 602, normalized size of antiderivative = 5.23

$$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - 12A\sqrt{\frac{1}{2}-\frac{\cos(dx+c}{2}}\right.}{}$$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
[Out] -2/5*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+40*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2
*c)^6-20*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1
```

```
/2*c)*sin(1/2*d*x+1/2*c)^4+12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)
^2-40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^2+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))+10*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1
/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{-i\sqrt{2}(3A+5C)b^{\frac{5}{2}}\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+\cdots}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="fricas")
```

```
[Out] 1/5*(-I*sqrt(2)*(3*A + 5*C)*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(3*A
+ 5*C)*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*b^2*cos(d*x + c)^2 +
A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^6\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^6\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^6}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^6,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^6, x)
```

3.60 $\int (b\cos(c+dx))^{5/2} (A+C\cos^2(c+dx)) \sec^7(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) \sec^7(c+dx)\,dx = \frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b^6*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b^4*(5*A+7*C)*sin(d*x+c)/d
/(b*cos(d*x+c))^(3/2)+2/21*b^3*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*c
os(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2720}

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) \sec^7(c+dx)\,dx = \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] (2*b^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*A*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (
2*b^4*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^7 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(b^5(5A+7C)\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{1}{21}\left(b^3(5A+7C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx
\end{aligned}$$

$$= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{\left(b^3(5A+7C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{21\sqrt{b\cos(c+dx)}}$$

$$= \frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{(b\cos(c+dx))^{5/2}\sec^5(c+dx)\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C)\sin\right.}{21d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*El
lipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))
/(21*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(127) = 254.

Time = 3018.44 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.60

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{6b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2(}}{3\sqrt{-b\left(2\left(\sin^4\right.\right.}}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(C*(-1/6
*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b
*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x
+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.22

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{-i\sqrt{2}(5A+7C)b^{\frac{5}{2}}\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\cdots}{\cdots}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm
="fricas")
```

```
[Out] 1/21*(-I*sqrt(2)*(5*A + 7*C)*b^(5/2)*cos(d*x + c)^4*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(5*A + 7*C)*b^(5/2)*cos(d*x
+ c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((5*A
+ 7*C)*b^2*cos(d*x + c)^2 + 3*A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^7\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^7\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^7}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^7,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^7, x)
```

3.61 $\int \frac{\cos^4(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 147

$$\begin{aligned}\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{10(11A+9C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d\sqrt{b\cos(c+dx)}} \\ &\quad+\frac{10(11A+9C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{231bd} \\ &\quad+\frac{2(11A+9C)(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d}+\frac{2C(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d}\end{aligned}$$

```
[Out] 2/77*(11*A+9*C)*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d+2/11*C*(b*cos(d*x+c))
^(9/2)*sin(d*x+c)/b^5/d+10/231*(11*A+9*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*
cos(d*x+c))^(1/2)+10/231*(11*A+9*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used

$= \{16, 3093, 2715, 2721, 2720\}$

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2(11A+9C)\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^3d} + \frac{10(11A+9C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{231bd} + \frac{10(11A+9C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^5d}$$

```
[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (10*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[
b*Cos[c + d*x]]) + (10*(11*A + 9*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(231
*b*d) + (2*(11*A + 9*C)*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(77*b^3*d) + (
2*C*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{7/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^4}\\
&= \frac{2C(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d}+\frac{(11A+9C)\int (b\cos(c+dx))^{7/2}\,dx}{11b^4}\\
&= \frac{2(11A+9C)(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d}\\
&\quad +\frac{2C(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d}+\frac{(5(11A+9C))\int (b\cos(c+dx))^{3/2}\,dx}{77b^2}\\
&= \frac{10(11A+9C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{231bd}+\frac{2(11A+9C)(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d}\\
&\quad +\frac{2C(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d}+\frac{1}{231}(5(11A+9C))\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{10(11A+9C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{231bd}+\frac{2(11A+9C)(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d}\\
&\quad +\frac{2C(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d}+\frac{\left(5(11A+9C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{231\sqrt{b\cos(c+dx)}}\\
&= \frac{10(11A+9C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d\sqrt{b\cos(c+dx)}}\\
&\quad +\frac{10(11A+9C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{231bd}\\
&\quad +\frac{2(11A+9C)(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d}+\frac{2C(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{80(11A+9C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(572A+531C+12(11A+16C)\cos(2(c+dx))+}{1848d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
[Out] (80*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (572*A + 53
1*C + 12*(11*A + 16*C)*Cos[2*(c + d*x)] + 21*C*Cos[4*(c + d*x)])*Sin[2*(c +
 d*x)])/(1848*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(155) = 310.

Time = 12.53 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.37

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(1344C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3360C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(528A+3\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos\right.\right.}}$

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/231*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1344*C*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-3360*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^10+(528*A+3792*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-792*A-
2328*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(616*A+924*C)*sin(1/2*d*x+1
/2*c)^4*cos(1/2*d*x+1/2*c)+(-176*A-186*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)+55*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*co
s(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx =$$
$$-\frac{5\sqrt{2}(11iA+9iC)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-11iA-9iC)\sqrt{}}{}$$

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/231*(5*sqrt(2)*(11*I*A + 9*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-11*I*A - 9*I*C)*sqrt(b)*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(21*C*cos(d*x + c)^4 + 3
*(11*A + 9*C)*cos(d*x + c)^2 + 55*A + 45*C)*sqrt(b*cos(d*x + c))*sin(d*x +
c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

3.62 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/9*C*(b*cos(d*x+c))^(
7/2)*sin(d*x+c)/b^4/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/co
s(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3093, 2715, 2721, 2719}

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^2d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^4d}$$

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt
[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b
^2*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^3} \\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx}{9b^3} \\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} \\
&\quad + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{(9A+7C)\int \sqrt{b\cos(c+dx)}\,dx}{15b}
\end{aligned}
$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{\left((9A+7C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{15b\sqrt{\cos(c+dx)}}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{6(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(18A+19C+5C\cos(2(c+dx)))\sin(c+dx)}{45d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^
2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*d*Sqrt[b*Cos[c + d
*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(127) = 254.

Time = 11.14 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.79

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-72A-29\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\right.\right.}}$

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
```

```
1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*C)
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1
/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{\cos^3(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{3\sqrt{2}(-9i\,A-7i\,C)\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))}{}$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I*C
)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*sqrt(b
*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^3\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

3.63 $\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 112

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))
^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3093, 2715, 2721, 2720}

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21bd} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}$$

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*C
os[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d)
+ (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}\\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}+\frac{(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx}{7b^2}\\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd}\\
&\quad+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}+\frac{1}{21}(7A+5C)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} \\
&\quad + \frac{\left((7A+5C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}} \\
&= \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$
\begin{aligned}
&\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{4(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(14A+13C+3C\cos(2(c+dx)))\sin(2(c+dx))}{42d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C
+ 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(124) = 248.

Time = 10.06 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.62

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(28A+56C)\left(\right.\right.}{21\sqrt{-}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\,d}}$

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*
```

```
c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}(-7i\,A-5i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(7i\,A+5i\,C)\sqrt{b}\text{weierst}}{}$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^2 + 7*A + 5*C)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

3.64 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos
(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3093, 2721, 2719}

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}$$

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[
Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx}{b}\\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}+\frac{(5A+3C)\int \sqrt{b\cos(c+dx)}\,dx}{5b}\\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}+\frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}}\\
&= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5bd\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos
[c + d*x]]*Sin[2*(c + d*x)]))/(5*b*d*Sqrt[Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(96) = 192$.

Time = 8.46 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.25

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{}\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}{}$

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERB
OSE)
[Out] 2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*cos(1/2*d*
x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)
^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{b\cos(dx+c)}C\cos(dx+c)\sin(dx+c)+\sqrt{2}(5iA+3iC)\sqrt{b}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInver}}{}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(5*I*A +
3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

3.65 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(d*
x+c)*(b*cos(d*x+c))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3093, 2721, 2720}

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}+\frac{1}{3}(3A+C)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}+\frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\
&= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int\frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*
x)])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(91) = 182.

Time = 5.91 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.15

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{d}{}\cdots\right.\right.\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\cdots\right.}$
parts	$\frac{2A\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert\sqrt{2}\right)}{d\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2(\mathrm{si}\cdots\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\cdots\right.\right.\right.}}$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x
+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(3i\,A+i\,C)\sqrt{b}\text{weierstra}\cdots}{3\,bd}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos(dx+c)^2+A}{\sqrt{b\cos(dx+c)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos(dx+c)^2+A}{\sqrt{b\cos(dx+c)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd} + \frac{2A\sqrt{\cos(c+dx)}\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{\cos(c+dx)}\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/2),x)
[Out] (2*C*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2)
*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*C*cos(c + d*x
)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))
```

3.66 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 71

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/
2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3091, 2721, 2719}

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c
 + d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{(A-C)\int \sqrt{b\cos(c+dx)}\,dx}{b} \\
&= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}} \\
&= -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.54 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.79

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{(A+C\cos^2(c+dx))\left(2(A-C)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sec(c)\sin(dx+\arctan(\tan(c)\right.}{}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] ((A + C*Cos[c + d*x]^2)*(2*(A - C)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, C
os[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]] + Csc[c]*(-3*(
A - C)*Cos[c - d*x - ArcTan[Tan[c]]]*Sec[c] - (A - C)*Cos[c + d*x + ArcTan[
Tan[c]]]*Sec[c] + 2*((2*A - C)*Cos[d*x] - C*Cos[2*c + d*x])*Sqrt[Sec[c]^2])
*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + C*C
os[2*(c + d*x)])*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(91) = 182$.

Time = 8.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.00

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}$
parts	$-\frac{2A\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\right.\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d
*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.65

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \frac{\sqrt{2}(-i\,A + i\,C)\sqrt{b}\cos{(dx+c)}\,\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos{(dx+c)} + i\,\sin{(dx+}}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] (sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A - I*C)*sqr
t(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b*d*cos(
d*x + c))
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(A + C\cos^2{(c + dx)})\sec{(c + dx)}}{\sqrt{b\cos{(c + dx)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos{(dx + c)}^2 + A\right)\sec{(dx + c)}}{\sqrt{b\cos{(dx + c)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos{(dx + c)}^2 + A\right)\sec{(dx + c)}}{\sqrt{b\cos{(dx + c)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)), x)
```

3.67 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 73

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3091, 2721, 2720}

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.46 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.93

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx =$$

$$-\frac{4b(A+C\cos^2(c+dx))\left((A+3C)\cos^2(c+dx)\sqrt{\cos^2(dx-\arctan(\cot(c)))}\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\right.\right.}{3d(b\cos(c+dx))^{3/2}(2A+C+C\cos(2(c+dx}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-4*b*(A + C*Cos[c + d*x]^2)*((A + 3*C)*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcTa
n[Cot[c]]]^2]*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]] - A*Sqrt[Csc[c]^2]*Sin[c + d*x]))/(3*
d*(b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2*(c + d*x)])*Sqrt[Csc[c]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(89) = 178$.

Time = 7.86 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.99

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+3C)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+A\sqrt{}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{d}{2}\right.\right.\right.}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\right.}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \frac{\sqrt{2}(-iA - 3iC)\sqrt{b}\cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \sqrt{2}(iA + 3i}{3\,bd\,\mathrm{co}}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*
cos(d*x + c))*A*sin(d*x + c))/(b*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(A + C\cos^2{(c + dx)})\sec^2{(c + dx)}}{\sqrt{b\cos{(c + dx)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/sqrt(b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(C\cos{(dx + c)}^2 + A)\sec{(dx + c)}^2}{\sqrt{b\cos{(dx + c)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(C\cos{(dx + c)}^2 + A)\sec{(dx + c)}^2}{\sqrt{b\cos{(dx + c)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^2\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)), x)
```

3.68 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 112

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*C)*sin(d*x+c)/d/(b*c
os(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2719}

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt
[Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*
(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^3 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}(b(3A+5C))\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{(3A+5C)\int\sqrt{b\cos(c+dx)}\,dx}{5b} \\
&= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} \\
&\quad - \frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.60 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.10

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{(C+A\sec^2(c+dx))\left(2(3A+5C)\cos^2(c+dx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sec(c)\sin(dx+\right.}{}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] ((C + A*Sec[c + d*x]^2)*(2*(3*A + 5*C)*Cos[c + d*x]^2*HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[
c]]] + Csc[c]*(-((3*A + 5*C)*Cos[c + d*x]^2*(3*Cos[c - d*x - ArcTan[Tan[c]]
] + Cos[c + d*x + ArcTan[Tan[c]]])*Sec[c]) + (2*(4*A + 5*C)*Cos[d*x] + (A +
 5*C)*Cos[2*c + d*x] + (3*A + 5*C)*Cos[2*c + 3*d*x])*Sqrt[Sec[c]^2])*Sqrt[S
in[d*x + ArcTan[Tan[c]]]^2]))/(5*d*Sqrt[b*Cos[c + d*x]]*(2*A + C + C*Cos[2*
(c + d*x)])*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(124) = 248.

Time = 12.40 (sec) , antiderivative size = 564, normalized size of antiderivative = 5.04

method	result
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}E\left(\mathrm{co}\right.\right.}{}$
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\mathrm{c}\right.\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5*A*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*si
n(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d-2*C*(-2*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.24

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \frac{\sqrt{2}(-3i\,A - 5i\,C)\sqrt{b}\cos{(dx + c)}^3\ \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos{(dx + c)} + i\,\sin$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)), x)
```

3.69 $\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 110

$$\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b*(5*A+7*C)*sin(d*x+c)/d/(
b*cos(d*x+c))^(3/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x
+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3091, 2716, 2721, 2720}

$$\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*C
os[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b*
(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3091

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^4 \int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}\left(b^2(5A+7C)\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{1}{21}(5A+7C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{\left((5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

$$\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C+3A\sec^2(c+dx))\tan(c+dx)\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c + d*x]],x]
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C + 
 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(122) = 244.

Time = 11.20 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.75

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{6b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2(\cdots}}{3\sqrt{-b\left(2\left(\sin^4\left(\cdots\right.\right.\right.}}\right.\right.}{}$
parts	$-\frac{A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{28b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^4}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\cdots\right.\right.\right.}}{21b\left(\cos^2\left(\cdots\right.\right.}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\,d\cdots}}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/6*cos
(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*(-1/56*cos(1/2*d*x+1/2*c)/b*(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)
```

```
^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1
/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23

$$\int \frac{(A + C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{2}(-5i\,A - 7i\,C)\sqrt{b}\cos{(dx+c)}^4\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)} + i\,\sin{(dx+c)}) + \sqrt{2}(5i\,A + 7}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*co
s(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^4(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^4}{\sqrt{b\cos\left(dx + c\right)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^4(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^4}{\sqrt{b\cos\left(dx + c\right)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^4(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)^4\,\sqrt{b\,\cos\left(c + d\,x\right)}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2)), x)
```

3.70 $\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 147

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2(7A+9C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{2(7A+9C)\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/9*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+2/45*b^2*(7*A+9*C)*sin(d*x+c)/d
/(b*cos(d*x+c))^(5/2)+2/15*(7*A+9*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2/15
*(7*A+9*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3091, 2716, 2721, 2719}

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{2(7A+9C)\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} - \frac{2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-2*(7*A + 9*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqr
t[Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (2
*b^2*(7*A + 9*C)*Sin[c + d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (2*(7*A + 9*
C)*Sin[c + d*x])/(15*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b^5 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{11/2}}\,dx \\ &= \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}\left(b^3(7A+9C)\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx \\ &= \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{1}{15}(b(7A+9C))\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\end{aligned}$$

$$
\begin{aligned}
&= \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} \\
&\quad + \frac{2(7A+9C)\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} - \frac{(7A+9C)\int\sqrt{b\cos(c+dx)}\,dx}{15b} \\
&= \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{2(7A+9C)\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} \\
&\quad - \frac{\left((7A+9C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{15b\sqrt{\cos(c+dx)}} \\
&= -\frac{2(7A+9C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} \\
&\quad + \frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{2(7A+9C)\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$
\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx
= \frac{-6(7A+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 6(7A+9C)\sin(c+dx) + 2\sec(c+dx)\left(7A+9C+5A\sec^2\right.}{45d\sqrt{b\cos(c+dx)}}
$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[b*Cos[c + d*x]],x]
[Out] (-6*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*(7*A + 9*C
)*Sin[c + d*x] + 2*Sec[c + d*x]*(7*A + 9*C + 5*A*Sec[c + d*x]^2)*Tan[c + d*
x])/(45*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. 2(155) = 310.

Time = 16.57 (sec) , antiderivative size = 731, normalized size of antiderivative = 4.97

method	result	size
default	Expression too large to display	731
parts	Expression too large to display	782

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/5*C/b/sin(1
/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*A*(-1/144*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/18
0*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*
(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*(EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))))/sin(1/2*d*x+1/2*c
)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09

$$\int \frac{(A + C\cos^2(c + dx))\sec^5(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx =$$

$$-\frac{3\sqrt{2}(7i\,A + 9i\,C)\sqrt{b}\cos{(dx + c)}^5\,\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos{(dx + c)} + i\,\text{si}}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(7*I*A + 9*I*C)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*
(-7*I*A - 9*I*C)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(7*A + 9*C)*cos(d*x +
 c)^4 + (7*A + 9*C)*cos(d*x + c)^2 + 5*A)*sqrt(b*cos(d*x + c))*sin(d*x + c)
)/(b*d*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5/(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^5}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^5}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^5\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2)), x)
```

3.71 $\int \frac{\cos^4(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/9*C*(b*cos(d*x+c))^(
7/2)*sin(d*x+c)/b^5/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/
cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2719}

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^3d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^5d}$$

```
[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sq
rt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45
*b^3*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^4}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}+\frac{(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx}{9b^4}\\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d}\\
&\quad+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}+\frac{(9A+7C)\int \sqrt{b\cos(c+dx)}\,dx}{15b^2}\\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}\\
&\quad+\frac{\left((9A+7C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{15b^2\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15b^2d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{6(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\cos^2(c+dx)(18A+}{45bd\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^
2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*b*d*Sqrt[b*Cos[c +
 d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(127) = 254.

Time = 12.55 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.82

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-72A-\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{5b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\right.\right.}}$

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-160*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*
C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x
+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{3\sqrt{2}(-9i\,A-7i\,C)\sqrt{b}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3}{}$$

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I*C
)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*sqrt(b
*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm ="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

[In] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)

[Out] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)

3.72 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^4/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c
))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3093, 2715, 2721, 2720}

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}$$

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b
*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2
*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^3} \\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}+\frac{(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx}{7b^3} \\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} \\
&\quad+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}+\frac{(7A+5C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b} \\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} \\
&\quad+\frac{\left((7A+5C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{\cos^3(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{4(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(14A+13C}{42bd\sqrt{b\cos(c+dx)}}$$

[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]

[Out] (4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*b*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(127) = 254.

Time = 10.52 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.57

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(28A+56C)\left(\sin\right.\right.}{21b\sqrt{-b(}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$

[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(48*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-7i\,A-5i\,C)\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^2 + 7*A + 5*C)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

3.73 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 80

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos
(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3093, 2721, 2719}

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}$$

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqr
t[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}\\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{(5A+3C)\int \sqrt{b\cos(c+dx)}\,dx}{5b^2}\\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}}\\
&= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$
\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\cos(c+dx)\sin(2(c}{5bd\sqrt{b\cos(c+dx)}}
$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + C*Cos[c + d*x
]*Sin[2*(c + d*x)])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(96) = 192.

Time = 9.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.29

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{5b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}{}$

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(8*cos(1/2*
d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{b\cos(dx+c)}C\cos(dx+c)\sin(dx+c)+\sqrt{2}(5iA+3iC)\sqrt{}}{}$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(5*I*A +
3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

3.74 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 78

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3093, 2721, 2720}

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}$$

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Co
s[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b}\\
&= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b}\\
&= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}}\\
&= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx$$

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*
x)])/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(94) = 188$.

Time = 7.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.06

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\right.\right.\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{d}{2}\right.}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{d}{2}\right.\right.\right.}}{}$

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(4*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d
*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\text{si}}{}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

3.75 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=-\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3091, 2721, 2719}

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos
[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{(A-C)\int\sqrt{b\cos(c+dx)}\,dx}{b^2} \\
&= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}} \\
&= -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int\frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x]
)/(b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(94) = 188.

Time = 7.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.92

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$-\frac{2A\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\right.}\right.}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}}$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2
*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \frac{\sqrt{2}(-i\,A + i\,C)\sqrt{b}\cos\left(dx + c\right)\,\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4,}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A - I*C)*sqr
t(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^2*d*co
s(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\left(b\,\cos\left(c + d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2), x)
```

3.76 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3091, 2721, 2720}

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx\\ &= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b}\\ &= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}}\\ &= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.00 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.87

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{4(A+C\cos^2(c+dx))\left((A+3C)\cos^2(c+dx)\sqrt{\cos^2(dx-\arctan(\cot(c)))}\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{arc}\right.\right.}{3d(b\cos(c+dx))^{3/2}(2A+C+C\cos(2(c+dx)))}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-4*(A + C*Cos[c + d*x]^2)*((A + 3*C)*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcTan[
Cot[c]]]^2]*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Co
t[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]] - A*Sqrt[Csc[c]^2]*Sin[c + d*x]))/(3*d*
(b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2*(c + d*x)])*Sqrt[Csc[c]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(91) = 182$.

Time = 7.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.92

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+3C)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+A\sqrt{\cdots}\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\cdots\right)\right)\right)}\cdots}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx\cdots}{2}}\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\cdots\right)}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/b*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.55

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-i\,A-3i\,C)\sqrt{b}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(\cdots}{\cdots}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
```

```
c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*
cos(d*x + c))*A*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(A+C\cos^2{(c+dx)})\sec{(c+dx)}}{(b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(C\cos{(dx+c)}^2+A\right)\sec{(dx+c)}}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(C\cos{(dx+c)}^2+A\right)\sec{(dx+c)}}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos{(c+d\,x)}^2+A}{\cos{(c+d\,x)}\,{(b\,\cos{(c+d\,x)})}^{3/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)), x)
```

3.77 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 113

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*C)*sin(d*x+c)/b/d/(b*c
os(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*
x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2719}

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sq
rt[Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*
(3*A + 5*C)*Sin[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} - \frac{(3A+5C)\int \sqrt{b\cos(c+dx)}\,dx}{5b^2}
\end{aligned}$$

$$= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} - \frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int\frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(3A+5C)\sin(\ldots\right.}{5bd\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*
C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b*d*Sqrt[b*Cos[c + d*x]]
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(125) = 250.

Time = 12.03 (sec) , antiderivative size = 567, normalized size of antiderivative = 5.02

method	result
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\mathrm{co}\ldots\right.\right.}{}$
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\ldots\right.\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5*A*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(
1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d
```

```
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*
sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d-2*C/b*(-
2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.23

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-3i\,A - 5i\,C)\sqrt{b}\cos{(dx+c)}^3 \,\text{weierstrassZeta}(-4, 0, \text{weiers}}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A + C\cos^2{(c+dx)})\sec^2{(c+dx)}}{(b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c + d*x))**(3/2), x
)
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{(C\cos{(dx + c)}^2 + A)\sec{(dx + c)}^2}{(b\cos{(dx + c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{(C\cos{(dx + c)}^2 + A)\sec{(dx + c)}^2}{(b\cos{(dx + c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos{(c + d\,x)}^2 + A}{\cos{(c + d\,x)}^2\,(b\,\cos{(c + d\,x)})^{3/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)), x)
```

3.78 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 112

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*(5*A+7*C)*sin(d*x+c)/d/(b*
cos(d*x+c))^(3/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x
+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3091, 2716, 2721, 2720}

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*
(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^3 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}(b(5A+7C))\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{(5A+7C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b} \\
&= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{\left((5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b\sqrt{b\cos(c+dx)}} \\
&= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + (5A+7C\right.}{21bd\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C +
 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(124) = 248.

Time = 11.48 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.70

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{6b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2(\cdots}}{3\sqrt{-b\left(2\left(\sin^4\right(\cdots}}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-\cdots}\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(C*(-1/6*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)
^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1
/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.21

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-5i\,A-7i\,C)\sqrt{b}\cos\left(dx+c\right)^4\text{weierstrassPInverse}(-4,0,\cos$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*
cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^3}{\left(b\cos\left(dx + c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)^3\left(b\,\cos\left(c + d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2)), x)
```

3.79 $\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{\cos^5(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/9*C*(b*cos(d*x+c))^(
7/2)*sin(d*x+c)/b^6/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/
cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2719}

$$\int \frac{\cos^5(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^4d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^6d}$$

```
[In] Int[(Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sq
rt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45
*b^4*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^6*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^5} \\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}+\frac{(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx}{9b^5} \\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} \\
&\quad +\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}+\frac{(9A+7C)\int \sqrt{b\cos(c+dx)}\,dx}{15b^3} \\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} \\
&\quad +\frac{\left((9A+7C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{15b^3\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{\cos^5(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{6(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(18A+}{45b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
[Out] (6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^
2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*b^2*d*Sqrt[b*Cos[c
 + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(127) = 254.

Time = 12.09 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.82

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-72A-29\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{5b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\right.\right.\right.}}$

```
[In] int(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVE
RBOSE)
[Out] -2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-160*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+13
6*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d
*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{3\sqrt{2}(-9i\,A-7i\,C)\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+}{}$$

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I*C
)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*sqrt(b
*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**5*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^5}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^5(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^5}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^5\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((cos(c + d*x)^5*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^5*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.80 $\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^5/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x
+c))^(1/2)/b^3/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2720}

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}$$

```
[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt
[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b
^3*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^4}\\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}+\frac{(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx}{7b^4}\\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d}\\
&\quad+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}+\frac{(7A+5C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b^2}\\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}\\
&\quad+\frac{\left((7A+5C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^2\sqrt{b\cos(c+dx)}}
\end{aligned}$$

$$= \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{4(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(14A+13C}{42b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C
+ 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(127) = 254.

Time = 10.38 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.57

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(28A+56C)\right(}{21b^2\sqrt{-}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{}\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}$

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(48*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+
1/2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{\cos^4(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}(-7i\,A-5i\,C)\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin$$

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm ="fricas")

[Out] 1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse (-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^2 + 7*A + 5*C) *sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^4(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2+A\right)\cos{(dx+c)}^4}{(b\cos{(dx+c)})^{\frac{5}{2}}}\,dx$$

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)

Giac [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.81 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 80

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos
(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 3093, 2721, 2719}

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}$$

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqr
t[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx}{b^3} \\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}+\frac{(5A+3C)\int\sqrt{b\cos(c+dx)}\,dx}{5b^3} \\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}+\frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}} \\
&= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int\frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\cos(c+dx)\sin(2}{5b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
[Out] (2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + C*Cos[c + d*x
]*Sin[2*(c + d*x)])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(96) = 192.

Time = 8.90 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.29

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{5b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}-\right.\right.\right.}}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}{}$

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(8*cos(1/
2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{b\cos(dx+c)}C\cos(dx+c)\sin(dx+c)+\sqrt{2}(5iA+3iC)\sqrt{b}}{}$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(5*I*A +
3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.82 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 78

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*C*si
n(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3093, 2721, 2720}

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}$$

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*
x)])/(3*b^2d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(94) = 188.

Time = 7.94 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.06

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{d}{2}\right.\right.\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}\right.}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2C\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}}{}$

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2
*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\text{si}}{}$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.83 $\int \frac{\cos(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 74

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))
^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3091, 2721, 2719}

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos
[c + d*x]]) + (2*A*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx}{b} \\
&= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{(A-C)\int \sqrt{b\cos(c+dx)}\,dx}{b^3} \\
&= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^3\sqrt{\cos(c+dx)}} \\
&= -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x]
)/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(94) = 188.

Time = 7.69 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.92

method	result
default	$\frac{2\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))b+b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(2A\cos(\frac{dx}{2}+\frac{c}{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,E(\cos(\frac{dx}{2}+\frac{c}{2}}{b^2\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+}}$
parts	$-\frac{2A(-2\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))b+b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(\sin^2(\frac{dx}{2}+\frac{c}{2}))+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{-2(\sin^4(}{b^2\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}}$

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \frac{\cos(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}(-i\,A+i\,C)\sqrt{b}\cos{(dx+c)}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassP}}{}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] (sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A - I*C)*sqr
t(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^3*d*co
s(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.84 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3091, 2721, 2720}

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{(A+3C)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}\\
&= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int\frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2\left((A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
[Out] (2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x]
))/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(94) = 188.

Time = 6.58 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.77

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+3C)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+A\sqrt{\frac{1}{2}\cdots}\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\cdots\right)\right)\right)}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\cdots\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\cdots\right)}$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/b^2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}(-i\,A-3i\,C)\sqrt{b}\cos\left(dx+c\right)^2 \text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\cdots}{\cdots}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*
cos(d*x + c))*A*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2), x)
```

3.85 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^2/d/(b*c
os(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*
x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 3091, 2716, 2721, 2719}

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sq
rt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(3
*A + 5*C)*Sin[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b} \\
&= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}-\frac{(3A+5C)\int \sqrt{b\cos(c+dx)}\,dx}{5b^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} \\
&\quad - \frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}} \\
&= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$
\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(3A+5C)\sin(c\right.}{5b^2d\sqrt{b\cos(c+dx)}}
$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*
C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^2*d*Sqrt[b*Cos[c + d*x
]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(124) = 248$.

Time = 11.63 (sec) , antiderivative size = 567, normalized size of antiderivative = 5.06

method	result
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}E\left(\cos\right.\right.}{}$
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\right.\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/5*A*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(
1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d
```

```
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*
sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d-2*C/b^2*
(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1
/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.24

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \frac{\sqrt{2}(-3i\,A - 5i\,C)\sqrt{b}\cos{(dx + c)}^3\,\text{weierstrassZeta}(-4, 0, \text{weierstr}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)),x)

[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)), x)

3.86 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 113

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*(5*A+7*C)*sin(d*x+c)/b/d/(b*
cos(d*x+c))^(3/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d
*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3091, 2716, 2721, 2720}

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}+\frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt
[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*
(5*A + 7*C)*Sin[c + d*x])/(21*b*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{(5A+7C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b^2} \\
&= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{\left((5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\Big((5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C}{21b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C +
 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(125) = 250.

Time = 10.28 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.66

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{6b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(c}}{3\sqrt{-b(2(\sin^4(}}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-c}\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(C*(-1/6
*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b
*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x
+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.19

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}(-5i\,A - 7i\,C)\sqrt{b}\cos{(dx+c)}^4 \operatorname{weierstrassPInverse}(-4, 0, \cos$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*
cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2 + A\right)\sec{(dx+c)}^2}{\left(b\cos{(dx+c)}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos(dx+c)^2 + A\right)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos(c+d\,x)^2 + A}{\cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^{5/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2)), x)
```

3.87 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx=-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^3/d/(b
*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^4/d/cos(
d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3091, 2716, 2721, 2719}

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx=-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sq
rt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (2*
(3*A + 5*C)*Sin[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b^2} \\
&= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{(3A+5C)\int \sqrt{b\cos(c+dx)}\,dx}{5b^4} \\
&= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} \\
&\quad - \frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^4\sqrt{\cos(c+dx)}} \\
&= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(3A+5C)\sin(c+dx)+A\,\mathrm{se}\right.}{5b^3d\sqrt{b\cos(c+dx)}}$$

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]

[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(127) = 254.

Time = 11.80 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.93

method	result
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}E\left(\mathrm{co}\right.\right.}{}$
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\right.\right.}{}$

[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(7/2),x,method=_RETURNVERBOSE)

[Out] -2/5*A*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d-2*C/b^3*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\,dx = \frac{\sqrt{2}(-3i\,A - 5i\,C)\sqrt{b}\cos{(dx + c)}^3\,\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^4*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\,dx = \int \frac{C\cos{(dx + c)}^2 + A}{(b\cos{(dx + c)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2 + A}{\left(b\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + A}{\left(b\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2), x)
```

3.88 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^4d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(7/2)+2/21*(5*A+7*C)*sin(d*x+c)/b^3/d/(
b*cos(d*x+c))^(3/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(b*cos
(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3091, 2716, 2721, 2720}

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^4d\sqrt{b\cos(c+dx)}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^4*d*Sqrt
[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (2*
(5*A + 7*C)*Sin[c + d*x])/(21*b^3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}+\frac{(5A+7C)\int\frac{1}{(b\cos(c+dx))^{5/2}}\,dx}{7b^2}\\
&= \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}+\frac{(5A+7C)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b^4}\\
&= \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}+\frac{\left((5A+7C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^4\sqrt{b\cos(c+dx)}}\\
&= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^4d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.67

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{2\Big((5A+7C)\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + (5A+7C+3A\sec^2(c+}{21b^4d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C +
 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^4*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(127) = 254$.

Time = 10.55 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.60

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{6b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2(}}{3\sqrt{-b\left(2\left(\sin^4\right.\right.}}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-\frac{c}{}}\right.}{}$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4*(C*(-1/6
*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b
*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x
+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\, dx = \frac{\sqrt{2}(-5i\,A - 7i\,C)\sqrt{b}\cos{(dx + c)}^4\,\text{weierstrassPInverse}(-4, 0, \cos{(dx + c)} + i\,\text{si}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^5*d*
cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\, dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\, dx = \int \frac{C\cos{(dx + c)}^2 + A}{(b\cos{(dx + c)})^{\frac{9}{2}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\left(b\,\cos\left(c + d\,x\right)\right)^{9/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2), x)
```

3.89 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\ dx$

Optimal result

Integrand size = 35, antiderivative size = 116

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\ dx = \frac{(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(A+2C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*(A+2*C)*sin(d*
x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/5*C*sin(d*x+c)^5*(b*cos(d*
x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3092, 380}

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\ dx = -\frac{(A+2C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{C\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((A +
2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (C*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{b\cos(c+dx)}\text{Subst}\left(\int \left(1-x^2\right)\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{b\cos(c+dx)}\text{Subst}\left(\int \left(A\left(1+\frac{C}{A}\right)-(A+2C)x^2+Cx^4\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(A+2C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}(100A+89C+4(5A+7C)\cos(2(c+dx))+3C\cos(4(c+dx)))\sin(c+dx)}{120d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3*C*
Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.59 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

method	result
default	$\frac{\left(3C\left(\cos^4(dx+c)\right)+5A\left(\cos^2(dx+c)\right)+4C\left(\cos^2(dx+c)\right)+10A+8C\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{15d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\left(2+\cos^2(dx+c)\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}+\frac{C\left(3\left(\cos^4(dx+c)\right)+4\left(\cos^2(dx+c)\right)+8\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{15d\sqrt{\cos(dx+c)}}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}\left(\sqrt{\cos}(dx+c)\right)e^{6i(dx+c)}C}{80\left(e^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{\cos(dx+c)b}\left(\sqrt{\cos}(dx+c)\right)e^{2i(dx+c)}(6A+5C)}{8\left(e^{2i(dx+c)}+1\right)d}+\frac{i\sqrt{\cos(dx+c)b}\left(\sqrt{\cos}(dx+c)\right)(6A}{8\left(e^{2i(dx+c)}+1\right)d}$

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/15/d*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10*A+8*C)*sin(d*
x+c)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\left(3\,C\cos\left(dx+c\right)^4+\left(5\,A+4\,C\right)\cos\left(dx+c\right)^2+10\,A+8\,C\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(3*C*cos(d*x + c)^4 + (5*A + 4*C)*cos(d*x + c)^2 + 10*A + 8*C)*sqrt(b*
cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{C\sqrt{b}\left(3\sin(5\,dx+5\,c)+25\sin\left(\frac{3}{5}\arctan(\sin(5\,dx+5\,c),\cos(5\,dx+5\,c))\right)+150\sin\left(\frac{1}{5}\arctan(\sin(5\,d\right.\right.}{2}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/240*(C*sqrt(b)*(3*sin(5*d*x + 5*c) + 25*sin(3/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c))) + 150*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c
)))) + 20*A*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))))/d
```

Giac [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(200\,A\,\sin\left(2\,c+2\,d\,x\right)+20\,A\,\sin\left(4\,c+4\,d\,x\right)+175\,C\,\sin\left(2\,c+2\,d\,x\right)+2\right.}{240\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int(cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(200*A*sin(2*c + 2*d*x) + 20*A*s
in(4*c + 4*d*x) + 175*C*sin(2*c + 2*d*x) + 28*C*sin(4*c + 4*d*x) + 3*C*sin(
6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

3.90 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 113

$$
\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx
&= \frac{(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} \\
&\quad + \frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}
\end{aligned}
$$

```
[Out] 1/4*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*(4*A+3*C)*x*(b
*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/8*(4*A+3*C)*sin(d*x+c)*cos(d*x+c)^(1/
2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3093, 2715, 8}

$$
\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx
&= \frac{x(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d} \\
&\quad + \frac{C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}
\end{aligned}
$$

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + ((4*A + 3*C)*
Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Cos[c + d*
x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}+\frac{\left((4A+3C)\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}\\
&= \frac{(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}\\
&\quad+\frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}+\frac{\left((4A+3C)\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{\cos(c+dx)}}\\
&= \frac{(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}\\
&\quad+\frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)]
 + C*Sin[4*(c + d*x)]))/(32*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

method	result
default	$\frac{\sqrt{\cos(dx+c)b}\,\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{C\sqrt{\cos(dx+c)b}\,\left(2\sin(dx+c)\left(\cos^3(dx+c)\right)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{i(dx+c)}x(8A+6C)}{8\,\mathrm{e}^{2i(dx+c)}+8}-\frac{i\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{5i(dx+c)}C}{32\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{i\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{-i(}}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)d}$

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
[Out] 1/8/d*(cos(d*x+c)*b)^(1/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*cos(
d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.77

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\left[\frac{2\left(2\,C\cos\left(dx+c\right)^2+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(4\,A+3\,C\right)\sqrt{-b}\log\left(2\,b\,c\right.}{16\,d}\right.$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c) + (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/8*((2*C
*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{8\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))A\sqrt{b}+\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\text{co}\right.\right.\right.}{32\,d}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + (12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*sqr
t(b))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(95) = 190.

Time = 2.96 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.20

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{4\,A\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+3\,C\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+16\,A\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+12\,C\sqrt{b}dx\tan\left(\frac{1}{2}\right.}{}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/8*(4*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^8 + 3*C*sqrt(b)*d*x*tan(1/2*d*x +
 1/2*c)^8 + 16*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 12*C*sqrt(b)*d*x*tan(
1/2*d*x + 1/2*c)^6 - 8*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 - 10*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^7 + 24*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 18*C*sqrt(b)
*d*x*tan(1/2*d*x + 1/2*c)^4 - 8*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 6*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^5 + 16*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 12*C
*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 8*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 -
6*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 4*A*sqrt(b)*d*x + 3*C*sqrt(b)*d*x + 8*
A*sqrt(b)*tan(1/2*d*x + 1/2*c) + 10*C*sqrt(b)*tan(1/2*d*x + 1/2*c))/(d*tan(
1/2*d*x + 1/2*c)^8 + 4*d*tan(1/2*d*x + 1/2*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^
4 + 4*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(c+d\,x\right)+8\,C\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(3\,c+3\,d\,x\right)+9\,C\,\sin\left(3\,c\right.\right.}{32\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) + 3
2*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x) + 1)
)
```

3.91 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\, d$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{C\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] (A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*C*sin(d*x+c)^3
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {17, 3092}

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{C\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (C*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
```

```
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int\cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{\sqrt{b\cos(c+dx)}\text{Subst}\left(\int\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{C\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\begin{aligned}
&\int\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx\\
&= \frac{\sqrt{b\cos(c+dx)}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{\cos(c+dx)}}
\end{aligned}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*S
qrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result
default	$\frac{\left(C\left(\cos^{2}(dx+c)\right)+3A+2C\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{C\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)e^{4i(dx+c)}C}{12\left(e^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)e^{2i(dx+c)}(4A+3C)}{4\left(e^{2i(dx+c)}+1\right)d}+\frac{i\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)(4A}{4\left(e^{2i(dx+c)}+1\right)d}$

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*sin(d*x+c)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\left(C\cos\left(dx+c\right)^2+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqr
t(cos(d*x + c)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(68) = 136.

Time = 29.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.88

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \begin{cases} x\sqrt{b\cos\left(c\right)}\left(A+C\cos^{2}\left(c\right)\right)\sqrt{\cos\left(c\right)} & \text{for}\: d=0 \\ 0 & \text{for}\: c=-dx+\frac{\pi}{2}\vee c=- \\ \frac{A\sqrt{b\cos\left(c+dx\right)}\sin\left(c+dx\right)}{d\sqrt{\cos\left(c+dx\right)}}+\frac{2C\sqrt{b\cos\left(c+dx\right)}\sin^{3}\left(c+dx\right)}{3d\sqrt{\cos\left(c+dx\right)}}+\frac{C\sqrt{b\cos\left(c+dx\right)}\sin\left(c+dx\right)\cos^{\frac{3}{2}}\left(c+dx\right)}{d} & \text{otherwise} \end{cases}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(b*cos(c))*(A + C*cos(c)**2)*sqrt(cos(c)), Eq(d, 0)), (0,
Eq(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (A*sqrt(b*cos(c + d*x))*sin(c +
 d*x)/(d*sqrt(cos(c + d*x))) + 2*C*sqrt(b*cos(c + d*x))*sin(c + d*x)**3/(3*
d*sqrt(cos(c + d*x))) + C*sqrt(b*cos(c + d*x))*sin(c + d*x)*cos(c + d*x)**(
3/2)/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{C\sqrt{b}\left(\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)+12\,A\sqrt{b}\sin\left(dx+c\right)}{12\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(C*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))) + 12*A*sqrt(b)*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79987 vs. 2(64) = 128.

Time = 7.58 (sec) , antiderivative size = 79987, normalized size of antiderivative = 1080.91

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/96*(3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x +
 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 24*
C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
+ 1/2*c)^3*tan(1/3*c)^6*tan(c) - 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*
tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^4*tan(1/3*c)^4*tan(c)^2 - 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*ta
n(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 48*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1
/2*c)^3*tan(1/3*c)^6*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*ta
```

n(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 18*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1
/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*ta
n(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 18*C*sqrt(b)*d*
x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*t
an(1/3*c)^6 + 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c
)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x +
1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 18*C
*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
1/2*c)^4*tan(1/3*c)^6 - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c) - 72*C*sqrt(b)*d
*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*
tan(1/3*c)^4*tan(c) + 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c) + 144*C*sqrt(b)*d*x^4
*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(
1/3*c)^6*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1
/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c) + 144*C*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/
3*c)^6*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6
*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 24*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^
6*tan(c) + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*
tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^
6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^
4*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)
^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c)^2 + 48*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
*tan(1/3*c)^6*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 108*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
^2*tan(1/3*c)^6*tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1
/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 + 48*C*sqrt
(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
^3*tan(1/3*c)^6*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 54*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)
^4*tan(1/3*c)^6*tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/
2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 54*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1
/2*c)^2*tan(1/3*c)^4 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d

*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 27*C*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*
c)^4 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^
4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6 - 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 54*C*sqr
t(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2
*c)^2*tan(1/3*c)^6 - 108*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
 + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 144*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c
)^6 - 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1
/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*t
an(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 54*C*sqrt(b)*d
*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*
tan(1/3*c)^6 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*
c)^4*tan(1/3*c)^6 - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c) - 72*C*sqrt(b)*d*x^4*
tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1
/3*c)^2*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/
6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) + 432*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c
)^4*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c
)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c) + 432*C*sqrt(b)*d*x^4*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)
^4*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)
^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*t
an(c) - 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*ta
n(1/3*c)^6*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c) - 144*C*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*
c)^6*tan(c) + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*
c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c) - 144*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^
6*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2
*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c) + 432*C*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*
tan(c) - 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*
tan(1/3*c)^6*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
 + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 72*C*sqrt(b)*d*x^
4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1
/3*c)^6*tan(c) + 24*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6
*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 3*C*sqrt(b)*d*x^4*tan(1/2*d*
x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 54*C
*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +

1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^
3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 + 27
*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x
 + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)
^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 9
*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4
*tan(c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)
^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 162*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^
4*tan(c)^2 + 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c
)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 432*C*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*
c)^4*tan(c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*
c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*
c)^4*tan(c)^2 - 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/
6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 9*C*sqrt(b)*d*x^4*ta
n(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 96*C*s
qrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^
4*tan(1/3*c)^5*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^4*tan(1/3*c)^6*tan(c)^2 - 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c)^2 + 144*C*sqrt(b)*d*x^4*ta
n(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c
)^6*tan(c)^2 - 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)
^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6
*tan(c)^2 + 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)
^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 18*C*sqrt(b)*d*x^4*tan(1
/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 144*C*sqr
t(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2
*c)^3*tan(1/3*c)^6*tan(c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(
1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 - 24*C*sqr
t(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*
tan(1/3*c)^6*tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d
*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 +
 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c
)^6*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*t
an(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x +
1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)
^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d
*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c
)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4 + 54*C*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*
c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*ta

n(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c
)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 54*C*sqrt
(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*
c)^4*tan(1/3*c)^2 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
1/6*c)^6*tan(1/3*c)^4 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4 + 162*C*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*
c)^4 - 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*
c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 144*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*
c)^3*tan(1/3*c)^4 - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 162*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^
4 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/
3*c)^4 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/
2*d*x + 1/2*c)^4*tan(1/3*c)^5 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*ta
n(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6 + 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)
^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x +
1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 48*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2
*c)*tan(1/3*c)^6 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 - 324*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6
 + 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/
3*c)^6 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*
tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 + 24*C*sqr
t(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*
tan(1/3*c)^6 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*
c)^4*tan(1/3*c)^6 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x +
 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 9*C*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 96*C*sqrt(b)*tan(1/2*
d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6
- 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
+ 1/2*c)^4*tan(1/3*c)^6 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 24*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(c) - 24
*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
 + 1/2*c)^4*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c) + 432*C*sqrt(b)*d*x^4*
tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^2*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1
/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c) + 432*C*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/

3*c)^2*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/
6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) + 72*C*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
^2*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^
6*tan(1/3*c)^4*tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) - 432*C*sqrt(b)*d*
x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan
(1/3*c)^4*tan(c) + 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c) - 432*C*sqrt(b)*d*x^
4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^4*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1
/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c) + 1296*C*sqrt(b)*d*x^4*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1
/3*c)^4*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1
/2*c)^3*tan(1/3*c)^4*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*ta
n(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) + 216*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*
c)^4*tan(1/3*c)^4*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) - 72*C*sqrt(b)*d*x^4
*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6*tan(c) + 24*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c
) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1
/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c) - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c) + 24
*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*
tan(c) + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*
tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c) - 432*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan
(c) - 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan
(1/3*c)^6*tan(c) + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x +
1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c) - 72*C*sqrt(b)*d*x^4*
tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c) - 24*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*ta
n(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-
1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^
4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 48*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^4*tan(1/3*c)^6*tan(c) - 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 48*C*sqrt(b)*d*x^4*tan(1
/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 +
9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d
*x + 1/2*c)^4*tan(c)^2 - 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c)^2 + 144*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)

*tan(1/3*c)^2*tan(c)^2 - 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 324*C*sqrt(
b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^2*tan(1/3*c)^2*tan(c)^2 - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(
1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 + 144*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*
c)^3*tan(1/3*c)^2*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(
1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 162*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/
2*c)^4*tan(1/3*c)^2*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 320*C*sqrt(b)*tan(1/2*d*x + 1/
2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2
 + 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3
*c)^4*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/
6*c)^6*tan(1/3*c)^4*tan(c)^2 + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*t
an(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 144*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2
*c)*tan(1/3*c)^4*tan(c)^2 - 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(
1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 972*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/
2*c)^2*tan(1/3*c)^4*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 432*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*t
an(c)^2 + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*t
an(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1
/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^
2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/
3*c)^4*tan(c)^2 - 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x +
1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 27*C*sqrt(b)*d*x^4
*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 144
0*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/
2*c)^4*tan(1/3*c)^4*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2
*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 432*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*ta
n(1/3*c)^4*tan(c)^2 + 192*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/
6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 - 1440*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
^5*tan(c)^2 + 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c)^2 - 384*C*sqrt(b)*tan(1/2*d*x +
 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c
)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1
/3*c)^6*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x +
1/6*c)^4*tan(1/3*c)^6*tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*t
an(1/3*c)^6*tan(c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d
*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 144*C*sqrt(b)*d

*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(
1/3*c)^6*tan(c)^2 - 24*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c
)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 18*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 324*C*sqrt
(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*
c)^2*tan(1/3*c)^6*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(
-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 192*C*sqrt(b)*tan(1/2*d*x + 1/2
*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2
- 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
+ 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 288*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan
(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 48*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan
(c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan
(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/2
*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2
- 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
1/2*c)^3*tan(1/3*c)^6*tan(c)^2 - 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2
*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 320*C*sqrt(b
)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan
(1/3*c)^6*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6
*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 432*C*sqrt(b)*tan(1/2
*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6
*tan(c)^2 - 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan
(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/
2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 -
48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^4*tan(1/3*c)^6*tan(c)^2 + 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*ta
n(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2 + 48*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3 - 9*C*sqrt(b)*
d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4
+ 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/
2*d*x + 1/2*c)^4 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1
/6*c)^6*tan(1/3*c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d
*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 162*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c
)^2 - 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c
)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 - 144*C*sqr
t(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^3*tan(1/3*c)^2 - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 162*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2
- 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3
*c)^2 - 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/

2*d*x + 1/2*c)^4*tan(1/3*c)^3 - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*t
an(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c
)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4 - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x +
 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4 + 144*C
*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1
/2*c)*tan(1/3*c)^4 + 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
 + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 972*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c
)^4 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan
(1/3*c)^4 + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)
^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x +
 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 + 72*C*
sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
^3*tan(1/3*c)^4 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1
/2*c)^4*tan(1/3*c)^4 + 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d
*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 27*C*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 1440*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/
3*c)^4 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1
/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1
/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 192*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c
)^5 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2
*d*x + 1/2*c)^4*tan(1/3*c)^5 + 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/
2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 - 384*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
^5 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/
3*c)^6 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*t
an(1/3*c)^6 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6 - 144*C
*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x +
 1/2*c)*tan(1/3*c)^6 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x
 + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 24*C*sqrt(b)*tan(1/2*d*x +
 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 18*C*
sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 -
 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2
*d*x + 1/2*c)^2*tan(1/3*c)^6 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 192*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*t
an(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 - 384*A*sqrt(b)*
tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^6 - 288*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-
1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3
*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1
/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 + 72*C*sq
rt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3
*tan(1/3*c)^6 + 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*

tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 + 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(c) + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(c) + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(c) + 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c) - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c) + 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c) - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c) + 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) - 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) + 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c) - 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c) + 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2

```
*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c) -
216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*
c)^4*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*
c)^4*tan(1/3*c)^4*tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2
*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) + 216*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(
1/3*c)^4*tan(c) + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)
^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*tan(c) + 72*C*sqrt(b)*d
*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6*tan(c) - 24*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c)
 + 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c
)^6*tan(c) - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c
)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c) + 144*C*sqrt(b)*d*
x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c) - 43
2*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x
 + 1/2*c)^2*tan(1/3*c)^6*tan(c) + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2
*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c) + 24*C*sqrt(b)*d*
x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 72*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c
)^4*tan(1/3*c)^6*tan(c) + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
 + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 24*C*sqrt(b)*tan(1
/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 3*C*sqrt(b)
*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(c)^2 + 48*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^
4*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 108*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 144*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)
^3*tan(c)^2 + 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^
6*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)
^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 54*C*sqrt(b)*d
*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*
tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4
*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan(1/2*d*x +
1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*tan(c)^2 - 54*C*sqrt(b)*d*x^4*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c)^2 + 432*C
*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x +
 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*ta
n(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 162*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/
2*c)^2*tan(1/3*c)^2*tan(c)^2 + 972*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*t
```

an(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 54*C*
sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*t
an(c)^2 - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2
*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 + 432*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*
tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-
1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x +
1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 162*C*sqrt(b)*d*x^
4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan
(1/3*c)^2*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x
 + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*t
an(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 576*A
*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^4*tan(1/3*c)^2*tan(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d
*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 640*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(
1/3*c)^3*tan(c)^2 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6
*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2 - 5760*C*sqrt(b)*tan(1/
2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^
3*tan(c)^2 + 1280*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*t
an(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4*tan(c)^2 - 162*C*sqrt(b)*
d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c)^2 +
 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c)^2 + 432*C*sqr
t(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2
*c)*tan(1/3*c)^4*tan(c)^2 - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/
2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 72*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/
3*c)^4*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x +
1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 972*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 162
*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^
4*tan(c)^2 - 2880*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*t
an(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x +
 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c
)^2 + 864*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*
d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^3*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 + 432*C*sqrt(b)*d*x^4*
tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3
*c)^4*tan(c)^2 - 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^
4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x
 + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan
(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*t
an(1/3*c)^4*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d
*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4

*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 172
8*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/
2*c)^4*tan(1/3*c)^4*tan(c)^2 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1
/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 5760*C*sq
rt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4
*tan(1/3*c)^4*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1
/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 1008*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^
4*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan
(1/3*c)^5*tan(c)^2 - 2880*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/
6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 + 3456*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)
^5*tan(c)^2 - 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*t
an(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x +
 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c
)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^3*tan(-1/2
*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c)^2 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)
^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c)^2 - 1
728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/
2*c)^4*tan(1/3*c)^5*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2
*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)^4*tan(1/3*c)^6*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan
(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x +
 1/6*c)^4*tan(1/3*c)^6*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1
/2*d*x + 1/6*c)^5*tan(1/3*c)^6*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c
)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*
x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c)^2 + 48*C*sqrt(b)*d*
x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 14
4*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x
+ 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/
2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 48*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/
3*c)^6*tan(c)^2 + 108*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x +
 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2
*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 640*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(
c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/
2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 864*C*sqrt(b)*tan(1/2*d*x + 1/2*c)
^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 7
68*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1
/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 96*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/
3*c)^6*tan(c)^2 + 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/
2*c)^3*tan(1/3*c)^6*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*

d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 - 144*C*sqrt(b
)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan
(1/3*c)^6*tan(c)^2 - 24*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2
*c)^3*tan(1/3*c)^6*tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^4*tan
(1/3*c)^6*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*
c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*
x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*ta
n(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-
1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 1280*C*sqrt(b)*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2
- 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x +
1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/
2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 96*C*sqrt(
b)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 3
*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6 - 48*C*sqrt(
b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
) + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1
/2*d*x + 1/2*c)^2 - 108*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*
c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3 - 48*C*sqrt(b)*d*x^4*ta
n(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3 - 9*C*sqr
t(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2
*c)^4 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*ta
n(-1/2*d*x + 1/2*c)^4 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d
*x + 1/2*c)^4 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c
)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x +
1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2 - 432*C*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 +
144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*
x + 1/2*c)*tan(1/3*c)^2 + 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/
2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 972*C*sqrt(b)*d*x^4
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(
1/3*c)^2 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^
2*tan(1/3*c)^2 + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1
/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 - 432*C*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 +
72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1
/2*c)^3*tan(1/3*c)^2 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*
x + 1/2*c)^4*tan(1/3*c)^2 + 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(
1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 27*C*sqrt(b)*d*x^
4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 1440*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*t
an(1/3*c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*t
an(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3

*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 640*C*sqrt(b
)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan
(1/3*c)^3 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*ta
n(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*
tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 + 1280*C*sqrt(b
)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan
(1/3*c)^3 - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^
2*tan(1/3*c)^4 + 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1
/6*c)^4*tan(1/3*c)^4 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^
4 - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-
1/2*d*x + 1/2*c)*tan(1/3*c)^4 + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*ta
n(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4 + 72*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^
4 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^4 - 972*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2
*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1
/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 2880*C*sqrt(b)*tan(1/2*d*x +
 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 115
2*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^2*tan(1/3*c)^4 + 864*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/
6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 144*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 432*C*sqrt(b)*d*x^4*ta
n(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c
)^4 + 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*
d*x + 1/2*c)^3*tan(1/3*c)^4 + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 + 54*C*sqrt(b)*d*x^4*ta
n(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 27*C*sqrt(b)*d*
x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 4800*C*sq
rt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4
*tan(1/3*c)^4 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^
4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*
c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 5760*C*s
qrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^
4*tan(1/3*c)^4 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*
tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 96*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^5 - 2880*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(
1/3*c)^5 + 3456*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan
(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 - 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 + 1440*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1
/3*c)^5 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^3*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 - 1728*C*sqrt(b)*

tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3
*c)^5 + 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3
*c)^5 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 54*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6 - 9*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6 + 96*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^6 - 192*A*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6 - 144*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6 - 48*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 144*C*sqrt(b)*d*x^4*ta
n(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^
6 + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x
 + 1/2*c)*tan(1/3*c)^6 + 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x +
1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 - 108*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 54*C*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 - 640*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3
*c)^6 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1
/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 864*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1
/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 - 768*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c
)^6 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*
x + 1/2*c)^2*tan(1/3*c)^6 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x +
 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 - 48*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 + 72*C*sqrt(b)*tan(1/2*d
*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 +
 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x +
 1/2*c)^3*tan(1/3*c)^6 + 24*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
 1/2*c)^3*tan(1/3*c)^6 - 3*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*
c)^6 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*
x + 1/2*c)^4*tan(1/3*c)^6 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*
x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 1008*C*sqrt(b)*tan(1/2*
d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6
+ 1280*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x
 + 1/2*c)^4*tan(1/3*c)^6 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x +
 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 1008*C*sqrt(b)*tan(1/2*d*x
 + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 96*
C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 24*
C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(c) + 72*C
*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x +
 1/2*c)*tan(c) - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1
/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(c) + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c) - 144*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2
*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*
tan(-1/2*d*x + 1/2*c)^3*tan(c) + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2

*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(c) - 24*C*sqrt(b)*d*x^4
*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(c) - 72*C*sqrt(b)*d*x^4
*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(
c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/
2*d*x + 1/2*c)^4*tan(c) + 24*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x
+ 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*tan(c) + 72*C*sqrt(b)*d*x^4*
tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c) + 216*C*sqr
t(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2
*c)*tan(1/3*c)^2*tan(c) - 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c) + 72*C*sqrt(b)*
d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c) + 12
96*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d
*x + 1/2*c)^2*tan(1/3*c)^2*tan(c) - 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*
c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c) - 72*
C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2
*tan(c) + 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^
2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c) - 72*C*sqrt(b)*
d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) +
216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*
x + 1/2*c)^4*tan(1/3*c)^2*tan(c) + 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan
(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) + 144*C*sqr
t(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*
tan(1/3*c)^2*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*
x + 1/6*c)^2*tan(1/3*c)^4*tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)
*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/
2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c) + 72*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) - 1296*C*sqrt(
b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c
)*tan(1/3*c)^4*tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2
*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c
)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c) - 1296*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^
4*tan(c) + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)
^3*tan(1/3*c)^4*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/
2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) + 216*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan
(c) + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*
d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*ta
n(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) - 24*C*sqrt(b)*d*x^4*tan(1/2*d*x
+ 1/2*c)^3*tan(1/3*c)^6*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*ta
n(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c
)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6*tan(c) - 48*C*sqrt(b)*tan(1/2*d*x +

1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c) - 144*C*sqrt(b)*d*x^4*
tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c) + 72*C*sqr
t(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)
 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3
*c)^6*tan(c) - 24*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(
c) + 24*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
^6*tan(c) + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan
(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c
)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(c)^2 - 18*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(c)^2 + 144*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(c)^2 -
48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
 + 1/2*c)*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
+ 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 324*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 18*
C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 1
44*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d
*x + 1/2*c)^3*tan(c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d
*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 - 24*C*sqrt(b)*tan(1/2*d*x +
 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 + 3*C*sqr
t(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 54*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1
/2*c)^4*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x +
1/2*c)^4*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c
)^5*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3
*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 48*C*sqrt(b)*tan
(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2
 + 192*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
 + 1/2*c)^2*tan(1/3*c)*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan
(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 1728*C*sq
rt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4
*tan(1/3*c)*tan(c)^2 - 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1
/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 27*C*sqrt(b)*d*x^4*ta
n(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2*tan(c)^2 - 162*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*tan
(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c)^2 + 43
2*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*
x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)
*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 72*C*
sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
*tan(1/3*c)^2*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2
*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 972*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1
/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^
2 - 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(

1/3*c)^2*tan(c)^2 + 2880*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6
*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 1152*A*sqrt(b)*tan(1/
2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^
2*tan(c)^2 - 2016*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*t
an(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 + 432*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3
*tan(1/3*c)^2*tan(c)^2 - 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 - 144*C*sqrt(b)*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*
c)^2*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/
2*c)^4*tan(1/3*c)^2*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*ta
n(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 4800*C*sqrt(b)*tan(1/2*d*x +
1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)
^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*
d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^
3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 57
60*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1
/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 432*C*sqrt(b
)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1
/3*c)^2*tan(c)^2 - 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c
)^6*tan(1/3*c)^3*tan(c)^2 + 9600*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d
*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 - 11520*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*ta
n(1/3*c)^3*tan(c)^2 + 2560*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1
/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 - 4800*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c
)^3*tan(c)^2 + 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^
3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2 - 19200*C*sqrt(b)*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3*t
an(c)^2 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(-1
/2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2 - 320*C*sqrt(b)*tan(1/2*d*x + 1/6*c
)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1
/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 162*C*sqrt(b)*d*x^4*tan(1/2*d*x +
1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4*tan(c)^2 + 27*C*sqrt(b)*d*x^4
*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^4*tan(c)^2 - 576*A*sqrt(b)*tan
(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c)^2 + 432*C*sq
rt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c)^2 +
144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)*tan(1/3*c
)^4*tan(c)^2 - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c
)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 216*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c
)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*

d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 + 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*
c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 162*C*sqrt(b)*d*x^4*ta
n(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 9600*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c
)^2*tan(1/3*c)^4*tan(c)^2 - 3456*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d
*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 23328*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*ta
n(1/3*c)^4*tan(c)^2 + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x +
1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 1152*A*sqrt(b)*tan
(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)
^4*tan(c)^2 - 2016*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 - 216*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/
3*c)^4*tan(c)^2 - 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)
^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x
 + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 + 9*C*sqrt(b)*d*x
^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d
*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan
(c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1
/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*
c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 -
 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x
 + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan
(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 9936*C*
sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4
*tan(1/3*c)^4*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x
 + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*t
an(1/2*d*x + 1/6*c)^4*tan(1/3*c)^5*tan(c)^2 + 1728*C*sqrt(b)*tan(1/2*d*x +
1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^5*tan(c)^2 - 384*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^5*tan(c)^2 + 2880*C*sqr
t(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*
tan(1/3*c)^5*tan(c)^2 - 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 + 11520*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^5*tan(c)^2 - 3456*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)
^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 + 192*C*sqrt(b)*tan(1/2*d*
x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 - 96*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c)^2 + 1728*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)
^4*tan(1/3*c)^5*tan(c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*
x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c)^2 + 5760*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/
3*c)^5*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*
c)^4*tan(1/3*c)^5*tan(c)^2 - 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(

1/3*c)^6*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*t
an(c)^2 - 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(1
/3*c)^6*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c
)^4*tan(1/3*c)^6*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*
x + 1/6*c)^4*tan(1/3*c)^6*tan(c)^2 - 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*d*x + 1/6*c)^5*tan(1/3*c)^6*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1
/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c)^2 - 48*C*sqrt(b)*tan(1/2*d
*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c)^2 - 48*C*sqrt(b)*d*x
^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 72*C*
sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)
*tan(1/3*c)^6*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x +
 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 24*C*sqrt(b)*tan(1/
2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 18*C*sqrt(b)
*d*x^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 192*C*sqrt(b)*tan(1/
2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*
tan(c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan
(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 2016*C*sqrt(b)*tan(1/2*d*x + 1
/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^
2 + 2560*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d
*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*t
an(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 2016*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)
^2*tan(1/3*c)^6*tan(c)^2 + 192*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*
x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 24*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*ta
n(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1
/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^
2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^
6*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*t
an(1/3*c)^6*tan(c)^2 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1
/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 576*A*sqrt(b
)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1
/3*c)^6*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^
2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 320*C*sqrt(b)*tan(1/2*d*x
 + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 9*C*sqrt(b)*d*x
^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4 + 18*C*sqrt(b)*d*x^4*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x
+ 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c) + 48*C*sqrt(b)*d*x^
4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c) + 54*C*
sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x +
1/2*c)^2 - 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^
4*tan(-1/2*d*x + 1/2*c)^2 + 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-
1/2*d*x + 1/2*c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
 + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3 + 24*C*sqrt(b)*tan(1/2*d

*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3 - 3*C*sqrt(b)*
d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4 + 54*C*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4 - 9*C*
sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4 - 96*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4 - 19
2*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^4 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/
2*d*x + 1/2*c)^4 + 192*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c
)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) - 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c
)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) + 1728*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*t
an(1/3*c) - 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^4*tan(1/3*c) - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4
*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2 + 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x
+ 1/6*c)^6*tan(1/3*c)^2 - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/
2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 432*C*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)
^2 + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*
x + 1/2*c)*tan(1/3*c)^2 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/
2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 972*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 162*C*sqrt(b)
*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 2880*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c
)^2*tan(1/3*c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*
c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 2016*C*sqrt(b)*tan(1/2*d*x + 1/
2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 144*C*
sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 -
432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d
*x + 1/2*c)^3*tan(1/3*c)^2 + 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d
*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 + 144*C*sqrt(b)*tan(1/2*
d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2
+ 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3
*c)^2 - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*t
an(1/3*c)^2 - 4800*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^
3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 9936*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*t
an(1/3*c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*t
an(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 432*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3
*c)^2 - 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3
*c)^3 + 9600*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1
/2*d*x + 1/2*c)^2*tan(1/3*c)^3 - 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan

(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 + 2560*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/
3*c)^3 - 4800*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-
1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 + 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*ta
n(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 - 19200*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(
1/3*c)^3 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(-
1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 - 320*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4
*tan(1/3*c)^4 + 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/
6*c)^2*tan(1/3*c)^4 - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^
4 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)
^4 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)
^4 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)
^4 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)*tan(1
/3*c)^4 + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*t
an(-1/2*d*x + 1/2*c)*tan(1/3*c)^4 + 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*ta
n(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4 + 144*C*sqrt(b)*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)
^4 - 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan
(1/3*c)^4 + 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c
)^2*tan(1/3*c)^4 + 9600*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*
c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 3456*A*sqrt(b)*tan(1/2*d*x + 1/
2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 23328*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*
c)^2*tan(1/3*c)^4 + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/
6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 1152*A*sqrt(b)*tan(1/2*d*x +
1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 2016*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^
2*tan(1/3*c)^4 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/
2*c)^3*tan(1/3*c)^4 + 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/
6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 + 432*C*sqrt(b)*tan(1/2*d*x + 1
/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 + 72*C*
sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 9*C*s
qrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 1440*C*sqrt(b)*tan(1/2*
d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 -
1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x +
1/2*c)^4*tan(1/3*c)^4 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 19200*C*sqrt(b)*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 -
1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x +
1/2*c)^4*tan(1/3*c)^4 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1
/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 1440*C*sqrt(b)*tan(1/2*d*x +
1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 1440*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^5 + 1728*C*sqrt(b)*tan(1/2*d*

x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^5 - 384*C*sqrt(b)*tan(1/2*d*
x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^5 + 2880*C*sqrt(b)*tan(1/2*d
*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 -
11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x
+ 1/2*c)^2*tan(1/3*c)^5 + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d
*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 - 3456*C*sqrt(b)*tan(1/2
*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 +
192*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5
- 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5
+ 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x +
1/2*c)^4*tan(1/3*c)^5 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
+ 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 + 5760*C*sqrt(b)*tan(1/2*d*
x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 - 14
40*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 +
18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 - 9*C*sqrt(b)*d*x^4*
tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6 - 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*
tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^6 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*
tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*
tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6 - 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^6 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*ta
n(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1
/2*d*x + 1/6*c)^6*tan(1/3*c)^6 + 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*ta
n(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(
1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 144*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6
+ 24*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 +
18*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 192*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c
)^6 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2
*d*x + 1/2*c)^2*tan(1/3*c)^6 - 2016*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/
2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 2560*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c
)^6 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d
*x + 1/2*c)^2*tan(1/3*c)^6 - 2016*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*
x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 192*C*sqrt(b)*tan(1/2*d
*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 24*C*sqrt(b)*tan(1/2*d
*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 + 144*C*sqrt(b)*tan(1/2*
d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6
+ 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6
- 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6
- 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6
- 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x +
1/2*c)^4*tan(1/3*c)^6 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1
/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 432*C*sqrt(b)*tan(1/2*d*x +
1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 320*C*

sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 72*C*
sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(c) + 24*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(c) + 72*C*sqrt
(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*
c)*tan(c) - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)
^4*tan(-1/2*d*x + 1/2*c)*tan(c) + 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6
*tan(-1/2*d*x + 1/2*c)*tan(c) + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*
tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(c) - 432*C*sqrt(b)*d*x^4
*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c)
 - 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(c)
 + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1
/2*d*x + 1/2*c)^3*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1
/2*d*x + 1/2*c)^3*tan(c) - 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1
/2*d*x + 1/2*c)^4*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*
d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x +
1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(c) + 48*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*ta
n(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*ta
n(1/3*c)^2*tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x +
1/6*c)^4*tan(1/3*c)^2*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c
)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c) - 1296*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*
tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*t
an(1/3*c)^2*tan(c) + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*
x + 1/2*c)^2*tan(1/3*c)^2*tan(c) - 1296*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c
)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c) + 432*
C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2
*tan(c) - 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^
3*tan(1/3*c)^2*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*
x + 1/2*c)^4*tan(1/3*c)^2*tan(c) + 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan
(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) + 432*C*sqr
t(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*
tan(1/3*c)^2*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^4*tan(1/3*c)^2*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(
1/3*c)^4*tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/
6*c)^2*tan(1/3*c)^4*tan(c) - 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d
*x + 1/6*c)^4*tan(1/3*c)^4*tan(c) - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c) - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x
+ 1/2*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) + 216*C*sqrt(b)*d*x^4*
tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) - 432*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c
) - 72*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c) + 72*C*s
qrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) +
 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x +

1/2*c)^4*tan(1/3*c)^4*tan(c) + 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1
/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) + 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)*tan(1/3*c)^6*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
+ 1/6*c)^2*tan(1/3*c)^6*tan(c) - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*d*x + 1/6*c)^4*tan(1/3*c)^6*tan(c) - 24*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6
*tan(1/3*c)^6*tan(c) + 24*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^
6*tan(c) + 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(
1/3*c)^6*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)
^4*tan(1/3*c)^6*tan(c) + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d
*x + 1/6*c)^2*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
d*x + 1/6*c)^4*tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(c)^2
 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1
/2*d*x + 1/2*c)*tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2
*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 24*C*sqrt(b)*tan(1/2*d*x +
 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 18*C*sqrt
(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 324*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1
/2*c)^2*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x +
 1/2*c)^2*tan(c)^2 - 192*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6
*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)
^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 96*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c)
^2 - 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^3*tan(
c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(
-1/2*d*x + 1/2*c)^3*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 - 48*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 - 18*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 9*C*
sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 320
*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2
*c)^4*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^
4*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*
tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 384*C*sqrt(b)*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2
 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
 1/2*c)^4*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c
)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*
tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)*tan(c)^2 - 2880*C*sqrt(b)*tan(1/2*d*x + 1
/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2
+ 3456*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x
 + 1/2*c)^2*tan(1/3*c)*tan(c)^2 - 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(
1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2 + 1440*C*sqr
t(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*
tan(1/3*c)*tan(c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1
/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 5760*C*sqrt(b)*tan(1/

2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*
tan(c)^2 - 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(-
1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^
6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 162*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/
2*c)^4*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^2*tan(c)^2 - 576*A*sqrt(b)*tan(1/2
*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c)^2 - 1008*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c)^2 + 14
4*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2
*tan(c)^2 - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2
*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 216*C*sqrt(b)*tan(1/2*d*x +
1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2
 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
 + 1/2*c)*tan(1/3*c)^2*tan(c)^2 + 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^
2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 162*C*sqrt(b)*d*x^4*tan(1
/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 9600*C*sq
rt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2
*tan(1/3*c)^2*tan(c)^2 - 3456*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 19872*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^2*tan(c)^2 - 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6
*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 1152*A*sqrt(b)*tan(1/
2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*
tan(c)^2 + 864*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1
/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x +
 1/2*c)*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 - 216*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)
^2*tan(c)^2 - 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*t
an(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1
/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 + 9*C*sqrt(b)*d*x^4*t
an(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x +
 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^
2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d
*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^
3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 19
200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x +
1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/
2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 11664*C*sq
rt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*t
an(1/3*c)^2*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x +
 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan
(1/2*d*x + 1/6*c)^4*tan(1/3*c)^3*tan(c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/
2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^3*tan(c)^2 + 1280*C*sqrt(b)*tan(1/

2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^3*tan(c)^2 - 9600*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*t
an(1/3*c)^3*tan(c)^2 + 38400*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x +
 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 - 38400*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/
3*c)^3*tan(c)^2 + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)
^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 - 640*C*sqrt(b)*tan(1/2*d*
x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 + 320*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2 - 5760
*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c
)^4*tan(1/3*c)^3*tan(c)^2 + 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2 - 19200*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan
(1/3*c)^3*tan(c)^2 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1
/2*c)^4*tan(1/3*c)^3*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*t
an(1/3*c)^4*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)
^4*tan(c)^2 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*
tan(1/3*c)^4*tan(c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x +
 1/6*c)^4*tan(1/3*c)^4*tan(c)^2 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*ta
n(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c)^2 + 5760*C*sqrt(b)*tan(1/2*d*x + 1
/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^4*tan(c)^2 - 576*A*sqrt(b)*tan(1/
2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c)^2 - 1008*C*sqrt(b
)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c)^2 - 144*C
*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(
c)^2 - 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2
*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 72*C*s
qrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 -
 54*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 2880*C*
sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2
*tan(1/3*c)^4*tan(c)^2 - 3456*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 19872*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^4*tan(c)^2 - 38400*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6
*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 3456*A*sqrt(b)*tan(1/
2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*
tan(c)^2 + 19872*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(
-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 2880*C*sqrt(b)*tan(1/2*d*x + 1/
6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 72*C*sqrt(b)*tan(1/2
*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 - 432*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*ta
n(1/3*c)^4*tan(c)^2 - 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1
/2*c)^3*tan(1/3*c)^4*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1
/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*
c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 5760*C*sqrt(b)*tan(1/2

*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*t
an(c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1
/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2
*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 +
4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*
tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan
(1/3*c)^5*tan(c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/
6*c)^3*tan(1/3*c)^5*tan(c)^2 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*d*x + 1/6*c)^4*tan(1/3*c)^5*tan(c)^2 - 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c
)*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^5*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x +
1/6*c)^6*tan(1/3*c)^5*tan(c)^2 - 192*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(
-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 + 3456*C*sqrt(b)*tan(1/2*d*x + 1/
2*c)^3*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 -
11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x
+ 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*ta
n(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 - 2880*C
*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)
^2 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c
)^5*tan(c)^2 - 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)*tan
(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1
/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c)^2 + 3*C*sqrt(b)*d*x^4*t
an(1/3*c)^6*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/
6*c)*tan(1/3*c)^6*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d
*x + 1/6*c)^2*tan(1/3*c)^6*tan(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3
*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*tan(c)^2 + 1280*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^6*tan(c)^2 - 576*A*sqrt(b)*tan
(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6*tan(c)^2 - 1008*C*sqr
t(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6*tan(c)^2 + 96
*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^6*tan(c)^2 - 24*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 144*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan
(1/3*c)^6*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2
*c)*tan(1/3*c)^6*tan(c)^2 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d
*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*t
an(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 768*C*sqrt(b)*tan(1/2*d*x +
1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2
- 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x
+ 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 864*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1
/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 640*C*sqr
t(b)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 -
48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*t
an(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1
/3*c)^6*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)
^4*tan(1/3*c)^6*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x
+ 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1

/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1
/2*c)^4*tan(1/2*d*x + 1/6*c)^2 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*d*x + 1/6*c)^4 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6 - 144*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1
/2*c) + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan
(-1/2*d*x + 1/2*c) + 24*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*
c)^6*tan(-1/2*d*x + 1/2*c) + 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(
-1/2*d*x + 1/2*c)^2 - 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*
x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6
*c)^4*tan(-1/2*d*x + 1/2*c)^2 - 192*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/
2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*
c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2 - 96*C*sqrt(b)*tan(1/2*
d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2 + 48*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^3 - 144*C*sqrt(b)*d*x^
4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3 + 72*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*
c)^3 + 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*
d*x + 1/2*c)^3 + 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1
/2*c)^4 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4
+ 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x
+ 1/2*c)^4 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*ta
n(-1/2*d*x + 1/2*c)^4 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x +
 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4 - 192*A*sqrt(b)*tan(1/2*d*x +
1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4 - 144*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4 + 96*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c) - 2880*C*sqrt(b
)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan
(1/3*c) + 3456*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(
-1/2*d*x + 1/2*c)^2*tan(1/3*c) - 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) + 1440*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*
x + 1/2*c)^4*tan(1/3*c) + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
 + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) - 1728*C*sqrt(b)*tan(1/2*d*x
 + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) + 96*C*
sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) - 9*C*sqr
t(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 162*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2 - 27*C*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*
tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*
tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3
*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^3*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 432*C*sqrt(b)*d*x^4*tan(1/2*d*x
 + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 216*C

*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c
)*tan(1/3*c)^2 + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^
6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 - 324*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 162*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 9600*C*sqrt(b)*tan(1/2
*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2
 - 3456*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*
x + 1/2*c)^2*tan(1/3*c)^2 + 19872*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*
d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 11520*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)
^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*
x + 1/2*c)^2*tan(1/3*c)^2 + 864*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x
 + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 144*C*sqrt(b)*d*x^4*tan(1
/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 + 216*C*sqrt(b)*tan(1/
2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^
2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*
x + 1/2*c)^3*tan(1/3*c)^2 + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*
x + 1/2*c)^3*tan(1/3*c)^2 - 9*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^4*tan(1
/3*c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1
/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(
1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 11664*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/
3*c)^2 + 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(
1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 11664*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*
c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4*tan(1/
3*c)^2 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1
/3*c)^3 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(
1/3*c)^3 + 1280*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan
(1/3*c)^3 - 9600*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 + 38400*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3
*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 - 38400*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*t
an(1/3*c)^3 + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*t
an(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 - 640*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*
tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 + 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4
*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)
^3*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 + 19200*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*t
an(1/3*c)^3 - 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*t
an(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4
*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^2*tan(1/3*c)^4 - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^
4 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)

^4 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c
)^4 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(1/3
*c)^4 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(1/
3*c)^4 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*
c)^4 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c
)^4 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)*tan(1/
3*c)^4 + 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1
/2*d*x + 1/2*c)*tan(1/3*c)^4 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2
*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4 + 72*C*sqrt(b)*tan(1/2*d
*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4 + 54*C*sqrt(b)*d*x^4*tan(-
1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 2880*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan
(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 3456*A*sqrt(b)*tan
(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*
c)^4 + 19872*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1
/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 38400*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan
(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 3456*A*sqrt(b)*t
an(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*
c)^4 + 19872*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2
*d*x + 1/2*c)^2*tan(1/3*c)^4 - 2880*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(-1
/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1
/2*d*x + 1/2*c)^3*tan(1/3*c)^4 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 + 216*C*sqrt(b)*tan(
1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 576*A*sqrt(b)*tan
(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 1008*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 5760*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/
3*c)^4 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/
2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/
2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 4800*C*sqrt(b)*tan(
1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 1440*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^5 - 5760*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^5 + 5760*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^5 - 1728*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^5 + 96*C*sqrt(b)*ta
n(1/2*d*x + 1/6*c)^6*tan(1/3*c)^5 - 192*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 + 3456*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*
tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 - 11520*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(
1/3*c)^5 + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(
-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 - 2880*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*t
an(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 - 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*t
an(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 + 1440*C*sqrt(b)*t
an(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 - 3*C*sqrt(b)*d*
x^4*tan(1/3*c)^6 + 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)

*tan(1/3*c)^6 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2
*tan(1/3*c)^6 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^
2*tan(1/3*c)^6 + 1280*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)
^3*tan(1/3*c)^6 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4
*tan(1/3*c)^6 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*
tan(1/3*c)^6 + 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^6 + 24*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 144*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(
1/3*c)^6 + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/
3*c)^6 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^6 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^6 - 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/
2*d*x + 1/2*c)^2*tan(1/3*c)^6 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2
*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 864*C*sqrt(b)*tan(1/
2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6
- 640*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6
 + 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6
 + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6
 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6
- 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 +
 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 7
2*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(c) + 72
*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(c) - 24*C*
sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(c) + 24*C*sqrt(b)
*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)*tan(c) - 432*C*sqrt(b)*
d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*t
an(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan
(c) + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^2*ta
n(c) - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(
-1/2*d*x + 1/2*c)^2*tan(c) + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan
(-1/2*d*x + 1/2*c)^3*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan
(-1/2*d*x + 1/2*c)^3*tan(c) + 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-
1/2*d*x + 1/2*c)^4*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
 + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c) + 144*C*sqrt(b)*tan(1/2*d*x + 1/
2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(c) + 24*C*sqrt(b)
*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c) - 72*C*sqrt(b)*d*x^4
*tan(1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d
*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2*tan(c) - 216*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*tan(c) - 144*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c) - 432*
C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*t
an(c) + 216*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*ta
n(1/3*c)^2*tan(c) - 432*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x +
 1/2*c)^2*tan(1/3*c)^2*tan(c) - 72*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^3*
tan(1/3*c)^2*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/

2*c)^4*tan(1/3*c)^2*tan(c) + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d
*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) + 216*C*sqrt(b)*t
an(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) + 72*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) - 216*C*sqrt(b)*tan(1/
2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4*tan(c) - 432*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c) - 72*C*s
qrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c) + 72*C*sqrt(b)*d*x^4*tan(
-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c) + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^
2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) + 216*C*sqrt(b)*tan(1/2*d*x +
 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) - 24*C*sqrt(b)*tan(1/
2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/
6*c)^4*tan(1/3*c)^6*tan(c) + 24*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c
)^6*tan(c) + 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(c)^2 - 54*C*sqrt(
b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(c)^2 + 9*C*sqrt(
b)*d*x^4*tan(1/2*d*x + 1/6*c)^4*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c
)^4*tan(1/2*d*x + 1/6*c)^5*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*
tan(1/2*d*x + 1/6*c)^6*tan(c)^2 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1
/2*d*x + 1/6*c)^6*tan(c)^2 + 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(
-1/2*d*x + 1/2*c)*tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1
/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x
 + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 48*C*sq
rt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*t
an(c)^2 + 108*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^
2*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c
)^2*tan(c)^2 + 640*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*
tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*ta
n(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 2016*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2
+ 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x
+ 1/2*c)^2*tan(c)^2 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*
c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 288*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*
tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 48*C*sqrt(b)*d*x^
4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 - 72*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2
- 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x
+ 1/2*c)^3*tan(c)^2 - 24*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^3*tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 96*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^
4*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*ta
n(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1
/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 1280*C*sqrt(b)*tan(1/2
*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 5
76*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2
*c)^4*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*

```
tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(
-1/2*d*x + 1/2*c)^4*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/
2*d*x + 1/6*c)^4*tan(1/3*c)*tan(c)^2 + 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^
3*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)*tan(c)^2 - 384*C*sqrt(b)*tan(1/2*d*x +
1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)*tan(c)^2 + 2880*C*sqrt(b)*tan(1/
2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)*
tan(c)^2 - 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^3*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2 + 11520*C*sqrt(b)*tan(1/2*d*x + 1
/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2
- 3456*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x +
 1/2*c)^2*tan(1/3*c)*tan(c)^2 + 192*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1
/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4
*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 1728*C*sqrt(b)*tan(1/2*d*x +
 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2
- 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x
 + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1
/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 - 1440*C*sqrt
(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 - 54
*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 27*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2*tan(c)^2 - 4800*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^2*tan(c)^2 - 1728*A*sqr
t(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*tan(c)^2 +
9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*t
an(c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(
1/3*c)^2*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)
^6*tan(1/3*c)^2*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x +
 1/6*c)^6*tan(1/3*c)^2*tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*
tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 216*C*sqrt(b)*tan(1/2*d*x + 1
/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2
- 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x
+ 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1
/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1
/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 2880*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1
/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 3456*A*sqrt
(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*t
an(1/3*c)^2*tan(c)^2 - 23328*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x +
 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 38400*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/
3*c)^2*tan(c)^2 - 3456*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^
4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 23328*C*sqrt(b)*tan(1/2*d
*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan
(c)^2 + 2880*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^2*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c
)^3*tan(1/3*c)^2*tan(c)^2 - 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*
x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 - 216*C*sqrt(b)*
```

tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 - 576*
A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c
)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*
c)^2*tan(c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x
+ 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)
^2 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*
x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 4800*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^3*tan(c)^2 + 19200*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^3*tan(c)^2 - 19200*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^3*tan(c)
^2 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^
3*tan(c)^2 - 320*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^3*tan(c)^2 + 6
40*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*ta
n(c)^2 - 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(-1
/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 + 38400*C*sqrt(b)*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2
- 38400*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x
+ 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 + 9600*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 - 1280*C*sqrt(b)*tan(1/2*d*x +
1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2 + 5760*C*sqrt(b)*tan
(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3
*tan(c)^2 - 4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*t
an(1/3*c)^3*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/3*c)^4*tan(c)^2 - 1440*C*sqr
t(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^4*tan(c)^2 - 17
28*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4*tan
(c)^2 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(1/
3*c)^4*tan(c)^2 - 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*
c)^3*tan(1/3*c)^4*tan(c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*
x + 1/6*c)^4*tan(1/3*c)^4*tan(c)^2 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*ta
n(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d*x + 1
/6*c)^5*tan(1/3*c)^4*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/
2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 216*C
*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2
- 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)
^4*tan(c)^2 - 2016*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^2
*tan(1/3*c)^4*tan(c)^2 + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
+ 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 3456*A*sqrt(b)*ta
n(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c
)^4*tan(c)^2 - 23328*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*
tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 9600*C*sqrt(b)*tan(1/2*d*x
+ 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 144*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 - 216*C*

sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^
2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4
*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^4*tan(
1/3*c)^4*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*
c)^4*tan(1/3*c)^4*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)
^5*tan(c)^2 + 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*ta
n(1/3*c)^5*tan(c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1
/6*c)^2*tan(1/3*c)^5*tan(c)^2 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2
*d*x + 1/6*c)^3*tan(1/3*c)^5*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)
^4*tan(1/3*c)^5*tan(c)^2 + 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*
x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 - 3456*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*ta
n(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 + 2880*C*s
qrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2
 - 96*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5*tan(c)^2 - 192*A*sqrt(
b)*tan(1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 - 48*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 - 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan
(1/2*d*x + 1/6*c)*tan(1/3*c)^6*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c
)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*tan(c)^2 - 320*C*sqrt(b)*tan(1
/2*d*x + 1/6*c)^3*tan(1/3*c)^6*tan(c)^2 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)
^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x +
 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 384*A*sqrt(b)*tan(1
/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 288*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 192
*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)
^2 - 24*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 - 3*C*sqrt(
b)*d*x^4*tan(1/2*d*x + 1/2*c)^4 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x + 1/6*c)^2 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^4 - 96*C*s
qrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^5 - 192*A*sqrt(b)*tan(1/
2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)
^3*tan(1/2*d*x + 1/6*c)^6 - 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(-
1/2*d*x + 1/2*c) + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1
/6*c)^2*tan(-1/2*d*x + 1/2*c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c) + 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c) - 108*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x +
1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2 + 640*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan
(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x +
1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2 - 2016*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2 + 768*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c
)^2 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*
x + 1/2*c)^2 - 288*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*ta
n(-1/2*d*x + 1/2*c)^2 - 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*
x + 1/2*c)^3 + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*t

an(-1/2*d*x + 1/2*c)^3 + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3 + 24*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3 - 3*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^4 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4 - 1280*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4 - 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^4 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c) + 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c) - 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c) + 2880*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) - 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) - 3456*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) + 192*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) + 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) - 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 27*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2 - 4800*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 54*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 2880*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 3456*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 23328*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 38400*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 3456*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^

2 - 23328*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*
x + 1/2*c)^2*tan(1/3*c)^2 + 2880*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*
d*x + 1/2*c)^2*tan(1/3*c)^2 + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*
d*x + 1/2*c)^3*tan(1/3*c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 + 216*C*sqrt(b)*tan(1/2
*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 - 576*A*sqrt(b)*tan(1/
2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 432*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 5760*C*sqrt(b)*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*
x + 1/2*c)^4*tan(1/3*c)^2 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x
 + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 4800*C*sqrt(b)*tan(1/2*d
*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 4800*C*sqrt(b)*tan(1/2
*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^3 + 19200*C*sqrt(b)*tan(1
/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^3 - 19200*C*sqrt(b)*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^3 + 5760*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^3 - 320*C*sqrt(b)*tan(
1/2*d*x + 1/6*c)^6*tan(1/3*c)^3 + 640*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(
-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 - 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*t
an(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 + 38400*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^3 - 38400*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(-
1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 + 9600*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan
(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 - 1280*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*t
an(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*t
an(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 - 4800*C*sqrt(b)*t
an(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 - 9*C*sqrt(b)*d*
x^4*tan(1/3*c)^4 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*
c)*tan(1/3*c)^4 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c
)^2*tan(1/3*c)^4 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*
c)^2*tan(1/3*c)^4 - 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/
6*c)^3*tan(1/3*c)^4 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6
*c)^4*tan(1/3*c)^4 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*
c)^4*tan(1/3*c)^4 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^4 + 72
*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4 + 432*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*
c)*tan(1/3*c)^4 + 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c
)*tan(1/3*c)^4 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c
)^2*tan(1/3*c)^4 - 2016*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2
*c)^2*tan(1/3*c)^4 + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1
/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 3456*A*sqrt(b)*tan(1/2*d*x + 1
/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 - 23328*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^
2*tan(1/3*c)^4 + 9600*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c
)^2*tan(1/3*c)^4 + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*

c)^3*tan(1/3*c)^4 + 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2
*c)^3*tan(1/3*c)^4 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*
c)^4*tan(1/3*c)^4 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c
)^4*tan(1/3*c)^4 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c
)^4*tan(1/3*c)^4 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 + 1728*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^5 - 5760*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^5 + 5760*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^5 - 1440*C
*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^5 + 768*C*sqrt(b)*tan(1/2*d*x +
1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 - 3456*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 + 2880*C
*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 - 96*C
*sqrt(b)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^5 - 192*A*sqrt(b)*tan(1/2*d*x +
1/2*c)^3*tan(1/3*c)^6 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 -
384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^6 - 5
76*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6 + 432
*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6 - 320*C
*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^6 + 48*C*sqrt(b)*tan(1/2*d*x + 1
/2*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 72*C*sqrt(b)*tan(1/2*d*x + 1/6
*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*
c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 - 288*C*sqrt(b)*tan(1/2*d*x + 1/2*c
)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 192*C*sqrt(b)*tan(1/2*d*x + 1/6*c)
*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 24*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^
3*tan(1/3*c)^6 - 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(c) + 72*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(c) - 72*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(c) - 48*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(c) - 144*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(c) - 144*C*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(c) - 24*C*sqrt(b)*d*x^4*tan(-1/2*
d*x + 1/2*c)^3*tan(c) + 24*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x +
1/2*c)^4*tan(c) + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)
^2*tan(-1/2*d*x + 1/2*c)^4*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan
(-1/2*d*x + 1/2*c)^4*tan(c) + 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1
/3*c)^2*tan(c) - 216*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^
2*tan(1/3*c)^2*tan(c) - 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x +
1/6*c)^4*tan(1/3*c)^2*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*
c)^2*tan(c) + 72*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)
+ 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2
*tan(c) + 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(
1/3*c)^2*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) -
432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4*t
an(c) - 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c) + 72*C*sqr
t(b)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) - 48*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^2*tan(1/3*c)^6*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(1

/3*c)^6*tan(c) - 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(c)^2 + 9*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2*tan(c)^2 + 320*C*sqrt(b)*tan(1/2*d*x +
1/2*c)^4*tan(1/2*d*x + 1/6*c)^3*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*
c)^3*tan(1/2*d*x + 1/6*c)^4*tan(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^
3*tan(1/2*d*x + 1/6*c)^4*tan(c)^2 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*d*x + 1/6*c)^5*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*
d*x + 1/6*c)^6*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x +
1/6*c)^6*tan(c)^2 - 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x +
1/2*c)*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^
2*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan
(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 24*C*sqrt(b)*tan(1/2*d
*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 18*C*sqrt(b)*d*x^4*tan(-1/2*
d*x + 1/2*c)^2*tan(c)^2 - 192*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
+ 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/
2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 864*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*t
an(c)^2 - 2560*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(
-1/2*d*x + 1/2*c)^2*tan(c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*
d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 864*C*sqrt(b)*tan(1/2*d*x
 + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 192*C*s
qrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 24*C*sqrt(
b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 - 144*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(c
)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2
- 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 1
44*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 384*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)
^4*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan
(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2
*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 320*C*sqrt(b)*tan(1/2*d*
x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x
+ 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)*tan(c)^2 - 5760*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)*tan(c)^2 + 5760*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)*tan(c)^2 - 172
8*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)*tan(c)^2
 + 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)*tan(c)^2 - 192*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2 + 3456*C
*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^
2*tan(1/3*c)*tan(c)^2 - 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
+ 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2 + 11520*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c
)*tan(c)^2 - 2880*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*
tan(1/3*c)*tan(c)^2 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1
/2*c)^4*tan(1/3*c)*tan(c)^2 - 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d
*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 1440*C*sqrt(b)*ta

n(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 9*C*sqrt
(b)*d*x^4*tan(1/3*c)^2*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan
(1/2*d*x + 1/6*c)*tan(1/3*c)^2*tan(c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*
c)^3*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2*tan(c)^2 - 11664*C*sqrt(b)*tan(1/2
*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2*tan(c)^2 + 19200*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^2*tan(c)^2 - 1
728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*tan(
c)^2 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*
c)^2*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(1/3*c)^2*tan(c)^2
 - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*t
an(c)^2 - 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-
1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)
^4*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x
 + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 864*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 11520
*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c
)^2*tan(1/3*c)^2*tan(c)^2 - 3456*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x
 + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 19872*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/
3*c)^2*tan(c)^2 - 9600*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*
c)^2*tan(1/3*c)^2*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*
d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 - 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2
*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x
+ 1/2*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 1008*C*sqrt(b)*tan
(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 1440*C*sq
rt(b)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 +
320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2 - 5760*C*sqrt(b)
*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^3*tan(c)^2 + 19200*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^3*tan(c)
^2 - 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)
^3*tan(c)^2 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^3*tan(c)^2 -
 2560*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3
*tan(c)^2 + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)*tan(-
1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 - 9600*C*sqrt(b)*tan(1/2*d*x + 1/6
*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 + 320*C*sqrt(b)*tan(-1/
2*d*x + 1/2*c)^4*tan(1/3*c)^3*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)
^3*tan(1/3*c)^4*tan(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/3*c)
^4*tan(c)^2 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*ta
n(1/3*c)^4*tan(c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6
*c)^2*tan(1/3*c)^4*tan(c)^2 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*
d*x + 1/6*c)^2*tan(1/3*c)^4*tan(c)^2 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^
3*tan(1/3*c)^4*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x
 + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(
-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*
c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 864*C*sqrt(b)*tan(1/2*d*

x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 2880*C*sqrt(b)*t
an(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 72*C*sq
rt(b)*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 + 384*C*sqrt(b)*tan(1/2
*d*x + 1/2*c)^2*tan(1/3*c)^5*tan(c)^2 - 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)
*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^5*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x +
1/6*c)^2*tan(1/3*c)^5*tan(c)^2 - 192*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^2*tan(
1/3*c)^5*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^
2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 + 96*C*sqrt(b)
*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^6*tan(c)^2 - 24*C*sqrt(b)*tan(-1/2*d*x + 1
/2*c)*tan(1/3*c)^6*tan(c)^2 + 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2 - 9
*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^2 + 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c
)^4*tan(1/2*d*x + 1/6*c)^3 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d
*x + 1/6*c)^4 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^
4 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^5 - 192*A*sqr
t(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6 - 144*C*sqrt(b)*tan(1/2*d*
x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6 + 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)
*tan(-1/2*d*x + 1/2*c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
1/6*c)^2*tan(-1/2*d*x + 1/2*c) + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c) + 24*C*sqrt(b)*tan(1/2*d*x + 1/6*c)
^6*tan(-1/2*d*x + 1/2*c) + 18*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)^2 - 192
*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c
)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(-1/2
*d*x + 1/2*c)^2 + 864*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)
^2*tan(-1/2*d*x + 1/2*c)^2 - 2560*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c
)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2 + 864*C*sqrt(b)*tan(1/2*d*
x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2 - 192*C*sqrt(b)*t
an(1/2*d*x + 1/6*c)^5*tan(-1/2*d*x + 1/2*c)^2 + 24*C*sqrt(b)*tan(1/2*d*x +
1/2*c)^4*tan(-1/2*d*x + 1/2*c)^3 + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan
(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3 + 72*C*sqrt(b)*tan(1/2*d*x + 1/
6*c)^4*tan(-1/2*d*x + 1/2*c)^3 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-
1/2*d*x + 1/2*c)^4 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/
2*c)^4 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/2
*d*x + 1/2*c)^4 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2
*tan(-1/2*d*x + 1/2*c)^4 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x
+ 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4 + 320*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*t
an(-1/2*d*x + 1/2*c)^4 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
+ 1/6*c)^2*tan(1/3*c) - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x +
1/6*c)^3*tan(1/3*c) + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x +
1/6*c)^4*tan(1/3*c) - 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6
*c)^5*tan(1/3*c) + 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c) - 192*C*s
qrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) + 3456*C*s
qrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*
tan(1/3*c) - 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*
tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) + 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*t

an(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) - 2880*C*sqrt(b)*t
an(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) + 384*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) - 1728*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
+ 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
- 9*C*sqrt(b)*d*x^4*tan(1/3*c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*ta
n(1/2*d*x + 1/6*c)*tan(1/3*c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan
(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*t
an(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2 + 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*
tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*
tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5
*tan(1/3*c)^2 + 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)*t
an(1/3*c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*t
an(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*ta
n(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 864*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*t
an(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 11520*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^
2*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 3456*A*sqrt(b
)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1
/3*c)^2 + 19872*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-
1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 9600*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan
(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*ta
n(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 + 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*t
an(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*ta
n(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*ta
n(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*ta
n(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*t
an(1/3*c)^3 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*ta
n(1/3*c)^3 + 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*
tan(1/3*c)^3 - 19200*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*
tan(1/3*c)^3 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^3 - 2560*C*
sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 + 11520
*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^
2*tan(1/3*c)^3 - 9600*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c
)^2*tan(1/3*c)^3 + 320*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^3 - 576
*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 1008*C*sqrt(b)*tan(1/2*d*x
 + 1/2*c)^3*tan(1/3*c)^4 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*
x + 1/6*c)*tan(1/3*c)^4 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x +
 1/6*c)^2*tan(1/3*c)^4 - 11664*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x +
 1/6*c)^2*tan(1/3*c)^4 + 4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^4
 + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4
+ 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4 -
 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 +
 864*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 -

2880*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 72*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 - 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^5 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^5 - 192*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^5 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*c)^6 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^6 + 24*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(c) - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(c) - 24*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^6*tan(c) + 24*C*sqrt(b)*d*x^4*tan(-1/2*d*x + 1/2*c)*tan(c) + 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c) + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) - 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2*tan(c) - 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*tan(c) + 72*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c) - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c) - 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4*tan(c) - 24*C*sqrt(b)*tan(1/3*c)^6*tan(c) + 3*C*sqrt(b)*d*x^4*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2*tan(c)^2 - 1280*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^4*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5*tan(c)^2 - 24*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 288*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 2016*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 640*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 + 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(1/3*c)*tan(c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)*tan(c)^2 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)*tan(c)^2 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)*tan(c)^2 + 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x +

1/2*c)^2*tan(1/3*c)*tan(c)^2 - 3456*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*
d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2 + 2880*C*sqrt(b)*t
an(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2 - 96*C*sq
rt(b)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d
*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*
tan(1/3*c)^2*tan(c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x +
 1/6*c)*tan(1/3*c)^2*tan(c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2
*d*x + 1/6*c)^2*tan(1/3*c)^2*tan(c)^2 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)
*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2*tan(c)^2 - 4800*C*sqrt(b)*tan(1/2*d*x
+ 1/6*c)^3*tan(1/3*c)^2*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan
(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 216*C*sqrt(b)*tan(1/2*d*x + 1/6*
c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 1152*A*sqrt(b)*tan(1/2*d
*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 - 2016*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 2880*C
*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2
 - 72*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2*tan(c)^2 - 1280*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^3*tan(c)^2 + 5760*C*sqrt(b)*tan(1/2*d
*x + 1/2*c)*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^3*tan(c)^2 - 4800*C*sqrt(b)*tan
(1/2*d*x + 1/6*c)^2*tan(1/3*c)^3*tan(c)^2 + 640*C*sqrt(b)*tan(-1/2*d*x + 1/
2*c)^2*tan(1/3*c)^3*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*c
)^4*tan(c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 1
440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^4*tan(c)^2 - 72*C*sqrt(b)*tan
(-1/2*d*x + 1/2*c)*tan(1/3*c)^4*tan(c)^2 - 96*C*sqrt(b)*tan(1/3*c)^5*tan(c)
^2 - 3*C*sqrt(b)*d*x^4 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
1/6*c) - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2 + 432*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^2 - 1280*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^3 - 576*A*sqrt(b)*tan(1/2*d*x + 1
/2*c)*tan(1/2*d*x + 1/6*c)^4 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d
*x + 1/6*c)^4 - 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^5 + 24*C*sqrt(b)*tan(1/2*
d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c) + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c) + 72*C*sqrt(b)*tan(1/2*d*x +
1/6*c)^4*tan(-1/2*d*x + 1/2*c) - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-
1/2*d*x + 1/2*c)^2 - 288*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(-1/2*d*x + 1/
2*c)^2 + 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(-1/2
*d*x + 1/2*c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^
2*tan(-1/2*d*x + 1/2*c)^2 - 2016*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x
 + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2 + 640*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*
tan(-1/2*d*x + 1/2*c)^2 + 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x
+ 1/2*c)^3 + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^3 -
192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^4 - 48*C*sqrt(b)*t
an(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^4 - 96*C*sqrt(b)*tan(1/2*d*x + 1/
6*c)*tan(-1/2*d*x + 1/2*c)^4 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(1/3*
c) + 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)*tan(1/3*c)
- 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c) +
 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c) - 14

40*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c) + 768*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) - 3456*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) + 2880*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) - 96*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c) - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 - 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^2 - 1728*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2 + 9936*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2 - 4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan(1/3*c)^2 + 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2 - 1152*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 - 2016*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 2880*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 72*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^2 - 1280*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 + 5760*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^3 - 4800*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^3 + 640*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^3 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*c)^4 + 432*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*c)^4 - 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^4 + 72*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^4 - 96*C*sqrt(b)*tan(1/3*c)^5 - 24*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^4*tan(c) - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^4*tan(c) + 24*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^4*tan(c) - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c) - 216*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2*tan(c) - 72*C*sqrt(b)*tan(1/3*c)^4*tan(c) - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(c)^2 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3*tan(c)^2 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2*tan(c)^2 + 320*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^3*tan(c)^2 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 384*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 192*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 - 24*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^3*tan(c)^2 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2 - 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)*tan(1/3*c)*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)*tan(c)^2 - 192*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)*tan(c)^2 - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^2*tan(c)^2 - 72*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^2*tan(c)^2 + 320*C*sqrt(b)*tan(1/3*c)^3*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 - 144*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 384*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1

```
/6*c) - 576*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2 - 1008*C*
sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^2 + 320*C*sqrt(b)*tan(1/2
*d*x + 1/6*c)^3 + 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)
 + 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c) - 384*A*sqrt(b
)*tan(1/2*d*x + 1/2*c)*tan(-1/2*d*x + 1/2*c)^2 - 96*C*sqrt(b)*tan(1/2*d*x +
 1/2*c)*tan(-1/2*d*x + 1/2*c)^2 - 192*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(-1
/2*d*x + 1/2*c)^2 + 24*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^3 + 384*C*sqrt(b)*ta
n(1/2*d*x + 1/2*c)^2*tan(1/3*c) - 1728*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1
/2*d*x + 1/6*c)*tan(1/3*c) + 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(1/3*
c) - 192*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c) - 576*A*sqrt(b)*tan(1
/2*d*x + 1/2*c)*tan(1/3*c)^2 - 1008*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(1/3*
c)^2 + 1440*C*sqrt(b)*tan(1/2*d*x + 1/6*c)*tan(1/3*c)^2 + 72*C*sqrt(b)*tan(
-1/2*d*x + 1/2*c)*tan(1/3*c)^2 + 320*C*sqrt(b)*tan(1/3*c)^3 - 48*C*sqrt(b)*
tan(1/2*d*x + 1/2*c)^2*tan(c) - 72*C*sqrt(b)*tan(1/2*d*x + 1/6*c)^2*tan(c)
- 72*C*sqrt(b)*tan(1/3*c)^2*tan(c) - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan
(c)^2 - 48*C*sqrt(b)*tan(1/2*d*x + 1/2*c)*tan(c)^2 - 96*C*sqrt(b)*tan(1/2*d
*x + 1/6*c)*tan(c)^2 - 24*C*sqrt(b)*tan(-1/2*d*x + 1/2*c)*tan(c)^2 - 96*C*s
qrt(b)*tan(1/3*c)*tan(c)^2 - 192*A*sqrt(b)*tan(1/2*d*x + 1/2*c) - 48*C*sqrt
(b)*tan(1/2*d*x + 1/2*c) - 96*C*sqrt(b)*tan(1/2*d*x + 1/6*c) + 24*C*sqrt(b)
*tan(-1/2*d*x + 1/2*c) - 96*C*sqrt(b)*tan(1/3*c) - 24*C*sqrt(b)*tan(c))/(d*
tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1
/3*c)^6*tan(c)^2 + d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2
*d*x + 1/2*c)^4*tan(1/3*c)^6 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6
*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2
*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2
+ 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4
*tan(1/3*c)^6*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*
tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 2*d*tan(1/2*d
*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 +
 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*
tan(1/3*c)^6 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d
*x + 1/2*c)^4*tan(1/3*c)^6 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c
)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c
)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 +
9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*t
an(1/3*c)^4*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*ta
n(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + d*tan(1/2*d*x + 1/2*c)^4*tan(
1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(1
/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 4*d*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)
^6*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*
x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
 + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + d*tan(1/2*d*x +
 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 3*d*tan(1/2*d*x +
```

1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 6*d
*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(
1/3*c)^4 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x +
1/2*c)^4*tan(1/3*c)^4 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
+ 1/6*c)^6*tan(1/3*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^
4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 4*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2
*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 3*d*tan(1/2*d*x + 1/
2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 6*d*ta
n(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3
*c)^6 + d*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + d*t
an(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c)
^2 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^2*tan(1/3*c)^2*tan(c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)
^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)
^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 3
*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c)^2 + 18
*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*ta
n(1/3*c)^4*tan(c)^2 + 12*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*ta
n(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*ta
n(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 18*d*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/
3*c)^4*tan(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/
3*c)^4*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3
*c)^6*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*
c)^6*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*
d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 12*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 2*d*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + d*tan(1/2*d*
x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 6*d*tan(1/2*d*
x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*ta
n(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*ta
n(c)^2 + d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2
*c)^4 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^2*tan(1/3*c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan
(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^
4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4 + 18*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 12*d*tan(1/2*d*x + 1
/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 9*d*t
an(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/
3*c)^4 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x +
1/2*c)^4*tan(1/3*c)^4 + 3*d*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*
tan(1/3*c)^4 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)
^6 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6 + 6*d*t

an(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 12*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 2*d*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + d*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 3*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 2*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 12*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 36*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 6*d*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 9*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6*tan(c)^2 + d*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 12*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 6*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 12*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4 + 18* d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 36*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 6*d*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 9*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6 + d*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6 + 2*d*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 12*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 6*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 3*d*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 4*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + d*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 36*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 6*d*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 9*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4*tan(c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 36*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 18*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 9*d*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + d*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*tan(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6*tan(c)^2 + 4*d*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 6*d*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + d*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 + d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2 + 4*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4 + 6*d*t

an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4 + d*ta
n(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4 + 9*d*tan(1/2*d*x + 1/2*c)^4*t
an(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
 + 1/6*c)^6*tan(1/3*c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)
^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 36*d*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 6*d*tan(1/2*d*x +
1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 9*d*tan(1/2*d*x + 1/6
*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan
(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
+ 1/6*c)^4*tan(1/3*c)^4 + 3*d*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^4 + 6*d*tan
(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 36*d*tan(1/2*d*x
 + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 1
8*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 6*d*tan(1
/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 9*d*tan(1/2*d*x +
1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + d*tan(1/2*d*x + 1/2*c)^4*ta
n(1/3*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6
 + 3*d*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^6 + 4*d*tan(1/2*d*x + 1/2*c)^2*tan
(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 6*d*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x
 + 1/2*c)^2*tan(1/3*c)^6 + d*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 3*d*tan
(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 12*d*tan(1/2*d*x + 1/2*c)
^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 2*d*tan(1/2*d*
x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + d*tan(1/2*d*x + 1/2*c)^4*ta
n(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1
/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^4*tan(-
1/2*d*x + 1/2*c)^4*tan(c)^2 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*
c)^2*tan(1/3*c)^2*tan(c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*
c)^4*tan(1/3*c)^2*tan(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^2*tan(c)
^2 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)
^2 + 36*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*
c)^2*tan(1/3*c)^2*tan(c)^2 + 18*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2
*c)^2*tan(1/3*c)^2*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2
*c)^4*tan(1/3*c)^2*tan(c)^2 + 9*d*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2
*c)^4*tan(1/3*c)^2*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c
)^2 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4*tan(c
)^2 + 9*d*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^4*tan(c)^2 + 12*d*tan(1/2*d*x +
 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 18*d*tan(1/2*d*x
+ 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 3*d*tan(-1/2*d*x
 + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^6
*tan(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6*tan(c)^2 + 2*d*tan(-1/2
*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^4 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6 + 6*d*ta

n(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2 + 12*d*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2 + 2*d
*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2 + d*tan(1/2*d*x + 1/2*c)^4*
tan(-1/2*d*x + 1/2*c)^4 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2
*tan(-1/2*d*x + 1/2*c)^4 + 3*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)
^4 + 9*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2 + 18*d*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2 + 3*d*tan(1/2*d*
x + 1/6*c)^6*tan(1/3*c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c
)^2*tan(1/3*c)^2 + 36*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-
1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 18*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x
+ 1/2*c)^2*tan(1/3*c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^
4*tan(1/3*c)^2 + 9*d*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3
*c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 18*d*tan(1/2*d*x + 1/2*c)
^2*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4 + 9*d*tan(1/2*d*x + 1/6*c)^4*tan(1/3
*c)^4 + 12*d*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 +
18*d*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 3*d*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^6
+ 3*d*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^6 + 2*d*tan(-1/2*d*x + 1/2*c)^2*tan
(1/3*c)^6 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2*tan(c)^2 + 6*
d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(c)^2 + d*tan(1/2*d*x +
1/6*c)^6*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(
c)^2 + 12*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/
2*c)^2*tan(c)^2 + 6*d*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(c)
^2 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 3*d*tan(
1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2
*c)^4*tan(1/3*c)^2*tan(c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6
*c)^2*tan(1/3*c)^2*tan(c)^2 + 9*d*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2*tan(c
)^2 + 12*d*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(
c)^2 + 18*d*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan
(c)^2 + 3*d*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 + 6*d*tan(1/2*d*x
 + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + 9*d*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^4
*tan(c)^2 + 6*d*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 + d*tan(1/3*c
)^6*tan(c)^2 + 3*d*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^2 + 6*d*tan(
1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4 + d*tan(1/2*d*x + 1/6*c)^6 + 2*d*
tan(1/2*d*x + 1/2*c)^4*tan(-1/2*d*x + 1/2*c)^2 + 12*d*tan(1/2*d*x + 1/2*c)^
2*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^2 + 6*d*tan(1/2*d*x + 1/6*c)
^4*tan(-1/2*d*x + 1/2*c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*
c)^4 + 3*d*tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4 + 3*d*tan(1/2*d*x
 + 1/2*c)^4*tan(1/3*c)^2 + 18*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)
^2*tan(1/3*c)^2 + 9*d*tan(1/2*d*x + 1/6*c)^4*tan(1/3*c)^2 + 12*d*tan(1/2*d*
x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 18*d*tan(1/2*d*x + 1/6*
c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2 + 3*d*tan(-1/2*d*x + 1/2*c)^4*tan
(1/3*c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + 9*d*tan(1/2*d*x + 1/6
*c)^2*tan(1/3*c)^4 + 6*d*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4 + d*tan(1/3*c
)^6 + d*tan(1/2*d*x + 1/2*c)^4*tan(c)^2 + 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/

```
2*d*x + 1/6*c)^2*tan(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^4*tan(c)^2 + 4*d*tan(1
/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + 6*d*tan(1/2*d*x + 1/6*
c)^2*tan(-1/2*d*x + 1/2*c)^2*tan(c)^2 + d*tan(-1/2*d*x + 1/2*c)^4*tan(c)^2
+ 6*d*tan(1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c)^2 + 9*d*tan(1/2*d*x + 1/6*
c)^2*tan(1/3*c)^2*tan(c)^2 + 6*d*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^2*tan(c
)^2 + 3*d*tan(1/3*c)^4*tan(c)^2 + d*tan(1/2*d*x + 1/2*c)^4 + 6*d*tan(1/2*d*
x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^4 + 4*d*tan(
1/2*d*x + 1/2*c)^2*tan(-1/2*d*x + 1/2*c)^2 + 6*d*tan(1/2*d*x + 1/6*c)^2*tan
(-1/2*d*x + 1/2*c)^2 + d*tan(-1/2*d*x + 1/2*c)^4 + 6*d*tan(1/2*d*x + 1/2*c)
^2*tan(1/3*c)^2 + 9*d*tan(1/2*d*x + 1/6*c)^2*tan(1/3*c)^2 + 6*d*tan(-1/2*d*
x + 1/2*c)^2*tan(1/3*c)^2 + 3*d*tan(1/3*c)^4 + 2*d*tan(1/2*d*x + 1/2*c)^2*t
an(c)^2 + 3*d*tan(1/2*d*x + 1/6*c)^2*tan(c)^2 + 2*d*tan(-1/2*d*x + 1/2*c)^2
*tan(c)^2 + 3*d*tan(1/3*c)^2*tan(c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^2 + 3*d*ta
n(1/2*d*x + 1/6*c)^2 + 2*d*tan(-1/2*d*x + 1/2*c)^2 + 3*d*tan(1/3*c)^2 + d*t
an(c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)\right)}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*si
n(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.92 $\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 90

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

```
[Out] A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*C*x*(b*cos(d*x+c))^(1/2)/cos(
d*x+c)^(1/2)+1/2*C*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 2715, 8}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (C*x*Sqrt[b*Cos[c + d*x]])/
(2*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(C\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}+\frac{\left(C\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{\cos(c+dx)}}\\
&= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$
\begin{aligned}
&\int\frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx\\
&=\frac{\sqrt{b\cos(c+dx)}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*Sq
rt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 6.90 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\sqrt{\cos(dx+c)b}\,(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))}{2d\sqrt{\cos(dx+c)}}$	54
risch	$\frac{\sqrt{\cos(dx+c)b}\,x(4A+2C)}{4\sqrt{\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)b}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	63
parts	$\frac{C\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{A\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	72

```
[In] int((A+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(cos(d*x+c)*b)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/
cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\left[\frac{2\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c)+(2A+C)\sqrt{-b}\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c}\right.}{4d}\right.$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*
sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(
d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x +
c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [A] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \begin{cases} \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}\sin^2(c+dx)}{2\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}{2} + \frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)\sqrt{\cos(c+dx)}}{2d} & \text{for } d \neq 0 \\ \frac{x\sqrt{b\cos(c)}(A+C\cos^2(c))}{\sqrt{\cos(c)}} & \text{otherwise} \end{cases}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Piecewise((A*x*sqrt(b*cos(c + d*x))/sqrt(cos(c + d*x)) + C*x*sqrt(b*cos(c +
 d*x))*sin(c + d*x)**2/(2*sqrt(cos(c + d*x))) + C*x*sqrt(b*cos(c + d*x))*co
s(c + d*x)**(3/2)/2 + C*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x)
)/(2*d), Ne(d, 0)), (x*sqrt(b*cos(c))*(A + C*cos(c)**2)/sqrt(cos(c)), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C\sqrt{b}+8\,A\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{4\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C*sqrt(b) + 8*A*sqrt(b)*arctan(sin(d*
x + c)/(cos(d*x + c) + 1)))/d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x
)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(C*sin(2*c + 2*d*x) + 4*A*d*x + 2*C*d*x))/(4*d*cos(
c + d*x)^(1/2))
```

3.93 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 68

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{A\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+C*sin(d*x+c)*
(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3093, 3855}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{A\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (C*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{A\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}(A\text{arctanh}(\sin(c+dx))+C\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
[Out] (Sqrt[b*Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Sqrt[C
os[c + d*x]])
```

Maple [A] (verified)

Time = 7.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\left(2A\,\operatorname{arctanh}\left(\cot\left(dx+c\right)-\csc\left(dx+c\right)\right)-\sin\left(dx+c\right)C\right)\sqrt{\cos\left(dx+c\right)b}}{d\sqrt{\cos\left(dx+c\right)}}$
parts	$\frac{C\sin\left(dx+c\right)\sqrt{\cos\left(dx+c\right)b}}{d\sqrt{\cos\left(dx+c\right)}}-\frac{2A\,\operatorname{arctanh}\left(\cot\left(dx+c\right)-\csc\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)b}}{d\sqrt{\cos\left(dx+c\right)}}$
risch	$-\frac{i\sqrt{\cos\left(dx+c\right)b}\,C\,{\mathrm e}^{i\left(dx+c\right)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{i\sqrt{\cos\left(dx+c\right)b}\,C\,{\mathrm e}^{-i\left(dx+c\right)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{\sqrt{\cos\left(dx+c\right)b}\,A\ln\left({\mathrm e}^{i\left(dx+c\right)}+i\right)}{\sqrt{\cos\left(dx+c\right)}\,d}-\frac{\sqrt{\cos\left(dx+c\right)b}\,A\ln\left({\mathrm e}^{i\left(dx+c\right)}-i\right)}{\sqrt{\cos\left(dx+c\right)}\,d}$

```
[In] int((A+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-sin(d*x+c)*C)*(cos(d*x+c)*b)^(1/2)
/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\left[\frac{A\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\mathrm{co}}}{2\,d\cos\left(dx+c\right)}\right.$$
$$\left.-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}\right]$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqr
t(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos{(c+dx)}}(A+C\cos^2{(c+dx)})}{\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)

[Out] Integral(sqrt(b*cos(c + d*x))*(A + C*cos(c + d*x)**2)/cos(c + d*x)**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{A\sqrt{b}\left(\log\left(\cos{(dx+c)}^2+\sin{(dx+c)}^2+2\,\sin{(dx+c)}+1\right)-\log\left(\cos{(dx+c)}^2+\sin{(dx+c)}^2-2\,\sin{(}\right.\right.}{2\,d}$$

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] 1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 2*C*sqrt(b)*sin(d*x + c))/d

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(C\cos{(dx+c)}^2+A)\sqrt{b\cos{(dx+c)}}}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x)
```

3.94 $\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 59

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+C*x*(b*cos(d*x+c))^(1/
2)/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3091, 8}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*Sqrt[b*Cos[c + d*x]]*Sin
[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(C\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$
\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}(Cdx\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{3}{2}}(c+dx)}
$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x
]^(3/2))
```

Maple [A] (verified)

Time = 8.61 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\sqrt{\cos(dx+c)b}\,(C\cos(dx+c)(dx+c)+A\sin(dx+c))}{d\cos(dx+c)^{\frac{3}{2}}}$	45
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}+\frac{C\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	59
risch	$\frac{Cx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{2i\sqrt{\cos(dx+c)b}\,A}{\sqrt{\cos(dx+c)}\,d(\mathrm{e}^{2i(dx+c)}+1)}$	61

```
[In] int((A+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(cos(d*x+c)*b)^(1/2)*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(3/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.14

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \left[\frac{C\sqrt{-b}\cos(dx+c)^2\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2\,\sqrt{}}{2\,d\cos(dx+c)^2}\right.$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))
*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*sqrt(b)*arctan(s
qrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)
^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c
)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(C\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\frac{A\sqrt{b}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1}\right)}{d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
[Out] 2*(C*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*sqrt(b)*sin(2*d*x
+ 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/
d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(5/2), x
)
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\left(2\,c+2\,d\,x\right)+A\,1\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),x)
[Out] ((b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x)
+ C*d*x + C*d*x*cos(2*c + 2*d*x)))/(d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x)
+ 1))
```

3.95 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 78

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/2*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*(A+2*C)*arctan
h(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3091, 3855}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*
x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{\left((A+2C)\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}\\
&= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int\frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\left((A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx)\right)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),
x]

[Out] (Sqrt[b*Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*S
in[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 9.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.47

method	result
default	$\frac{\left(-A\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+A\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-4C\left(\cos^{2}(dx+c)\right)\operatorname{arctanh}(\cot(dx\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\left(-\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)\right)\sqrt{\cos(dx+c)b}}{2d\cos(dx+c)^{\frac{5}{2}}}-\frac{2C\,\mathrm{a}}{}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}\,A\left(\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}-\frac{\sqrt{\cos(dx+c)b}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)b}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)}\,d}$

```
[In] int((A+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+A*cos(d*x+c)^2*ln(-cot(
d*x+c)+csc(d*x+c)+1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(
d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\,\sqrt{b\cos\left(dx+\right.}}{4\,d\cos\left(dx+c\right)^{3}}\right.$$
$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{3}-\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+\right.}{2\,d\cos\left(dx+c\right)^{3}}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt
(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x +
 c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(66) = 132.

Time = 0.46 (sec) , antiderivative size = 728, normalized size of antiderivative = 9.33

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
[Out] 1/4*(2*C*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
 - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) - (4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
 + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2
*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*
x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2), x)
```

3.96 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 79

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/3*(2*A+3*C)*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3091, 3852, 8}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + ((2*A + 3*
C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{\left((2A+3C)\sqrt{b\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{3\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}-\frac{\left((2A+3C)\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{3d\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int\frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*Cos
[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 8.82 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\left(2A\left(\cos^2(dx+c)\right)+3C\left(\cos^2(dx+c)\right)+A\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\cos(dx+c)^{\frac{7}{2}}}$	54
parts	$\frac{A\left(2\left(\cos^2(dx+c)\right)+1\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{C\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}$	73
risch	$\frac{2i\sqrt{\cos(dx+c)b}\left(3C\,e^{4i(dx+c)}+6A\,e^{2i(dx+c)}+6C\,e^{2i(dx+c)}+2A+3C\right)}{3\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^3}$	81

```
[In] int((A+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*sin(d*x+c)*(cos(d*x+c)*b)^(1/2)
/cos(d*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*c
os(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(67) = 134$.

Time = 0.43 (sec) , antiderivative size = 350, normalized size of antiderivative = 4.43

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2\left(\frac{2\,((3\,\cos(2\,dx+2\,c)+1)\sin(6\,dx+6\,c)+3\,(3\,\cos(2\,dx+2\,c)+1)\sin(4}{2\,(3\,\cos(4\,dx+4\,c)+3\,\cos(2\,dx+2\,c)+1)\cos(6\,dx+6\,c)+\cos(6\,dx+6\,c)^2+6\,(3\,\cos(2\,dx+2\,c)+1)\cos(4\,dx+4\,c)+9\,\cos(4\,dx+4\,c)^2+9\,\cos(2\,dx+2\,c}\right.}{}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(2*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) +
 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x +
4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c
) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*
x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(
4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^
2 + 6*cos(2*d*x + 2*c) + 1) + 3*C*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x
)
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,A\,\sin\left(6\,c+6\,d\,x\right)+15\,C\,\sin\left(2\,c+2\,d\right.\right.}{}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*c
 + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*c
+ 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(4*c
 + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c + 4
*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x)
 + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.97 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/4*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*(3*A+4*C)*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/8*(3*A+4*C)*arctanh(sin(d*
x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3091, 3853, 3855}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c +
d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (
(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{\left((3A+4C)\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}\\
&\quad+\frac{\left((3A+4C)\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{8\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}((3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{9}{2}}(c+dx)}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2)
,x]
[Out] (Sqrt[b*Cos[c + d*x]]*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (
2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

Maple [A] (verified)

Time = 8.64 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.48

method	result
default	$-\frac{(3A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)-3A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+4C(\cos^4(dx+c))\ln(-\cot(dx-}{}$
parts	$-\frac{A(3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)-3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-3(\cos^2(dx+c))\sin(dx+c)-2\sin}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}\,(3A\,e^{7i(dx+c)}+4C\,e^{7i(dx+c)}+11A\,e^{5i(dx+c)}+4C\,e^{5i(dx+c)}-11A\,e^{3i(dx+c)}-4C\,e^{3i(dx+c)}-3A\,e^{i(dx+c)}-4C\,e^{i(dx+c)})}{4\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^4}$

```
[In] int((A+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
[Out] -1/8/d*(3*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-3*A*cos(d*x+c)^4*ln(-
cot(d*x+c)+csc(d*x+c)+1)+4*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-4*C*
cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*A*sin(d*x+c)*cos(d*x+c)^2-4*C*c
os(d*x+c)^2*sin(d*x+c)-2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(9/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \left[\frac{(3\,A+4\,C)\sqrt{b}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,((3\,A+4}{16\,d\cos\left(dx+c\right)^5}\right.$$
$$-\frac{(3\,A+4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left((3\,A+4\,C)\cos\left(dx+c\right)^2+2\,A\right)\sqrt{b\,c}}{8\,d\cos\left(dx+c\right)^5}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*((3*A + 4*C)*sqr
t(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c
))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2318 vs. $2(104) = 208$.

Time = 0.51 (sec) , antiderivative size = 2318, normalized size of antiderivative = 19.00

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44
*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x
+ 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*
x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)
^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*
cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(
4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(
3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x +
6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*
sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos
(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*
(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x +
8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1
)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
 + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
```

```
*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c
) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) +
 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*
x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)
^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2
*d*x + 2*c) + 1) + 4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d
*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*
c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d
*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +
2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2
*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(co
s(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(b)/(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(C\cos(dx+c)^2+A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(11/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^{11/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(11/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(11/2), x)
```

3.98 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 119

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{b(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b(A+2C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] b*(A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b*(A+2*C)*si
n(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/5*b*C*sin(d*x+c)^5*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3092, 380}

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{b(A+2C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{b(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bC\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (b*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*(
A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (b
*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int\left(1-x^2\right)\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int\left(A\left(1+\frac{C}{A}\right)-(A+2C)x^2+Cx^4\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{b(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\\
&\quad -\frac{b(A+2C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{3/2}(100A+89C+4(5A+7C)\cos(2(c+dx))+3C\cos(4(c+dx)))\sin}{120d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3*
C*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 8.97 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60

method	result
default	$\frac{b\left(3C\left(\cos^4(dx+c)\right)+5A\left(\cos^2(dx+c)\right)+4C\left(\cos^2(dx+c)\right)+10A+8C\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{15d\sqrt{\cos(dx+c)}}$
parts	$\frac{Ab\left(2+\cos^2(dx+c)\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\sqrt{\cos(dx+c)}}+\frac{Cb\left(3\left(\cos^4(dx+c)\right)+4\left(\cos^2(dx+c)\right)+8\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{15d\sqrt{\cos(dx+c)}}$
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)e^{6i(dx+c)}C}{80\left(e^{2i(dx+c)}+1\right)d}-\frac{ib\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)e^{2i(dx+c)}(6A+5C)}{8\left(e^{2i(dx+c)}+1\right)d}+\frac{ib\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c\right.}{8\left(e^{2i(dx+c)}+1\right)d}$

```
[In] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/15*b/d*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10*A+8*C)*(cos
(d*x+c)*b)^(1/2)*sin(d*x+c)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{\left(3\,Cb\cos\left(dx+c\right)^4+(5\,A+4\,C)b\cos\left(dx+c\right)^2+2\,(5\,A+4\,C)b\right)\sqrt{b\cos\left(dx+c\right)}\sin}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(3*C*b*cos(d*x + c)^4 + (5*A + 4*C)*b*cos(d*x + c)^2 + 2*(5*A + 4*C)*b
)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{20\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+\left(3\,b\right.}{}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algor
ithm="maxima")
```

```
[Out] 1/240*(20*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))*A*sqrt(b) + (3*b*sin(5*d*x + 5*c) + 25*b*sin(3/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 150*b*sin(1/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c))))*C*sqrt(b))/d
```

Giac [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(200\,A\,\sin\left(2\,c+2\,d\,x\right)+20\,A\,\sin\left(4\,c+4\,d\,x\right)+17\right.}{240\,d\,\left(\cos\left(2\,c+2\,d\,x\right)\right.}$$

```
[In] int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(200*A*sin(2*c + 2*d*x) + 20*A
*sin(4*c + 4*d*x) + 175*C*sin(2*c + 2*d*x) + 28*C*sin(4*c + 4*d*x) + 3*C*si
n(6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

3.99 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 35, antiderivative size = 116

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{b(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

```
[Out] 1/4*b*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*b*(4*A+3*C)*
x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/8*b*(4*A+3*C)*sin(d*x+c)*cos(d*x+
c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3093, 2715, 8}

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{bx(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}+\frac{bC\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (b*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*(4*A + 3
*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*C*Cos[
c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}+\frac{\left(b(4A+3C)\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{4\sqrt{\cos(c+dx)}} \\
&= \frac{b(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} \\
&\quad+\frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} \\
&\quad+\frac{\left(b(4A+3C)\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{b(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} \\
&\quad + \frac{b(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} \\
&\quad + \frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$
\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{3/2}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d\cos^{\frac{3}{2}}(c+dx)}
$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x
)] + C*Sin[4*(c + d*x)]))/(32*d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 9.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result
default	$\frac{b\sqrt{\cos(dx+c)b}\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8d\sqrt{\cos(dx+c)}}$
parts	$\frac{Ab\sqrt{\cos(dx+c)b}\left(\cos(dx+c)\sin(dx+c)+dx+c\right)}{2d\sqrt{\cos(dx+c)}}+\frac{Cb\sqrt{\cos(dx+c)b}\left(2\sin(dx+c)\left(\cos^3(dx+c)\right)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{b\sqrt{\cos(dx+c)b}\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{i(dx+c)}x(8A+6C)}{8\,\mathrm{e}^{2i(dx+c)}+8}-\frac{ib\sqrt{\cos(dx+c)b}\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{5i(dx+c)}C}{32\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{ib\sqrt{\cos(dx+c)b}\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{-i\cdots}}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)d}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8*b/d*(cos(d*x+c)*b)^(1/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*co
s(d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/cos(d*x+c)^(1/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.80

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \left[\frac{(4A+3C)\sqrt{-b}b\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*((4*A + 3*C)*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c
))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b*cos(d*x + c)^2
+ (4*A + 3*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d, 1
/8*((4*A + 3*C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*c
os(d*x + c)^(3/2))) + (2*C*b*cos(d*x + c)^2 + (4*A + 3*C)*b)*sqrt(b*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{8\left(2(dx+c)b+b\sin(2dx+2c)\right)A\sqrt{b}+\left(12(dx+c)b+b\sin(4dx+4c)+8b\sin\left(\tfrac{1}{2}\right.\right.}{32d}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + (12*(d*x + c)*b +
b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*C*sqrt(b))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(98) = 196$.

Time = 3.04 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.13

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\,dx = \frac{\left(4\,A\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+3\,C\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+16\,A\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/8*(4*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^8 + 3*C*sqrt(b)*d*x*tan(1/2*d*x +
 1/2*c)^8 + 16*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 12*C*sqrt(b)*d*x*tan(
1/2*d*x + 1/2*c)^6 - 8*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 - 10*C*sqrt(b)*tan(
1/2*d*x + 1/2*c)^7 + 24*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 18*C*sqrt(b)
*d*x*tan(1/2*d*x + 1/2*c)^4 - 8*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 6*C*sqrt
(b)*tan(1/2*d*x + 1/2*c)^5 + 16*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 12*C
*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 8*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 -
6*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 4*A*sqrt(b)*d*x + 3*C*sqrt(b)*d*x + 8*
A*sqrt(b)*tan(1/2*d*x + 1/2*c) + 10*C*sqrt(b)*tan(1/2*d*x + 1/2*c))*b/(d*ta
n(1/2*d*x + 1/2*c)^8 + 4*d*tan(1/2*d*x + 1/2*c)^6 + 6*d*tan(1/2*d*x + 1/2*c
)^4 + 4*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\,dx = \frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(c+d\,x\right)+8\,C\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(3\,c+\right.\right.}{32\,d\,\left(\mathrm{co}\right.}$$

```
[In] int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c
+ d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) +
 32*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x) +
1))
```

3.100 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 76

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{b(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{bC\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] b*(A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b*C*sin(d*x+
c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {17, 3092}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{b(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{bC\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*C
*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\ &= \frac{b(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{bC\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int\frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{b\sqrt{b\cos(c+dx)}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d
*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{b\left(C\left(\cos^{2}(dx+c)\right)+3A+2C\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}$	48
risch	$\frac{b\sqrt{\cos(dx+c)b}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{\cos(dx+c)b}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}$	73
parts	$\frac{Ab\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{Cb\left(2+\cos^{2}(dx+c)\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\sqrt{\cos(dx+c)}}$	73

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3*b/d*(C*cos(d*x+c)^2+3*A+2*C)*sin(d*x+c)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{\left(Cb\cos\left(dx+c\right)^{2}+\left(3\,A+2\,C\right)b\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+\right.}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*b*cos(d*x + c)^2 + (3*A + 2*C)*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/
(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{12\,Ab^{\frac{3}{2}}\sin\left(dx+c\right)+\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\,\arctan\left(\mathrm{si}\right.\right.\right.}{12\,d}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(12*A*b^(3/2)*sin(d*x + c) + (b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)),
 x)
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\,C\,\sin\left(c+d\,x\right)+C\right.}{12\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + C*sin(3*c
 + 3*d*x)))/(12*d*cos(c + d*x)^(1/2))
```

3.101 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 93

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

```
[Out] A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b*C*x*(b*cos(d*x+c))^(1/2)/
cos(d*x+c)^(1/2)+1/2*b*C*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2715, 8}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bC\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*C*x*Sqrt[b*Cos[c + d*x
]])/(2*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*S
in[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(bC\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\left(bC\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{\cos(c+dx)}}$$

$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.56

$$\int\frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
[Out] ((b*Cos[c + d*x])^(3/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*
Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 9.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{b\sqrt{\cos(dx+c)b}\,(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))}{2d\sqrt{\cos(dx+c)}}$	55
risch	$\frac{b\sqrt{\cos(dx+c)b}\,x(4A+2C)}{4\sqrt{\cos(dx+c)}}+\frac{b\sqrt{\cos(dx+c)b}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	65
parts	$\frac{Cb\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{Ab\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	74

[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x,method=_RETU RNVERBOSE)

[Out] 1/2*b/d*(cos(d*x+c)*b)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/cos(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.77

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}Cb\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(2\,A+C\right.}{}\right.$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor ithm="fricas")

[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{8\,Ab^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(2\,(dx+c)b+b\sin\left(2\,dx+2\,c\right)\right)}{4\,d}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
[Out] 1/4*(8*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (2*(d*x + c)*b +
 b*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x)
[Out] (b*(b*cos(c + d*x))^(1/2)*(C*sin(2*c + 2*d*x) + 4*A*d*x + 2*C*d*x))/(4*d*co
s(c + d*x)^(1/2))
```

3.102 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 70

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{A\text{barctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*b*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b*C*sin(d*x
+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3093, 3855}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{A\text{barctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bC\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (A*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (
b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{A\text{barctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$
\int\frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{3/2}(A\text{arctanh}(\sin(c+dx))+C\sin(c+dx))}{d\cos^{\frac{3}{2}}(c+dx)}
$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Cos[
c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 9.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result
default	$-\frac{b(2A\,\mathrm{arctanh}(\cot(dx+c)-\csc(dx+c))-\sin(dx+c)C)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$
parts	$-\frac{2A\,\mathrm{arctanh}(\cot(dx+c)-\csc(dx+c))b\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{bC\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{ib\sqrt{\cos(dx+c)b}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{\cos(dx+c)b}\,A\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{\cos(dx+c)b}\,A\ln(\mathrm{e}^{i(dx+c)}+}{\sqrt{\cos(dx+c)}\,d}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -b/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-sin(d*x+c)*C)*(cos(d*x+c)*b)^(1/2)
/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.91

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\left[\frac{Ab^{\frac{3}{2}}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin}{\cos(dx+c)^3}\right.}{2\,d}\right.$$

$$\left.-\frac{A\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)-\sqrt{b\cos(dx+c)}Cb\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)}\right]$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*b^(3/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)), -(A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,Cb^{\frac{3}{2}}\sin\left(dx+c\right)+\left(b\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\right.\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
[Out] 1/2*(2*C*b^(3/2)*sin(d*x + c) + (b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*
sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c)
+ 1))*A*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2), x)
```

3.103 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 61

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b*C*x*(b*cos(d*x+c))
^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3091, 8}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (b*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(bC\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{3/2}(Cdx\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d
*x]^(5/2))
```

Maple [A] (verified)

Time = 8.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{b\sqrt{\cos(dx+c)b}\,(C\cos(dx+c)(dx+c)+A\sin(dx+c))}{d\cos(dx+c)^{\frac{3}{2}}}$	46
parts	$\frac{Ab\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}+\frac{Cb\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	61
risch	$\frac{bCx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{2ib\sqrt{\cos(dx+c)b}\,A}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}$	63

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] b/d*(cos(d*x+c)*b)^(1/2)*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(3/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.08

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left[\frac{C\sqrt{-b}b\cos\left(dx+c\right)^2\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx\right.}\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(C*sqrt(-b)*b*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c
))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*b^(3/2)*arct
an(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
+ c)^2 + sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(Cb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + \frac{Ab^{\frac{3}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,c}\right.}{d}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
[Out] 2*(C*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*b^(3/2)*sin(2*d*x
+ 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/
d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(C\cos(dx+c)^2 + A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\left(2\right.\right.}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+\right.\right.}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x
) + C*d*x + C*d*x*cos(2*c + 2*d*x)))/(d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x
) + 1))
```

3.104 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 80

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{b(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/2*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*b*(A+2*C)*ar
ctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3091, 3855}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{b(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (b*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c +
d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

<u>Rule 17</u>

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b(A+2C)\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}} \\
&= \frac{b(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}\,((A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+d}{2d\cos^{\frac{7}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A
*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))
```

Maple [A] (verified)

Time = 9.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

method	result
default	$\frac{b\left(-A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-4C\left(\cos^2(dx+c)\right)\operatorname{arctanh}(\cot(dx}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{Ab\left(-\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)\right)\sqrt{\cos(dx+c)b}}{2d\cos(dx+c)^{\frac{5}{2}}}-\frac{2C}{}$
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\,A\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}-\frac{b\sqrt{\cos(dx+c)b}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{\cos(dx+c)b}\,(A+2C)\ln\left(e^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)}\,d}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2*b/d*(-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+A*cos(d*x+c)^2*ln(-co
t(d*x+c)+csc(d*x+c)+1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+A*si
n(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.70

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\left[\frac{(A+2\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{}}{\cos(dx}\right.}{}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\sqrt{b\cos\left(dx+c\right)}Ab\sqrt{\cos\left(dx+c\right)}\sin\left(dx+\right.}{2\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*
sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d
*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(68) = 136$.

Time = 0.48 (sec) , antiderivative size = 761, normalized size of antiderivative = 9.51

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
[Out] 1/4*(2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log
(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) - (4*(b*s
in(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*c
os(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4
*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x
+ 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) +
b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
 - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*
c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*
(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*
x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*s
in(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(9/2),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2), x)
```

3.105 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 81

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/3*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/3*b*(2*A+3*C)*
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3091, 3852, 8}

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b*(2*A
+ 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int(A+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(b(2A+3C)\sqrt{b\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{3\sqrt{\cos(c+dx)}} \\
&= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} \\
&\quad - \frac{\left(b(2A+3C)\sqrt{b\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{3d\sqrt{\cos(c+dx)}} \\
&= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/
2),x]
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*C
os[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 8.64 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{b\left(2A\left(\cos^2(dx+c)\right)+3C\left(\cos^2(dx+c)\right)+A\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}$	55
parts	$\frac{Ab\left(2\left(\cos^2(dx+c)\right)+1\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{C\sqrt{\cos(dx+c)b}\,b\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}$	75
risch	$\frac{2ib\sqrt{\cos(dx+c)b}\left(3C\,e^{4i(dx+c)}+6A\,e^{2i(dx+c)}+6C\,e^{2i(dx+c)}+2A+3C\right)}{3\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^{3}}$	82

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
[Out] 1/3*b/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*(cos(d*x+c)*b)^(1/2)*sin(d*x+
c)/cos(d*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{\left((2\,A+3\,C)b\cos\left(dx+c\right)^2+Ab\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+\right.}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
[Out] 1/3*((2*A + 3*C)*b*cos(d*x + c)^2 + A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/
(d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(69) = 138.

Time = 0.46 (sec) , antiderivative size = 355, normalized size of antiderivative = 4.38

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\left(\frac{3\,Cb^{\frac{3}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1} - \frac{\ldots}{2\,(3\,\cos(4\,dx+4\,c)+3\,\mathrm{c}\ldots}\right.}{\ldots}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
[Out] 2/3*(3*C*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) - 2*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*
cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x +
6*c) - 3*(3*b*cos(2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*cos(4
*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2
 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*c
os(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c
) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(11/2)
, x)
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.69

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\right.\right.}{}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4
*c + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*
c + 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(4
*c + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c +
 4*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*
x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.106 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 125

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\frac{b(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/4*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*b*(3*A+4*C)*
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/8*b*(3*A+4*C)*arctanh(
sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3091, 3853, 3855}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\frac{b(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{b(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2),x]
```

```
[Out] (b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c
+ d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2))
```

```
 + (b*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2
))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(b(3A+4C)\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}} \\
&= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{\left(b(3A+4C)\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{8\sqrt{\cos(c+dx)}} \\
&= \frac{b(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.65

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\frac{b\sqrt{b\cos(c+dx)}((3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)}{8d\cos^{\frac{9}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/
2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 +
 (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

Maple [A] (verified)

Time = 8.69 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.45

method	result
default	$\frac{b\left(3A\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-3A\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+4C\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)\right.}{}$
parts	$-\frac{A\left(3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)-3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-3\left(\cos^2(dx+c)\right)\sin(dx+c)-2\sin\right.}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib\sqrt{\cos(dx+c)}\,b\left(3A\,e^{7i(dx+c)}+4C\,e^{7i(dx+c)}+11A\,e^{5i(dx+c)}+4C\,e^{5i(dx+c)}-11A\,e^{3i(dx+c)}-4C\,e^{3i(dx+c)}-3A\,e^{i(dx+c)}-4C\,e^{i(dx+c)}\right.}{4\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/8*b/d*(3*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*A*cos(d*x+c)^4*ln(
-cot(d*x+c)+csc(d*x+c)-1)+4*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-4*C
*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*A*sin(d*x+c)*cos(d*x+c)^2+4*C*
cos(d*x+c)^2*sin(d*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(9/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.08

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\left[\frac{(3A+4C)b^{\frac{3}{2}}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}}{\cos(}\right.}{}\right.$$

$$-\frac{(3A+4C)\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left((3A+4C)b\cos(dx+c)^2+2Ab\right)\sqrt{b\cos}}{8d\cos(dx+c)^5}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*b*cos(d*x + c)^2 + 2*A*b)*sqrt(b*cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*((3*A + 4*C)
*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d
*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*b*cos(d*x + c)^2 + 2*A*b)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2434 vs. $2(107) = 214$.

Time = 0.52 (sec) , antiderivative size = 2434, normalized size of antiderivative = 19.47

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] -1/16*((12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c)
+ 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) +
4*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*
b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2
 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2
 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*
sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*c
os(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2
```

```
*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
 + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x +
4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) +
2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2
+ 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2
+ 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*s
in(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*
d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
+ 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4
*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2
*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12
*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*
cos(2*d*x + 2*c) + b)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*
b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) +
4*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4
*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x
+ 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2
 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d
*x + 2*c) + 1) + 4*(4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*s
in(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*
cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x
 + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x +
 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 +
2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*l
og(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2
```

```
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) +
 b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x +
 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*C*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4
*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(13/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{13/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(13/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(13/2), x)
```

3.107 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 35, antiderivative size = 125

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{b^2(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b^2(A+2C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*(A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*(A+2*C
)*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/5*b^2*C*sin(d*x+c)
^5*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3092, 380}

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=-\frac{b^2(A+2C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (b^2*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b
^2*(A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
+ (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(1-x^2\right)\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(A\left(1+\frac{C}{A}\right)-(A+2C)x^2+Cx^4\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b^2(A+2C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{5/2}(100A+89C+4(5A+7C)\cos(2(c+dx))+3C\cos(4(c+dx)))\sin(}{120d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),
x]
[Out] ((b*Cos[c + d*x])^(5/2)*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3*
C*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 8.43 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

method	result
default	$\frac{b^2\left(3C\left(\cos^4(dx+c)\right)+5A\left(\cos^2(dx+c)\right)+4C\left(\cos^2(dx+c)\right)+10A+8C\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{15d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\,b^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}+\frac{C\,b^2\left(3\left(\cos^4(dx+c)\right)+4\left(\cos^2(dx+c)\right)+8\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{15d\sqrt{\cos(dx+c)}}$
risch	$-\frac{ib^2\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{6i(dx+c)}C}{80\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{ib^2\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{2i(dx+c)}(6A+5C)}{8\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{ib^2\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+\right.}{8\left(\mathrm{e}^{2i(dx+c)}+1\right)d}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
[Out] 1/15*b^2/d*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10*A+8*C)*si
n(d*x+c)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\left(3\,Cb^2\cos\left(dx+c\right)^4+\left(5\,A+4\,C\right)b^2\cos\left(dx+c\right)^2+2\left(5\,A+4\,C\right)b^2\right)\sqrt{b\cos\left(dx+c\right)}\,\mathrm{si}}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(3*C*b^2*cos(d*x + c)^4 + (5*A + 4*C)*b^2*cos(d*x + c)^2 + 2*(5*A + 4*
C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{20\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
[Out] 1/240*(20*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))))*A*sqrt(b) + (3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*arc
tan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 150*b^2*sin(1/5*arctan2(sin(5*d
*x + 5*c), cos(5*d*x + 5*c))))*C*sqrt(b))/d
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) \, dx = \frac{b^2 \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (200 A \sin(2c+2dx) + 20 A \sin(4c+4dx) + 175}{240 d (\cos(2c+2dx) +}$$

```
[In] int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(200*A*sin(2*c + 2*d*x) + 20
*A*sin(4*c + 4*d*x) + 175*C*sin(2*c + 2*d*x) + 28*C*sin(4*c + 4*d*x) + 3*C*
sin(6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

3.108 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

```
[Out] 1/4*b^2*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*b^2*(4*A+3
*C)*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/8*b^2*(4*A+3*C)*sin(d*x+c)*co
s(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3093, 2715, 8}

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2x(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d} + \frac{b^2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b^2*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*(4*A
 + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*
C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}\\
&\quad+\frac{\left(b^2(4A+3C)\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}\\
&= \frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}\\
&\quad+\frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}\\
&\quad+\frac{\left(b^2(4A+3C)\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= \frac{b^2(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.55

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{(b\cos(c+dx))^{5/2}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+}{32d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x
)] + C*Sin[4*(c + d*x)]))/(32*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 7.87 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

method	result
default	$\frac{b^2\sqrt{\cos(dx+c)b}\,\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{b^2\sqrt{\cos(dx+c)b}\,x(8A+6C)}{16\sqrt{\cos(dx+c)}}+\frac{b^2\sqrt{\cos(dx+c)b}\,C\sin(4dx+4c)}{32\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{\cos(dx+c)b}\,(A+C)\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$
parts	$\frac{A\,b^2\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{C\,b^2\sqrt{\cos(dx+c)b}\,\left(2\sin(dx+c)\left(\cos^3(dx+c)\right)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right.}{8d\sqrt{\cos(dx+c)}}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8*b^2/d*(cos(d*x+c)*b)^(1/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*
cos(d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/cos(d*x+c)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.80

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{(4\,A+3\,C)\sqrt{-b}b^2\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right.}\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*((4*A + 3*C)*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b^2*cos(d*x + c
)^2 + (4*A + 3*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
)/d, 1/8*((4*A + 3*C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqr
t(b)*cos(d*x + c)^(3/2))) + (2*C*b^2*cos(d*x + c)^2 + (4*A + 3*C)*b^2)*sqrt
(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{8\left(2\left(dx+c\right)b^2+b^2\sin\left(2\,dx+2\,c\right)\right)A\sqrt{b}+\left(12\left(dx+c\right)b^2+b\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + (12*(d*x + c)*
b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))))*C*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)),
 x)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(2\,c+2\,d\,x\right)+8\,C\,\sin\left(2\,c+2\,d\right.}{32\,d\,\sqrt{\cos\left(c+d\right.}}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(8*A*sin(2*c + 2*d*x) + 8*C*sin(2*c + 2*d*x) +
C*sin(4*c + 4*d*x) + 16*A*d*x + 12*C*d*x))/(32*d*cos(c + d*x)^(1/2))
```

3.109 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 80

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b^2C\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*(A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*C*sin(
d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {17, 3092}

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{b^2C\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b^2*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b
^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b^2C\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int\frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d
*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 8.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{b^{2} \left(C \left(\cos^{2}\left(d x +c \right)\right)+3 A +2 C \right) \sin \left(d x +c \right) \sqrt{\cos \left(d x +c \right) b}}{3 d \sqrt{\cos \left(d x +c \right)}}$	50
risch	$\frac{b^{2} \sqrt{\cos \left(d x +c \right) b}\, \left(4 A +3 C \right) \sin \left(d x +c \right)}{4 \sqrt{\cos \left(d x +c \right)}\, d}+\frac{b^{2} \sqrt{\cos \left(d x +c \right) b}\, C \sin \left(3 d x +3 c \right)}{12 \sqrt{\cos \left(d x +c \right)}\, d}$	77
parts	$\frac{A \,b^{2} \sin \left(d x +c \right) \sqrt{\cos \left(d x +c \right) b}}{d \sqrt{\cos \left(d x +c \right)}}+\frac{C \,b^{2} \left(2+\cos^{2}\left(d x +c \right)\right) \sin \left(d x +c \right) \sqrt{\cos \left(d x +c \right) b}}{3 d \sqrt{\cos \left(d x +c \right)}}$	77

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3*b^2/d*(C*cos(d*x+c)^2+3*A+2*C)*sin(d*x+c)*(cos(d*x+c)*b)^(1/2)/cos(d*x+
c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{\left(Cb^2\cos\left(dx+c\right)^2+(3\,A+2\,C)b^2\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+\right.}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*b^2*cos(d*x + c)^2 + (3*A + 2*C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x +
 c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{12\,Ab^{\frac{5}{2}}\sin\left(dx+c\right)+\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\,\arctan\right.\right.}{12\,d}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
[Out] 1/12*(12*A*b^(5/2)*sin(d*x + c) + (b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\,C\,\sin\left(c+d\,x\right)+\right.}{12\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x)
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + C*sin(3
*c + 3*d*x)))/(12*d*cos(c + d*x)^(1/2))
```

3.110 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 99

$$\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

```
[Out] A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b^2*C*x*(b*cos(d*x+c))^(1
/2)/cos(d*x+c)^(1/2)+1/2*b^2*C*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(
1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2715, 8}

$$\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2C\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*C*x*Sqrt[b*Cos[c +
 d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(b^2C\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}\\
&\quad+\frac{\left(b^2C\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{5/2}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*
Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 8.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{b^{2}\sqrt{\cos(dx+c)b}\,(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))}{2d\sqrt{\cos(dx+c)}}$	57
risch	$\frac{b^{2}\sqrt{\cos(dx+c)b}\,x(4A+2C)}{4\sqrt{\cos(dx+c)}}+\frac{b^{2}\sqrt{\cos(dx+c)b}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	69
parts	$\frac{A\,b^{2}\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{C\,b^{2}\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}$	78

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2*b^2/d*(cos(d*x+c)*b)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+
c))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.73

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^{2}(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\left[\frac{2\sqrt{b\cos\left(dx+c\right)}Cb^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(2A+C\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A +
 C)*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*s
qrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*b^2*sqr
t(cos(d*x + c))*sin(d*x + c) + (2*A + C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c)
)*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.60

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{8\,Ab^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(2\,(dx+c)b^2+b^2\sin\left(2\,dx+2\right.\right.}{4\,d}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
[Out] 1/4*(8*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (2*(d*x + c)*b^2
 + b^2*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),x)

[Out] (b^2*(b*cos(c + d*x))^(1/2)*(C*sin(2*c + 2*d*x) + 4*A*d*x + 2*C*d*x))/(4*d*cos(c + d*x)^(1/2))

3.111 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{Ab^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*b^2*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b^2*C*sin
(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3093, 3855}

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{Ab^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (A*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) +
 (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(A\text{arctanh}(\sin(c+dx))+C\sin(c+dx))}{d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Cos[
c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 8.72 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

method	result
default	$-\frac{b^{2}\left(2A\ \operatorname{arctanh}\left(\cot\left(dx+c\right)-\csc\left(dx+c\right)\right)-\sin\left(dx+c\right)C\right)\sqrt{\cos\left(dx+c\right)b}}{d\sqrt{\cos\left(dx+c\right)}}$
parts	$-\frac{2A\ \operatorname{arctanh}\left(\cot\left(dx+c\right)-\csc\left(dx+c\right)\right)b^{2}\sqrt{\cos\left(dx+c\right)b}}{d\sqrt{\cos\left(dx+c\right)}}+\frac{b^{2}C\sin\left(dx+c\right)\sqrt{\cos\left(dx+c\right)b}}{d\sqrt{\cos\left(dx+c\right)}}$
risch	$-\frac{i b^{2}\sqrt{\cos\left(dx+c\right)b}\,C\,{\mathrm e}^{i\left(dx+c\right)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{i b^{2}\sqrt{\cos\left(dx+c\right)b}\,C\,{\mathrm e}^{-i\left(dx+c\right)}}{2\sqrt{\cos\left(dx+c\right)}\,d}-\frac{b^{2}\sqrt{\cos\left(dx+c\right)b}\,A\ln\left({\mathrm e}^{i\left(dx+c\right)}-i\right)}{\sqrt{\cos\left(dx+c\right)}\,d}+\frac{b^{2}\sqrt{\cos\left(dx+c\right)b}\,A\ln\left({\mathrm e}^{i\left(d\right.}\right.}{\sqrt{\cos\left(dx+c\right)}\,d}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -b^2/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-sin(d*x+c)*C)*(cos(d*x+c)*b)^(1/
2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.84

$$\int\frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^{2}(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\left[\frac{Ab^{\frac{5}{2}}\cos\left(dx+c\right)\log\left(-\frac{b\cos\left(dx+c\right)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\,s}{\cos\left(dx+c\right)^{3}}\right.}{2}\right.$$
$$\left.-\frac{A\sqrt{-b}b^{2}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}Cb^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}\right]$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*b^(5/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)), -(A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*b^2*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\,Cb^{\frac{5}{2}}\sin\left(dx+c\right)+\left(b^2\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\right.\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(2*C*b^(5/2)*sin(d*x + c) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 +
2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x +
 c) + 1))*A*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(7/2),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2), x)
```

3.112 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 65

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b^2*C*x*(b*cos(d*x
+c))^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3091, 8}

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (b^2*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(b^2C\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(Cdx\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{7}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d
*x]^(7/2))
```

Maple [A] (verified)

Time = 8.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{b^{2}\sqrt{\cos(dx+c)b}\,(C\cos(dx+c)(dx+c)+A\sin(dx+c))}{d\cos(dx+c)^{\frac{3}{2}}}$	48
parts	$\frac{A\,b^{2}\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}+\frac{C\,b^{2}\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	65
risch	$\frac{b^{2}Cx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{2ib^{2}\sqrt{\cos(dx+c)b}\,A}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)}$	67

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] b^2/d*(cos(d*x+c)*b)^(1/2)*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(
3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.98

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^{2}(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left[\frac{C\sqrt{-b}b^{2}\cos\left(dx+c\right)^{2}\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx\right.}\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(C*sqrt(-b)*b^2*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x +
 c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*b^(5/2)*
arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(
d*x + c)^2 + sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left(Cb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\frac{Ab^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(}\right.}{d}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
[Out] 2*(C*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*b^(5/2)*sin(2*d*x
+ 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/
d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(9/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{b^2\,\sqrt{b\,\cos(c+d\,x)}\,(A\,\sin(2\,c+2\,d\,x)+C\,d\,x+C\,d\,x\,\cos(2}{d\,\sqrt{\cos(c+d\,x)}\,\left(\cos(2\,c+2\right.}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d
*x) + C*d*x + C*d*x*cos(2*c + 2*d*x)))/(d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d
*x) + 1))
```

3.113 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 84

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{b^2(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/2*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*b^2*(A+2*C
)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3091, 3855}

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{b^2(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (b^2*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c
+ d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2
))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b^2(A+2C)\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\,((A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx}{2d\cos^{\frac{9}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/
2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A
*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(9/2))
```

Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
default	$\frac{b^2(-A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-4C(\cos^2(dx+c))\,\mathrm{arctanh}(\cot}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\,b^2(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c))\sqrt{\cos(dx+c)b}}{2d\cos(dx+c)^{\frac{5}{2}}} -$
risch	$-\frac{ib^2\sqrt{\cos(dx+c)b}\,A(e^{3i(dx+c)}-e^{i(dx+c)})}{\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^2} - \frac{b^2\sqrt{\cos(dx+c)b}\,(A+2C)\ln(e^{i(dx+c)}-i)}{2\sqrt{\cos(dx+c)}\,d} + \frac{b^2\sqrt{\cos(dx+c)b}\,(A+2C)\ln(e^{i(dx+c)}+i}{2\sqrt{\cos(dx+c)}\,d}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/2*b^2/d*(-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+A*cos(d*x+c)^2*ln(-
cot(d*x+c)+csc(d*x+c)+1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+A*
sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.64

$$\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \left[\frac{(A+2C)b^{\frac{5}{2}}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}}{\cos(}\right.}{}\right.$$

$$-\frac{(A+2C)\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\sqrt{b\cos(dx+c)}Ab^2\sqrt{\cos(dx+c)}\sin(dx}{2\,d\cos(dx+c)^3}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x +
c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*c
os(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(72) = 144$.

Time = 0.43 (sec) , antiderivative size = 821, normalized size of antiderivative = 9.77

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
[Out] 1/4*(2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2
*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) - (4*
(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d*x +
4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*
c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*s
in(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d
*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^
2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x + 4*c)
+ 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{11/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2), x)
```

3.114 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 85

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b^2(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/3*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/3*b^2*(2*A+3
*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3091, 3852, 8}

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b^2(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2),x]
```

```
[Out] (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b^2*(
2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(b^2(2A+3C)\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{3\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} - \frac{\left(b^2(2A+3C)\sqrt{b\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b^2(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{5/2}\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/
2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*C
os[c + d*x]^(7/2))
```

Maple [A] (verified)

Time = 7.66 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{b^2\left(2A\left(\cos^2(dx+c)\right)+3C\left(\cos^2(dx+c)\right)+A\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}$	57
parts	$\frac{A\,b^2\left(2\left(\cos^2(dx+c)\right)+1\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{C\sin(dx+c)\sqrt{\cos(dx+c)b}\,b^2}{d\cos(dx+c)^{\frac{3}{2}}}$	79
risch	$\frac{2ib^2\sqrt{\cos(dx+c)b}\left(3C\,e^{4i(dx+c)}+6A\,e^{2i(dx+c)}+6C\,e^{2i(dx+c)}+2A+3C\right)}{3\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^3}$	84

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/3*b^2/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*(cos(d*x+c)*b)^(1/2)*sin(d*
x+c)/cos(d*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\frac{\left((2\,A+3\,C)b^2\cos\left(dx+c\right)^2+Ab^2\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+\right.}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*b^2*cos(d*x + c)^2 + A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x +
 c)/(d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(73) = 146$.

Time = 0.42 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.32

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2\left(\frac{3\,Cb^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1} - \frac{}{2\,(3\,\cos(4\,dx+4\,c)+3}\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="maxima")
[Out] 2/3*(3*C*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) - 2*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*
b^2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(
6*d*x + 6*c) - 3*(3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*sqrt(b)
/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*
d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x +
4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(13/2)
, x)
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.59

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)\right.}{}$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos
(4*c + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(
4*c + 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin
(4*c + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c
 + 4*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*
d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.115 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 131

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{b^2(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b^2(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/4*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*b^2*(3*A+4
*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/8*b^2*(3*A+4*C)*ar
ctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3091, 3853, 3855}

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{b^2(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{b^2(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15/2),x]
```

```
[Out] (b^2*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[
c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9
```

```
/2)) + (b^2*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x
]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{\left(b^2(3A+4C)\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b^2(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}\\
&\quad+\frac{\left(b^2(3A+4C)\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{8\sqrt{\cos(c+dx)}}\\
&= \frac{b^2(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b^2(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\left((3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+\right.}{8d\cos^{\frac{13}{2}}(c+}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15/
2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 +
 (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(13/2)
)
```

Maple [A] (verified)

Time = 7.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.40

method	result
default	$\frac{b^2\left(3A\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-3A\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+4C\left(\cos^4(dx+c)\right)\ln(-\cot(d\right.}{}$
parts	$\frac{A\,b^2\left(-3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+3\left(\cos^2(dx+c)\right)\sin(dx+c)+\right.}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib^2\sqrt{\cos(dx+c)b}\,\left(3A\,e^{7i(dx+c)}+4C\,e^{7i(dx+c)}+11A\,e^{5i(dx+c)}+4C\,e^{5i(dx+c)}-11A\,e^{3i(dx+c)}-4C\,e^{3i(dx+c)}-3A\,e^{i(dx+c)}-4C\,e^{i(dx\right.}{4\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/8*b^2/d*(3*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*A*cos(d*x+c)^4*l
n(-cot(d*x+c)+csc(d*x+c)-1)+4*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-4
*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*A*sin(d*x+c)*cos(d*x+c)^2+4*
C*cos(d*x+c)^2*sin(d*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(
9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.06

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \left[\frac{(3A+4C)b^{\frac{5}{2}}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\cdots}{\cos(\cdots}\right.}{\cdots}\right.$$

$$-\frac{(3A+4C)\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left((3A+4C)b^2\cos(dx+c)^2+2Ab^2\right)\sqrt{b}\cdots}{8d\cos(dx+c)^5}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, algo
rithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*b^2*cos(d*x + c)^2 + 2*A*b^2)*sqrt(b*cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*((3*A +
4*C)*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt
(cos(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*b^2*cos(d*x + c)^2 + 2*A*b^2
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(15/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2662 vs. 2(113) = 226.

Time = 0.50 (sec) , antiderivative size = 2662, normalized size of antiderivative = 20.32

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, algo
rithm="maxima")
```

```
[Out] -1/16*((12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d*x
 + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin
```

(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b
^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c)
 + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*
x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*si
n(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 +
48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^
2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*
c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x +
4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x
 + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d
*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x
+ 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*
d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*
sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 +
 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c
) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2
*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x +
 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*
x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x +
 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(b^2*cos(8*d*x +
8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x +
2*c) + b^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(b^2*
cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*
cos(2*d*x + 2*c) + b^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 44*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4
*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*c
os(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x +
 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*
x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x +
4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c)
 + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 +
 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*

```
x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1) + 4*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*s
in(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(
b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)
^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b
^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x +
 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x
 + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x +
 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)
*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x +
4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*C*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + co
s(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{15}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(15/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{15/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(15/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(15/2), x)
```

3.116 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 113

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/4*C*cos(
d*x+c)^(7/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(d*x+c)^(
1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3093, 2715, 8}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*
x]^(7/2)*Sin[c + d*x])/(4*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}+\frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{b\cos(c+dx)}}\\
&= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{b\cos(c+dx)}}\\
&= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],
x]
[Out] (Sqrt[Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] +
 C*Sin[4*(c + d*x)]))/(32*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 8.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

method	result
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8d\sqrt{\cos(dx+c)b}}$
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)x(8A+6C)}{16\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(4dx+4c)}{32\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)(A+C)\sin(2dx+2c)}{4\sqrt{\cos(dx+c)b}\,d}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\left(\sqrt{\cos}(dx+c)\right)}{2d\sqrt{\cos(dx+c)b}}+\frac{C\left(2\sin(dx+c)\left(\cos^3(dx+c)\right)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)\left(\sqrt{\cos}(dx+c)\right)}{8d\sqrt{\cos(dx+c)b}}$

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
[Out] 1/8/d*cos(d*x+c)^(1/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*cos(d*x+
c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.83

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \left[\frac{2\left(2C\cos(dx+c)^2+4A+3C\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)-(4A+3C)\sqrt{-b}\log\left(2b\cos\right.}{16bd}\right.$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/8*(
(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(
sqrt(b)*cos(d*x + c)^(3/2))))/(b*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{\sqrt{b}} + \frac{\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)C}{\sqrt{b}}}{32\,d}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + (12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/sqr
t(b))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x
)
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(c+d\,x\right)+8\,C\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(3\,c+3\,d\,x\right)+9\,C\,\sin\left(3\,c\right.\right.}{32\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int((cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) + 3
2*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*b*d*(cos(2*c + 2*d*x) +
1))
```

3.117 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{C\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (A+C)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-1/3*C*sin(d*x+c)^3
*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {17, 3092}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} - \frac{C\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) - (C*Sqr
t[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\text{Subst}\left(\int\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}\\ &= \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{C\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqr
t[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 8.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\left(C\left(\cos^{2}\left(dx+c\right)\right)+3A+2C\right)\left(\sqrt{\cos}\left(dx+c\right)\right)\sin\left(dx+c\right)}{3d\sqrt{\cos\left(dx+c\right)b}}$	47
risch	$\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)\left(4A+3C\right)\sin\left(dx+c\right)}{4\sqrt{\cos\left(dx+c\right)b}\,d}+\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)C\sin\left(3dx+3c\right)}{12\sqrt{\cos\left(dx+c\right)b}\,d}$	71
parts	$\frac{A\sin\left(dx+c\right)\left(\sqrt{\cos}\left(dx+c\right)\right)}{d\sqrt{\cos\left(dx+c\right)b}}+\frac{C\left(2+\cos^{2}\left(dx+c\right)\right)\left(\sqrt{\cos}\left(dx+c\right)\right)\sin\left(dx+c\right)}{3d\sqrt{\cos\left(dx+c\right)b}}$	71

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*cos(d*x+c)^(1/2)*sin(d*x+c)/(cos(d*x+c)*b)^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int\frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^{2}(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\frac{\left(C\cos\left(dx+c\right)^{2}+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,bd\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*s
qrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int\frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^{2}(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{\sqrt{b}}+\frac{12\,A\sin(dx+c)}{\sqrt{b}}}{12\,d}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))))/sqrt(b) + 12*A*sin(d*x + c)/sqrt(b))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x
)
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(12\,A\,\sin(2\,c+2\,d\,x)+10\,C\,\sin(2\,c+2\,d\,x)+C\,\sin(4\,c+4\,d\,x))}{12\,b\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

```
[In] int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*si
n(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))
```

3.118 $\int \frac{\sqrt{\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 90

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)^(1/
2)/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2715, 8}

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (C*x*Sqrt[Cos[c + d*x]])/(2
*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Co
s[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int \left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$
\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{b\cos(c+dx)}}
$$

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*Sqrt
[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 8.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c)\right)}{2d\sqrt{\cos(dx+c)b}}$	54
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)x(4A+2C)}{4\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)b}\,d}$	63
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\sqrt{\cos(dx+c)b}}+\frac{C(\cos(dx+c)\sin(dx+c)+dx+c)\left(\sqrt{\cos}(dx+c)\right)}{2d\sqrt{\cos(dx+c)b}}$	72

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*cos(d*x+c)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/(cos
(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[\frac{2\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c)-(2A+C)\sqrt{-b}\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\right.}{4bd}\right.$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)*
sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(
d*x + c))*sin(d*x + c) - b))/(b*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(d*
x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b*d)]
```

Sympy [A] (verification not implemented)

Time = 16.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \begin{cases} \frac{Ax\sqrt{\cos{(c+dx)}}}{\sqrt{b\cos{(c+dx)}}} + \frac{Cx\sin^2{(c+dx)}\sqrt{\cos{(c+dx)}}}{2\sqrt{b\cos{(c+dx)}}} + \frac{Cx\cos^{\frac{5}{2}}{(c+dx)}}{2\sqrt{b\cos{(c+dx)}}} + \frac{C\sin{(c+dx)}\cos^{\frac{3}{2}}{(c+dx)}}{2d\sqrt{b\cos{(c+dx)}}} & \text{for } d \neq 0 \\ \frac{x(A+C\cos^2{(c)})\sqrt{\cos{(c)}}}{\sqrt{b\cos{(c)}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((A*x*sqrt(cos(c + d*x))/sqrt(b*cos(c + d*x)) + C*x*sin(c + d*x)**
2*sqrt(cos(c + d*x))/(2*sqrt(b*cos(c + d*x))) + C*x*cos(c + d*x)**(5/2)/(2*
sqrt(b*cos(c + d*x))) + C*sin(c + d*x)*cos(c + d*x)**(3/2)/(2*d*sqrt(b*cos(
c + d*x))), Ne(d, 0)), (x*(A + C*cos(c)**2)*sqrt(cos(c))/sqrt(b*cos(c)), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{\sqrt{b}} + \frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}}{4\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/sqrt(b) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/sqrt(b))/d
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(C\cos{(dx+c)}^2+A)\sqrt{\cos{(dx+c)}}}{\sqrt{b\cos{(dx+c)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x
)
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)+C\,\sin\left(3\,c+3\,d\,x\right)+8\,A\,d\,x\,\cos\left(c+d\,x\right)+4\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{4\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + C*sin(3*c + 3*
d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))/(4*b*d*(cos(2*c + 2*d*
x) + 1))
```

3.119 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 68

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x+c)*
cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {18, 3093, 3855}

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{A\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (C*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)} \int (A+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} \\
&= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} \\
&= \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(A\text{arctanh}(\sin(c+dx))+C\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Sqrt[b*C
os[c + d*x]])
```

Maple [A] (verified)

Time = 7.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{(2A\ \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-\sin(dx+c)C)\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}$	53
parts	$\frac{C\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}-\frac{2A\ \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}$	71
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(e^{i(dx+c)}+i\right)}{\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(e^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)b}\,d}+\frac{C\sin(2dx+2c)}{2d\sqrt{\cos(dx+c)}\sqrt{\cos(dx+c)b}}$	108

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-sin(d*x+c)*C)*cos(d*x+c)^(1/2)/(co
s(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.04

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[\frac{A\sqrt{b}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}C\sqrt{c}}{2\,bd\cos(dx+c)}\right.$$

$$\left.-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)-\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c)}{bd\cos(dx+c)}\right]$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x
 + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c)
)*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{A+C\cos^2{(c+dx)}}{\sqrt{b\cos{(c+dx)}}\sqrt{\cos{(c+dx)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
[Out] Integral((A + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*sqrt(cos(c + d*x))),
 x)
```

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{2C\sin(dx+c)}{\sqrt{b}}}{2d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 2*C*sin(d*x +
 c)/sqrt(b))/d
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos{(dx+c)}^2+A}{\sqrt{b\cos{(dx+c)}}\sqrt{\cos{(dx+c)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos{(c+d\,x)}^2 + A}{\sqrt{\cos{(c+d\,x)}}\,\sqrt{b\,\cos{(c+d\,x)}}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.120 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 59

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/2)/(
b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {18, 3091, 8}

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*Sin[c + d*x])/(d*Sqrt[Co
s[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
```

```
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{Cdx\cos(c+dx)+A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),
x]
[Out] (C*d*x*Cos[c + d*x] + A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c +
d*x]])
```

Maple [A] (verified)

Time = 7.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{C\cos(dx+c)(dx+c)+A\sin(dx+c)}{d\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	45
risch	$\frac{Cx(\sqrt{\cos}(dx+c))}{\sqrt{\cos(dx+c)b}}+\frac{i\mathrm{e}^{-i(dx+c)}A}{\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,d}$	57
parts	$\frac{A\sin(dx+c)}{d\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}+\frac{C(\sqrt{\cos}(dx+c))(dx+c)}{d\sqrt{\cos(dx+c)b}}$	59

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.24

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\left[-\frac{C\sqrt{-b}\cos(dx+c)^2\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2}{2\,bd\cos(dx+c)^2}\right.$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c)
)*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2), (C*sqrt(b)*arcta
n(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x +
 c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*
x + c)^2)]
```

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{A+C\cos^2{(c+dx)}}{\sqrt{b\cos{(c+dx)}}\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2))
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.44

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{2\left(\frac{C\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{A\sqrt{b}\sin(2\,dx+2\,c)}{b\cos(2\,dx+2\,c)^2+b\sin(2\,dx+2\,c)^2+2\,b\cos(2\,dx+2\,c)+b}\right)}{d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + A*sqrt(b)*sin(2*d*x
+ 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c)
+ b))/d
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{C\cos{(dx+c)}^2+A}{\sqrt{b\cos{(dx+c)}}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(3/2)),
 x)
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.42

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\, dx$$
$$= \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\left(2\,c+2\,d\,x\right)+A\,1\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{b\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x)
+ C*d*x + C*d*x*cos(2*c + 2*d*x)))/(b*d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x
) + 1))
```

3.121 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 78

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+2C)\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*arctan
h(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3091, 3855}

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+2C)\sqrt{\cos(c+dx)}\operatorname{arctanh}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*
x]]) + (A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left((A+2C)\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2\sqrt{b\cos(c+dx)}}\\
&= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int\frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),
x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x])/(2*d*Cos[
c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.47

method	result
default	$\frac{-A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-4C(\cos^2(dx+c))\operatorname{arctanh}(\cot(dx}{2d\sqrt{\cos(dx+c)b}\cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c))}{2d\sqrt{\cos(dx+c)b}\cos(dx+c)^{\frac{3}{2}}} - \frac{2C\operatorname{arctanh}(\cot(}{}$
risch	$-\frac{iA(e^{2i(dx+c)}-1)}{2\sqrt{\cos(dx+c)b}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)d} - \frac{(\sqrt{\cos}(dx+c))(A+2C)\ln(e^{i(dx+c)}-i)}{2\sqrt{\cos(dx+c)b}\,d} + \frac{(\sqrt{\cos}(dx+c))(A+2C)\ln(e^{i(dx+c)}+i}{2\sqrt{\cos(dx+c)b}\,d}$

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+A*cos(d*x+c)^2*ln(-cot(
d*x+c)+csc(d*x+c)+1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(
d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.81

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx$$
$$= \left[\frac{(A + 2C)\sqrt{b}\cos(dx + c)^3\log\left(-\frac{b\cos(dx+c)^3 - 2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c) - 2b\cos(dx+c)}{\cos(dx+c)^3}\right) + 2\sqrt{b\cos(dx}{4bd\cos(dx + c)^3}\right.$$
$$-\frac{(A + 2C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx + c)^3 - \sqrt{b\cos(dx + c)}A\sqrt{\cos(dx + c)}\sin(dx}{2bd\cos(dx + c)^3}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b
*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sq
rt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x
 + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(66) = 132.

Time = 0.43 (sec) , antiderivative size = 728, normalized size of antiderivative = 9.33

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
[Out] 1/4*(2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(c
os(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) - (4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
 + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2
*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*
x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*A/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos(dx + c)^2 + A}{\sqrt{b\cos(dx + c)}\cos(dx + c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(5/2)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos(c + d\,x)^2 + A}{\cos(c + d\,x)^{5/2}\,\sqrt{b\,\cos(c + d\,x)}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.122 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 79

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(
d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3091, 3852, 8}

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*
C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} - \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{3d\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sin(c+dx)\,(3(A+C)+A\tan^2(c+dx))}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),
x]
[Out] (Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[
b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.68 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\left(2A\left(\cos^{2}\left(dx+c\right)\right)+3C\left(\cos^{2}\left(dx+c\right)\right)+A\right)\sin\left(dx+c\right)}{3d\sqrt{\cos\left(dx+c\right)b}\,\cos\left(dx+c\right)^{\frac{5}{2}}}$	54
parts	$\frac{A\left(2\left(\cos^{2}\left(dx+c\right)\right)+1\right)\sin\left(dx+c\right)}{3d\sqrt{\cos\left(dx+c\right)b}\,\cos\left(dx+c\right)^{\frac{5}{2}}}+\frac{C\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)b}\,\sqrt{\cos\left(dx+c\right)}}$	73
risch	$\frac{i\left(3C\,{\mathrm e}^{3i\left(dx+c\right)}+\left(8A+9C\right)\cos\left(dx+c\right)+i\left(4A+3C\right)\sin\left(dx+c\right)\right)}{3\sqrt{\cos\left(dx+c\right)b}\,\sqrt{\cos\left(dx+c\right)}\,\left({\mathrm e}^{2i\left(dx+c\right)}+1\right)^{2}d}$	81

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*sin(d*x+c)/(cos(d*x+c)*b)^(1/2)
/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,bd\cos\left(dx+c\right)^{\frac{7}{2}}}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d
*cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(67) = 134.

Time = 0.60 (sec) , antiderivative size = 355, normalized size of antiderivative = 4.49

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx$$

$$= \frac{2\left(\frac{3\,C\sqrt{b}\sin(2\,dx+2\,c)}{b\cos(2\,dx+2\,c)^2+b\sin(2\,dx+2\,c)^2+2\,b\cos(2\,dx+2\,c)+b} + \frac{\cdots}{\left(2\,(3\,\cos(4\,dx+4\,c)+3\,\cos(2\,dx+2\,c)+1)\cos(6\,dx+6\,c)+\cos(6\,dx+6\,c)^2+6\,(3\,\cos\right.}\cdots\right.}{\cdots}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c
)^2 + 2*b*cos(2*d*x + 2*c) + b) + 2*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6
*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(
2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d*x + 4*
c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
+ 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6
*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\sqrt{b\cos\left(dx + c\right)}\cos\left(dx + c\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)),
 x)
```

Mupad [B] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.78

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,A\,\sin\left(6\,c+6\,d\,x\right)+15\,C\,\sin\left(2\,c+2\,d\,x\right)\right.}{}$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*c
 + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*c
+ 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(4*c
 + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c + 4
*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*b*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*
x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.123 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*A*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*sin(
d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*arctanh(sin(d*
x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {18, 3091, 3853, 3855}

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(3A+4C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{8d\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c +
d*x]]) + (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (
(3*A + 4*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} \\
&\quad+\frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{8\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

$$
= \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$
\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),
x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[
c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.48

method	result
default	$\frac{3A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-3A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+4C(\cos^4(dx+c))\ln(-\cot(dx+c}{8d\sqrt{\cos(d}}$
parts	$\frac{A(-3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+3(\cos^2(dx+c))\sin(dx+c)+2\,\mathrm{si}}{8d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{i(3A\,e^{6i(dx+c)}+4C\,e^{6i(dx+c)}+11A\,e^{4i(dx+c)}+4C\,e^{4i(dx+c)}-11A\,e^{2i(dx+c)}-4C\,e^{2i(dx+c)}-3A-4C)}{8\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^3 d}-\frac{(\sqrt{\cos}(dx+c))(3A+4C)}{8\sqrt{\cos(dx+c}}$

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*(3*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*A*cos(d*x+c)^4*ln(-c
ot(d*x+c)+csc(d*x+c)-1)+4*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-4*C*c
os(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*A*sin(d*x+c)*cos(d*x+c)^2+4*C*co
s(d*x+c)^2*sin(d*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.14

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{(3A+4C)\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left((3A+4C\right.}{16\,bd\cos(dx+c)^5}\right.$$
$$-\frac{(3A+4C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left((3A+4C)\cos(dx+c)^2+2A\right)\sqrt{b\cos}}{8\,bd\cos(dx+c)^5}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5), -1/8*((3*A + 4*C)*s
qrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x +
 c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2318 vs. 2(104) = 208.

Time = 0.48 (sec) , antiderivative size = 2318, normalized size of antiderivative = 19.00

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44
*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x
+ 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*
x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)
^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*
cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(
4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(
3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x +
6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*
sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos
(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*
(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x +
8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1
)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
 + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
```

```
*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c
) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*A/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c)
+ 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(
2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c
)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x +
2*c) + 1)*sqrt(b)) + 4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +
2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2
*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x
+ 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin
(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(
cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/((2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos{(dx + c)}^2 + A}{\sqrt{b\cos{(dx + c)}}\cos{(dx + c)}^{\frac{9}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(9/2)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^{9/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.124 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/4*C*co
s(d*x+c)^(7/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(d*x+
c)^(1/2)/b/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3093, 2715, 8}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c
+ d*x]^(7/2)*Sin[c + d*x])/(4*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4b\sqrt{b\cos(c+dx)}}$$

$$= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8b\sqrt{b\cos(c+dx)}}$$

$$= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.55

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C}{32d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] +
 C*Sin[4*(c + d*x)]))/(32*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 7.57 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

method	result	s
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8bd\sqrt{\cos(dx+c)b}}$	9
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)x(8A+6C)}{16b\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(4dx+4c)}{32b\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)(A+C)\sin(2dx+2c)}{4b\sqrt{\cos(dx+c)b}\,d}$	1
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)\left(\cos(dx+c)\sin(dx+c)+dx+c\right)}{2db\sqrt{\cos(dx+c)b}}+\frac{C\left(\sqrt{\cos}(dx+c)\right)\left(2\sin(dx+c)\left(\cos^3(dx+c)\right)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)}{8db\sqrt{\cos(dx+c)b}}$	1

```
[In] int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/b/d*cos(d*x+c)^(1/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*cos(d*
x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/(cos(d*x+c)*b)^(1/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.70

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{2\left(2\,C\cos\left(dx+c\right)^2+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\,s}{}\right.$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/8
*((2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)
/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(7/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{3}{2}}} + \frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\arctan(\sin(4\,dx+\right.}{b^{\frac{3}{2}}}}{32\,d}$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(3/2) + (12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/b^(
3/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(8\,A\,\sin(c+d\,x)+8\,C\,\sin(c+d$$

```
[In] int((cos(c + d*x)^(7/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) + 3
2*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*b^2*d*(cos(2*c + 2*d*x)
+ 1))
```

3.125 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 80

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{C\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (A+C)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-1/3*C*sin(d*x+c)
^3*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {17, 3092}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}-\frac{C\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) - (C*S
qrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int\cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\text{Subst}\left(\int\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}$$

$$= \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{C\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int\frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2
),x]
[Out] (Cos[c + d*x]^(3/2)*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*(b*
Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{\left(C\left(\cos^{2}\left(dx+c\right)\right)+3A+2C\right)\left(\sqrt{\cos}\left(dx+c\right)\right)\sin\left(dx+c\right)}{3bd\sqrt{\cos\left(dx+c\right)b}}$	50
risch	$\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)\left(4A+3C\right)\sin\left(dx+c\right)}{4b\sqrt{\cos\left(dx+c\right)b}\,d}+\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)C\sin\left(3dx+3c\right)}{12b\sqrt{\cos\left(dx+c\right)b}\,d}$	77
parts	$\frac{A\sin\left(dx+c\right)\left(\sqrt{\cos}\left(dx+c\right)\right)}{bd\sqrt{\cos\left(dx+c\right)b}}+\frac{C\left(2+\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)\left(\sqrt{\cos}\left(dx+c\right)\right)}{3db\sqrt{\cos\left(dx+c\right)b}}$	77

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/b/d*(C*cos(d*x+c)^2+3*A+2*C)*cos(d*x+c)^(1/2)*sin(d*x+c)/(cos(d*x+c)*b)
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.61

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\left(C\cos\left(dx+c\right)^2+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,b^2d\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*d
*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{3}{2}}}+\frac{12\,A\sin(dx+c)}{b^{\frac{3}{2}}}}{12\,d}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))))/b^(3/2) + 12*A*sin(d*x + c)/b^(3/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\sin\right.}{12\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int((cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*si
n(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.126 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 99

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)^(
1/2)/b/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 2715, 8}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]]
)/(2*b*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*d*S
qrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int\left(A+C\cos^2(c+dx)\right)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2b\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$
\int\frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\cos^{\frac{3}{2}}(c+dx)(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d(b\cos(c+dx))^{3/2}}
$$

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2
),x]
[Out] (Cos[c + d*x]^(3/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*(b*C
os[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 7.82 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{(\sqrt{\cos}(dx+c))(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))}{2bd\sqrt{\cos(dx+c)b}}$	57
risch	$\frac{(\sqrt{\cos}(dx+c))x(4A+2C)}{4b\sqrt{\cos(dx+c)b}}+\frac{(\sqrt{\cos}(dx+c))C\sin(2dx+2c)}{4b\sqrt{\cos(dx+c)b}\,d}$	69
parts	$\frac{A(\sqrt{\cos}(dx+c))(dx+c)}{d\sqrt{\cos(dx+c)b}\,b}+\frac{C(\sqrt{\cos}(dx+c))(\cos(dx+c)\sin(dx+c)+dx+c)}{2db\sqrt{\cos(dx+c)b}}$	78

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/b/d*cos(d*x+c)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/(c
os(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.71

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-(2A+C)\sqrt{-}}{}\right.$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)*
sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(
d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(
d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{3}{2}}} + \frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}}{4\,d}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(3/2) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/b^(3/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(C\,\sin(c+d\,x)+C\,\sin(3\,c+3$$
$$4\,b^2\,d\,(\cos(2\,c+2\,d$$

```
[In] int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + C*sin(3*c + 3*
d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))/(4*b^2*d*(cos(2*c + 2*
d*x) + 1))
```

3.127 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x+c
)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3093, 3855}

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{A\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (
C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\cos^{\frac{3}{2}}(c+dx)(A\operatorname{arctanh}(\sin(c+dx))+C\sin(c+dx))}{d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*(b*Cos[c
 + d*x])^(3/2))
```

Maple [A] (verified)

Time = 8.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\left(2A \,\operatorname{arctanh}\left(\cot \left(d x +c \right)-\csc \left(d x +c \right)\right)-\sin \left(d x +c \right) C \right) \left(\sqrt{\cos}\left(d x +c \right)\right)}{b d \sqrt{\cos \left(d x +c \right) b}}$
parts	$-\frac{2 A \,\operatorname{arctanh}\left(\cot \left(d x +c \right)-\csc \left(d x +c \right)\right) \left(\sqrt{\cos}\left(d x +c \right)\right)}{d b \sqrt{\cos \left(d x +c \right) b}}+\frac{C \sin \left(d x +c \right) \left(\sqrt{\cos}\left(d x +c \right)\right)}{b d \sqrt{\cos \left(d x +c \right) b}}$
risch	$-\frac{i \left(\sqrt{\cos}\left(d x +c \right)\right) C \,{\mathrm e}^{i \left(d x +c \right)}}{2 b \sqrt{\cos \left(d x +c \right) b}\, d}+\frac{i \left(\sqrt{\cos}\left(d x +c \right)\right) C \,{\mathrm e}^{-i \left(d x +c \right)}}{2 b \sqrt{\cos \left(d x +c \right) b}\, d}+\frac{\left(\sqrt{\cos}\left(d x +c \right)\right) A \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{b \sqrt{\cos \left(d x +c \right) b}\, d}-\frac{\left(\sqrt{\cos}\left(d x +c \right)\right) A \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{b \sqrt{\cos \left(d x +c \right) b}\, d}$

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/b/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-sin(d*x+c)*C)*cos(d*x+c)^(1/2)/(
cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.80

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{A\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos\left(dx+c\right)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2\,b\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{3}}\right)+2\,\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,b^{2}d\cos\left(dx+c\right)}, -\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin\left(dx+c\right)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b^{2}d\cos\left(dx+c\right)}\right]$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d
*x + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\mathrm{si}\right.\right.}{b^{\frac{3}{2}}}}{2\,d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 2*C*sin(d*x +
 c)/b^(3/2))/d
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

3.128 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 65

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/2)
/b/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {18, 3091, 8}

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b*d*S
qrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
```

```
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{\cos(c+dx)}(Cdx\cos(c+dx)+A\sin(c+dx))}{d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)
),x]
[Out] (Sqrt[Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*
x])^(3/2))
```

Maple [A] (verified)

Time = 8.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{C\cos(dx+c)(dx+c)+A\sin(dx+c)}{bd\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	48
risch	$\frac{Cx(\sqrt{\cos}(dx+c))}{b\sqrt{\cos(dx+c)b}}+\frac{i e^{-i(dx+c)}A}{b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,d}$	63
parts	$\frac{A\sin(dx+c)}{bd\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}+\frac{C(\sqrt{\cos}(dx+c))(dx+c)}{d\sqrt{\cos(dx+c)b}\,b}$	65

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/b/d*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.94

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{C\sqrt{-b}\cos(dx+c)^2\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\sqrt{\right.}}{}\right.$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c)
)*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2), (C*sqrt(b)*arc
tan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
 + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*co
s(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/((b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x)
)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(\frac{A\sqrt{b}\sin(2\,dx+2\,c)}{b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2}+\frac{C\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}\right)}{d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c
)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + C*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/b^(3/2))/d
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*sqrt(cos(d*x + c))
), x)
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\left(2\,c+2\,d\,x\right)\right)}{b^2\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x)
+ C*d*x + C*d*x*cos(2*c + 2*d*x)))/(b^2*d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d
*x) + 1))
```

3.129 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 84

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*arct
anh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {18, 3091, 3855}

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(A+2C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c +
d*x]]) + (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((A+2C)\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2b\sqrt{b\cos(c+dx)}}\\ &= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x])/(2*d*Sqrt
[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 8.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
default	$\frac{-A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-4C(\cos^2(dx+c))\operatorname{arctanh}(\cot(dx}{2bd\sqrt{\cos(dx+c)b}\cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c))}{2db\sqrt{\cos(dx+c)b}\cos(dx+c)^{\frac{3}{2}}} - \frac{2C\operatorname{arctanh}(\cot(}{}$
risch	$-\frac{iA(e^{2i(dx+c)}-1)}{2b\sqrt{\cos(dx+c)b}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)d} - \frac{(\sqrt{\cos}(dx+c))(A+2C)\ln(e^{i(dx+c)}-i)}{2b\sqrt{\cos(dx+c)b}\,d} + \frac{(\sqrt{\cos}(dx+c))(A+2C)\ln(e^{i(dx+c)}+}{2b\sqrt{\cos(dx+c)b}\,d}$

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/b/d*(-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+A*cos(d*x+c)^2*ln(-co
t(d*x+c)+csc(d*x+c)+1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+A*si
n(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.61

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{(A+2C)\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c}}{\cos(dx+c)^3}\right.}{4b^2d\cos}\right.$$

$$-\frac{(A+2C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\sqrt{b\cos(dx+c)}A\sqrt{\cos(dx+c)}\sin(dx+}{2b^2d\cos(dx+c)^3}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b
^2*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*
sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d
*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(72) = 144$.

Time = 0.43 (sec) , antiderivative size = 736, normalized size of antiderivative = 8.76

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
[Out] -1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c)
+ 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*A/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2
*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*
sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*sqrt(b)) - 2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2
*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) +
 1))/b^(3/2))/d
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(3/2)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^{3/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.130 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 85

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*A*sin(d*x+c)/b/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*si
n(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3091, 3852, 8}

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (A*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A +
3*C)*Sin[c + d*x])/(3*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{3b\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}-\frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\operatorname{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{3bd\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{\cos(c+dx)}\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*(b*Co
s[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 8.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{(2A(\cos^2(dx+c))+3C(\cos^2(dx+c))+A)\sin(dx+c)}{3bd\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}$	57
parts	$\frac{A(2(\cos^2(dx+c))+1)\sin(dx+c)}{3db\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}+\frac{C\sin(dx+c)}{db\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	79
risch	$\frac{i(3C\,e^{3i(dx+c)}+(8A+9C)\cos(dx+c)+i(4A+3C)\sin(dx+c))}{3b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^2 d}$	84

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/b/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*sin(d*x+c)/(cos(d*x+c)*b)^(1/
2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.59

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,b^2d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2
*d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(73) = 146.

Time = 0.42 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.47

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(\frac{3\,C\sqrt{b}\sin(2\,dx+2\,c)}{b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2}+\frac{\ldots}{\left(b\cos(6\,dx+6\,c)^2+9\,b\cos\ldots\right.}\right.}{\ldots}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + 2*((3*cos(2*d*x + 2*c) + 1)*sin(6
*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6
*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6*d*x
 + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*d*x +
 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c) +
b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*b*c
os(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + b)*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2)
), x)
```

Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.59

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,A\ldots\right.}{\ldots}$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*c
 + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*c
+ 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(4*c
 + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c + 4
*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*b^2*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*
d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.131 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 131

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*A*sin(d*x+c)/b/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*si
n(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*arctanh(si
n(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3091, 3853, 3855}

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(3A+4C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{8bd\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b*d*Sqrt[b*Cos[c
+ d*x]]) + (A*Sin[c + d*x])/(4*b*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

```
 + ((3*A + 4*C)*Sin[c + d*x])/(8*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]
])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{4b\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{8b\sqrt{b\cos(c+dx)}}\\
&= \frac{(3A+4C)\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\cos$$
$$8d\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[
c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 7.46 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.40

method	result
default	$\frac{3A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-3A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+4C(\cos^4(dx+c))\ln(-\cot(dx+c}{8bd\sqrt{\cos(c}}$
parts	$\frac{A(-3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+3(\cos^2(dx+c))\sin(dx+c)+2\sin}{8db\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{i(3A\,e^{6i(dx+c)}+4C\,e^{6i(dx+c)}+11A\,e^{4i(dx+c)}+4C\,e^{4i(dx+c)}-11A\,e^{2i(dx+c)}-4C\,e^{2i(dx+c)}-3A-4C)}{8b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^3 d}-\frac{(\sqrt{\cos}(dx+c))(3A+4C)}{8b\sqrt{\cos(dx+c}}$

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/b/d*(3*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*A*cos(d*x+c)^4*ln(
-cot(d*x+c)+csc(d*x+c)-1)+4*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-4*C
*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*A*sin(d*x+c)*cos(d*x+c)^2+4*C*
cos(d*x+c)^2*sin(d*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.99

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{(3A+4C)\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx-}}{\cos(dx+c)^3}\right.}{}\right.$$

$$-\frac{(3A+4C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left((3A+4C)\cos(dx+c)^2+2A\right)\sqrt{b\cos}}{8b^2d\cos(dx+c)^5}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^5), -1/8*((3*A + 4*C)
*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x
 + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2350 vs. $2(113) = 226$.

Time = 0.47 (sec) , antiderivative size = 2350, normalized size of antiderivative = 17.94

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44
*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x
+ 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*
x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)
^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*
cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(
4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(
```

3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) + 4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x +

```
2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d
*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*C/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*
d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)
^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) +
 b)*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2 + A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(7/2)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + A}{\cos\left(c+d\,x\right)^{7/2}\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.132 $\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/4*C*
cos(d*x+c)^(7/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(
d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3093, 2715, 8}

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(9/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b^2*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3
*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Co
s[c + d*x]^(7/2)*Sin[c + d*x])/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}+\frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.57

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+}{32b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(9/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] +
 C*Sin[4*(c + d*x)]))/(32*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.73 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

method	result
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8b^2d\sqrt{\cos(dx+c)b}}$
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)x(8A+6C)}{16b^2\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(4dx+4c)}{32b^2\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)(A+C)\sin(2dx+2c)}{4b^2\sqrt{\cos(dx+c)b}\,d}$
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)\left(\cos(dx+c)\sin(dx+c)+dx+c\right)}{2d\,b^2\sqrt{\cos(dx+c)b}}+\frac{C\left(\sqrt{\cos}(dx+c)\right)\left(2\sin(dx+c)\left(\cos^3(dx+c)\right)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)}{8d\,b^2\sqrt{\cos(dx+c)b}}$

```
[In] int(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/b^2/d*cos(d*x+c)^(1/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*cos(
d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/(cos(d*x+c)*b)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.70

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{2\left(2C\cos(dx+c)^2+4A+3C\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c}}{}\right.$$

```
[In] integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/8*((2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^3*d)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(9/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{5}{2}}} + \frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c\ldots}{b^{\frac{5}{2}}}}{32\,d}$$

[In] integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/b^(5/2))/d

Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2), x)

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(c+d\,x\right)+8\,C\,\sin\left(c+\right.\right.}{}$$

```
[In] int((cos(c + d*x)^(9/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) + 3
2*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*b^3*d*(cos(2*c + 2*d*x)
+ 1))
```

3.133 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 80

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{C\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (A+C)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-1/3*C*sin(d*x+
c)^3*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {17, 3092}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} - \frac{C\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) - (C
*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int\cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= -\frac{\sqrt{\cos(c+dx)}\text{Subst}\left(\int\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$
$$= \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{C\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int\frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*b^2*d
*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.64 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{\left(C\left(\cos^{2}(dx+c)\right)+3A+2C\right)\left(\sqrt{\cos}(dx+c)\right)\sin(dx+c)}{3b^{2}d\sqrt{\cos(dx+c)b}}$	50
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)(4A+3C)\sin(dx+c)}{4b^{2}\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(3dx+3c)}{12b^{2}\sqrt{\cos(dx+c)b}\,d}$	77
parts	$\frac{A\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{b^{2}d\sqrt{\cos(dx+c)b}}+\frac{C\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{3d\,b^{2}\sqrt{\cos(dx+c)b}}$	77

```
[In] int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/b^2/d*(C*cos(d*x+c)^2+3*A+2*C)*cos(d*x+c)^(1/2)*sin(d*x+c)/(cos(d*x+c)*
b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.61

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^{2}(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\left(C\cos\left(dx+c\right)^{2}+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,b^{3}d\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3*d
*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^{2}(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(7/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{5}{2}}}+\frac{12\,A\sin(dx+c)}{b^{\frac{5}{2}}}}{12\,d}$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))))/b^(5/2) + 12*A*sin(d*x + c)/b^(5/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\sin\right.}{12\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int((cos(c + d*x)^(7/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*si
n(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.134 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 99

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)
^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))
^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 2715, 8}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x
]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b
^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int\left(A+C\cos^2(c+dx)\right)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$
\int\frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4b^2d\sqrt{b\cos(c+dx)}}
$$

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*b^2*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.49 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)\left(C\cos\left(dx+c\right)\sin\left(dx+c\right)+2A\left(dx+c\right)+C\left(dx+c\right)\right)}{2b^{2}d\sqrt{\cos\left(dx+c\right)b}}$	57
risch	$\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)x\left(4A+2C\right)}{4b^{2}\sqrt{\cos\left(dx+c\right)b}}+\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)C\sin\left(2dx+2c\right)}{4b^{2}\sqrt{\cos\left(dx+c\right)b}\,d}$	69
parts	$\frac{A\left(\sqrt{\cos}\left(dx+c\right)\right)\left(dx+c\right)}{d\,b^{2}\sqrt{\cos\left(dx+c\right)b}}+\frac{C\left(\sqrt{\cos}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)\sin\left(dx+c\right)+dx+c\right)}{2d\,b^{2}\sqrt{\cos\left(dx+c\right)b}}$	78

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/b^2/d*cos(d*x+c)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/
(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.71

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(2A+C\right)\sqrt{-b}\right.}{}\right.$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)*
sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(
d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(
d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{5}{2}}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}}{4\,d}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(5/2) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/b^(5/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)+C\,\sin\left(3\,c+3\,d\right.\right.}{4\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)\right.}$$

```
[In] int((cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + C*sin(3*c + 3*
d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))/(4*b^3*d*(cos(2*c + 2*
d*x) + 1))
```

3.135 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x
+c)*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3093, 3855}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{A\sqrt{\cos(c+dx)}\operatorname{arctanh}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) +
 (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(A\text{arctanh}(\sin(c+dx))+C\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2
),x]
[Out] (Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(b^2*d*Sqrt
[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.92 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\left(2 A \,\operatorname{arctanh}\left(\cot \left(d x +c \right)-\csc \left(d x +c \right)\right)-\sin \left(d x +c \right) C \right) \left(\sqrt{\cos}\left(d x +c \right)\right)}{b^{2} d \sqrt{\cos \left(d x +c \right) b}}$
parts	$-\frac{2 A \left(\sqrt{\cos}\left(d x +c \right)\right) \operatorname{arctanh}\left(\cot \left(d x +c \right)-\csc \left(d x +c \right)\right)}{d \sqrt{\cos \left(d x +c \right) b}\, b^{2}}+\frac{C \sin \left(d x +c \right) \left(\sqrt{\cos}\left(d x +c \right)\right)}{b^{2} d \sqrt{\cos \left(d x +c \right) b}}$
risch	$-\frac{i \left(\sqrt{\cos}\left(d x +c \right)\right) C \,{\mathrm e}^{i \left(d x +c \right)}}{2 b^{2} \sqrt{\cos \left(d x +c \right) b}\, d}+\frac{i \left(\sqrt{\cos}\left(d x +c \right)\right) C \,{\mathrm e}^{-i \left(d x +c \right)}}{2 b^{2} \sqrt{\cos \left(d x +c \right) b}\, d}+\frac{\left(\sqrt{\cos}\left(d x +c \right)\right) A \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{b^{2} \sqrt{\cos \left(d x +c \right) b}\, d}-\frac{\left(\sqrt{\cos}\left(d x +c \right)\right) A \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right.}{b^{2} \sqrt{\cos \left(d x +c \right) b}\, d}$

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/b^2/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-sin(d*x+c)*C)*cos(d*x+c)^(1/2)
/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.80

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{A\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(d}{\cos(dx+c)^3}\right.}{2\,b^3d\cos}\right.$$
$$\left.-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b^3d\cos\left(dx+c\right)}\right]$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d
*x + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{b^{\frac{5}{2}}}+\frac{2\,C\sin(dx+c)}{b^{\frac{5}{2}}}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 2*C*sin(d*x +
 c)/b^(5/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(5/2),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.136 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 65

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/
2)/b^2/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 3091, 8}

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b^2
*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int\frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\cos^{\frac{3}{2}}(c+dx)(Cdx\cos(c+dx)+A\sin(c+dx))}{d(b\cos(c+dx))^{5/2}}$$

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*
x])^(5/2))
```

Maple [A] (verified)

Time = 7.56 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{C\cos(dx+c)(dx+c)+A\sin(dx+c)}{b^2 d\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	48
parts	$\frac{A\sin(dx+c)}{b^2 d\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}+\frac{C\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\,b^2\sqrt{\cos(dx+c)b}}$	65
risch	$\frac{Cx\left(\sqrt{\cos}(dx+c)\right)}{b^2\sqrt{\cos(dx+c)b}}+\frac{2i\left(\sqrt{\cos}(dx+c)\right)A}{b^2\sqrt{\cos(dx+c)b}\,d\left(e^{2i(dx+c)}+1\right)}$	67

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/b^2/d*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.94

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \left[-\frac{C\sqrt{-b}\cos\left(dx+c\right)^2\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\right.}{}\right.$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c)
)*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^2), (C*sqrt(b)*arc
tan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
 + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*co
s(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(\frac{A\sqrt{b}\sin(2\,dx+2\,c)}{b^3\cos(2\,dx+2\,c)^2+b^3\sin(2\,dx+2\,c)^2+2\,b^3\cos(2\,dx+2\,c)+b^3} + \frac{C\arctan\left(\frac{\sin(dx}{\cos(dx+}\right.}{b^{\frac{5}{2}}}\right.}{d}$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
[Out] 2*(A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c
)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + C*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/b^(5/2))/d
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(5/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(c+d\,x\right)+A\,\sin\left(3\,c+3\right.\right.}{b^3\,d\,\left(4\right.}$$

```
[In] int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(A*cos(c + d*x)*3i + A*sin(c +
 d*x) + A*cos(3*c + 3*d*x)*1i + A*sin(3*c + 3*d*x) + C*d*x*cos(3*c + 3*d*x)
 + 3*C*d*x*cos(c + d*x)))/(b^3*d*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3
))
```

3.137 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 84

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*ar
ctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {18, 3091, 3855}

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{(A+2C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c
+ d*x]]) + (A*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]
])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left((A+2C)\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$
\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos(c+dx)}((A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+A\sin(c}{2d(b\cos(c+dx))^{5/2}}
$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)
),x]

[Out] (Sqrt[Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin
[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 7.61 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
default	$\frac{-A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-4C(\cos^2(dx+c))\operatorname{arctanh}(\cot(d}{2b^2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c))}{2d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}} - \frac{2C(\sqrt{\cos}(dx+c)}{}$
risch	$-\frac{iA(e^{2i(dx+c)}-1)}{2b^2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)d} - \frac{(\sqrt{\cos}(dx+c))(A+2C)\ln(e^{i(dx+c)}-i)}{2b^2\sqrt{\cos(dx+c)b}\,d} + \frac{(\sqrt{\cos}(dx+c))(A+2C)\ln(e^{i(dx+c)}-}{2b^2\sqrt{\cos(dx+c)b}\,d}$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/b^2/d*(-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+A*cos(d*x+c)^2*ln(-
cot(d*x+c)+csc(d*x+c)+1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+A*
sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.61

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{(A+2\,C)\sqrt{b}\cos{(dx+c)}^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+}}{\cos(dx+c)^3}\right.}{4\,b^3d\,c}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos{(dx+c)}^3-\sqrt{b\cos{(dx+c)}}A\sqrt{\cos{(dx+c)}}\sin{(dx+}}{2\,b^3d\cos{(dx+c)}^3}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b
^3*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*
sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d
*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. 2(72) = 144.

Time = 0.42 (sec) , antiderivative size = 754, normalized size of antiderivative = 8.98

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
[Out] -1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c)
+ 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*A/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x
 + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(
2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)) - 2*C*(log(cos(d*x + c)^2 +
sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2
- 2*sin(d*x + c) + 1))/b^(5/2))/d
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{5}{2}}\sqrt{\cos\left(dx + c\right)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(5/2)*sqrt(cos(d*x + c))
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\sqrt{\cos\left(c + d\,x\right)}\,\left(b\,\cos\left(c + d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)), x)
```

3.138 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 85

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*
sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {18, 3091, 3852, 8}

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (A*Sin[c + d*x])/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A
+ 3*C)*Sin[c + d*x])/(3*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)} \int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{3b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad - \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\operatorname{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d(b\cos(c+dx))^{5/2}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*(b*Co
s[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 6.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{(2A(\cos^2(dx+c))+3C(\cos^2(dx+c))+A)\sin(dx+c)}{3b^2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}$	57
parts	$\frac{A(2(\cos^2(dx+c))+1)\sin(dx+c)}{3d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}+\frac{C\sin(dx+c)}{d\,b^2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	79
risch	$\frac{i(3C\,\mathrm{e}^{3i(dx+c)}+(8A+9C)\cos(dx+c)+i(4A+3C)\sin(dx+c))}{3b^2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(\mathrm{e}^{2i(dx+c)}+1)^2d}$	84

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/b^2/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*sin(d*x+c)/(cos(d*x+c)*b)^(
1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.59

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,b^3d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3
*d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(73) = 146$.

Time = 0.43 (sec) , antiderivative size = 412, normalized size of antiderivative = 4.85

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(\frac{3\,C\sqrt{b}\sin(2\,dx+2\,c)}{b^3\cos(2\,dx+2\,c)^2+b^3\sin(2\,dx+2\,c)^2+2\,b^3\cos(2\,dx+2\,c)+b^3}+\frac{\cdots}{\left(b^2\cos(6\,dx+6\,c)^2+9\,b^2\cdots\right.}\cdots\right.}{\cdots}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
[Out] 2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + 2*((3*cos(2*d*x + 2*c) + 1)*sin(6
*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6
*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos(6*d
*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin
(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3
*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*
(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) +
 b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2 + A}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2)
), x)
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.59

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\,\cos(c+d\,x)}\,(18\,A\,\sin(2\,c+2\,d\,x)+12\,A\,\sin(4\,c+4\,d\,x)+2\,A\,\mathrm{s}}{}$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*c
 + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*c
+ 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(4*c
 + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c + 4
*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*b^3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*
d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.139 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 131

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*
sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*arctan
h(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {18, 3091, 3853, 3855}

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{(3A+4C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b^2*d*Sqrt[b*Cos[
c + d*x]]) + (A*Sin[c + d*x])/(4*b^2*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*
```

```
x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c
+ d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad + \frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{8b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}}\\
&\quad + \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\cos}{8d\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)
),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[
c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 7.93 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.40

method	result
default	$-\frac{3A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)-3A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+4C(\cos^4(dx+c))\ln(-\cot(dx}{8b^2d\sqrt{\mathrm{co}}}$
parts	$\frac{A(-3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+3(\cos^2(dx+c))\sin(dx+c)+2\sin}{8d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{i(3A\,e^{6i(dx+c)}+4C\,e^{6i(dx+c)}+11A\,e^{4i(dx+c)}+4C\,e^{4i(dx+c)}-11A\,e^{2i(dx+c)}-4C\,e^{2i(dx+c)}-3A-4C)}{8b^2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^3d}+\frac{(\sqrt{\cos}(dx+c))(3A+4C)}{8b^2\sqrt{\cos(dx+}}$

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/b^2/d*(3*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-3*A*cos(d*x+c)^4*
ln(-cot(d*x+c)+csc(d*x+c)+1)+4*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-
4*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*A*sin(d*x+c)*cos(d*x+c)^2-4
*C*cos(d*x+c)^2*sin(d*x+c)-2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^
(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.99

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{(3A+4C)\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx-}}{\cos(dx+c)^3}\right.}{}\right.$$

$$-\frac{(3A+4C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-((3A+4C)\cos(dx+c)^2+2A)\sqrt{b\cos}}{8b^3d\cos(dx+c)^5}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^5), -1/8*((3*A + 4*C)
*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x
 + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2418 vs. $2(113) = 226$.

Time = 0.46 (sec) , antiderivative size = 2418, normalized size of antiderivative = 18.46

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44
*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x
+ 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*
x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)
^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*
cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(
4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(
```

3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)) + 4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +

```
2*c)) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x +
 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/((b^2*co
s(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^
2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(
2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqr
t(b)))/d
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{5}{2}}\cos\left(dx + c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(5/2)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)^{5/2}\left(b\,\cos\left(c + d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2)), x)
```

3.140 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^3d} - \frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{130b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/13*C*(b*cos(d*x+c))^(10/3)*sin(d*x+c)/b^3/d-3/130*(13*A+10*C)*(b*cos(d*x+
c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d
*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3093, 2722}

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13b^3d} - \frac{3(13A+10C)\sin(c+dx)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{130b^3d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^3*d) - (3*(13*A + 10*C)*(b
*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(130*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{7/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$
$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^3d} + \frac{(13A+10C)\int (b\cos(c+dx))^{7/3}\,dx}{13b^2}$$
$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^3d} - \frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{130b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(8A\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)+5C\cos^4(c+\right.}{80d}$$

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(8*A*Cos[c + d*x]^2*Hypergeometric2
F1[1/2, 5/3, 8/3, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^4*Hypergeometric2F1[1/
2, 8/3, 11/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(80*d)
```

Maple [F]

$$\int \left(\cos^2 (dx + c)\right) (\cos (dx + c) b)^{\frac{1}{3}} \left(A + C\left(\cos^2 (dx + c)\right)\right) dx$$

[In] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2),x)

[Out] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2),x)

Fricas [F]

$$\int \cos^2(c + dx) \sqrt[3]{b\cos(c + dx)} \left(A + C\cos^2(c + dx)\right) \, dx = \int \left(C\cos\left(dx + c\right)^2 + A\right)\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}\cos\left(dx + c\right)^2 \, dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm ="fricas")

[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c + dx) \sqrt[3]{b\cos(c + dx)} \left(A + C\cos^2(c + dx)\right) \, dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Maxima [F]

$$\int \cos^2(c + dx) \sqrt[3]{b\cos(c + dx)} \left(A + C\cos^2(c + dx)\right) \, dx = \int \left(C\cos\left(dx + c\right)^2 + A\right)\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}\cos\left(dx + c\right)^2 \, dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)

Giac [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\big(A+C\cos^2(c+dx)\big)\,dx$$
$$=\int \big(C\cos{(dx+c)}^2+A\big)(b\cos{(dx+c)})^{\frac{1}{3}}\cos{(dx+c)}^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\big(A+C\cos^2(c+dx)\big)\,dx$$
$$=\int \cos{(c+d\,x)}^2\,\left(C\cos{(c+d\,x)}^2+A\right)\,\left(b\,\cos{(c+d\,x)}\right)^{1/3}dx$$

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

3.141 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx = \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^2d} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b^2/d-3/70*(10*A+7*C)*(b*cos(d*x+c))
^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3093, 2722}

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10b^2d} - \frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{70b^2d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b^2*d) - (3*(10*A + 7*C)*(b*C
os[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(70*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b}\\
&= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^2d}+\frac{(10A+7C)\int (b\cos(c+dx))^{4/3}\,dx}{10b}\\
&= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^2d}\\
&\quad -\frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70b^2d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\left(13A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)+7C\cos^2(c+dx)\,\text{Hype}\right.}{91bd}$$

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(13*A*Hypergeometric2F1[1/2, 7/6, 1
3/6, Cos[c + d*x]^2] + 7*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 13/6, 19/6
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*b*d)
```

Maple [F]

$$\int \cos \left(d x+c\right) \left(\cos \left(d x+c\right) b\right)^{\frac{1}{3}} \left(A+C\left(\cos^{2}\left(d x+c\right)\right)\right)d x$$

```
[In] int(cos(d*x+c)*(cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

3.142 $\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(4/
3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3093, 2722}

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd} - \frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{28bd\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*(7*A + 4*C)*(b*Cos[c
 + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x
])/(28*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd}+\frac{1}{7}(7A+4C)\int\sqrt[3]{b\cos(c+dx)}\,dx\\ &= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd}\\ &\quad-\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=$$
$$-\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(5A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hyperge}\right.}{20d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3, 5/
3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3, Co
s[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*d)
```

Maple [F]

$$\int (\cos(dx+c)\,b)^{\frac{1}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int((cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2),x)
[Out] int((cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

3.143 $\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 87

$$\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec(c+dx)\,dx = \frac{3C\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{3(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d-3/4*(4*A+C)*(b*cos(d*x+c))^(1/3)*hy
pergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec(c+dx)\,dx = \frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4d} - \frac{3(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*(4*A + C)*(b*Cos[c + d
*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx\\
&= \frac{3C\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d}+\frac{1}{4}(b(4A+C))\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx\\
&= \frac{3C\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d}\\
&\quad -\frac{3(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$
$$-\frac{3b\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{7d(b\cos(c+dx))^{2/3}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (-3*b*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] +
C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[Si
n[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{1}{3}} \left(A + C\left(\cos^2 (dx+c)\right)\right) \sec (dx+c)\, dx$$

```
[In] int((cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] int((cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A + C\cos^2(c+dx)\right)\sec(c+dx)\, dx = \int \left(C\cos{(dx+c)}^2 + A\right)\left(b\cos{(dx+c)}\right)^{\frac{1}{3}} \sec{(dx+c)}\ dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A + C\cos^2(c+dx)\right)\sec(c+dx)\, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x), x)
```

3.144 $\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 91

$$\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx = \frac{3Ab\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)+3/8*(A-2*C)*(b*cos(d*x+c))^(4/3)*
hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3091, 2722}

$$\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx = \frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8bd\sqrt{\sin^2(c+dx)}} + \frac{3Ab\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}$$

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c +
 d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(8*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = b^2 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= \frac{3Ab\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{1}{2}(-A+2C)\int \sqrt[3]{b\cos(c+dx)}\,dx$$
$$= \frac{3Ab\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = -\frac{3b\csc(c+dx)\left(-2A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\right.}{4d(b\cos(c+dx))^{2/3}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]
+ C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[S
in[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{\frac{1}{3}} \left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right) \left(\sec^{2}\left(dx+c\right)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^{2}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^2,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^2, x)
```

3.145 $\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 92

$$\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} - \frac{3(2A+5C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)-3/5*(2*A+5*C)*(b*cos(d*x+c))^(1
/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} - \frac{3(2A+5C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) - (3*(2*A + 5*C)*(b*Cos
[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d
*x])/(5*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^3 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{8/3}}\,dx \\
&= \frac{3Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} + \frac{1}{5}(b(2A+5C))\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx \\
&= \frac{3Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} \\
&\quad - \frac{3(2A+5C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{3\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hype}\right.}{5d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-5/6, 1/2, 1
/6, Cos[c + d*x]^2]) + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/6, 1/2, 7/6,
Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(5*d)
```

Maple [F]

$$\int \left(\cos\left(dx+c\right)b\right)^{\frac{1}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^3\left(dx+c\right)\right)dx$$

```
[In] int((cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^3,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^3, x)
```

3.146 $\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right) dx=\frac{3C(b\cos(c+dx))^{11/3}\sin(c+dx)}{14b^3d}-\frac{3(14A+11C)(b\cos(c+dx))^{11/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{154b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/14*C*(b*cos(d*x+c))^(11/3)*sin(d*x+c)/b^3/d-3/154*(14*A+11*C)*(b*cos(d*x+
c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin
(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3093, 2722}

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right) dx=\frac{3C\sin(c+dx)(b\cos(c+dx))^{11/3}}{14b^3d}-\frac{3(14A+11C)\sin(c+dx)(b\cos(c+dx))^{11/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)}{154b^3d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(11/3)*Sin[c + d*x])/(14*b^3*d) - (3*(14*A + 11*C)*(b
*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Si
n[c + d*x])/(154*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{8/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$

$$= \frac{3C(b\cos(c+dx))^{11/3}\sin(c+dx)}{14b^3d} + \frac{(14A+11C)\int (b\cos(c+dx))^{8/3}\,dx}{14b^2}$$

$$= \frac{3C(b\cos(c+dx))^{11/3}\sin(c+dx)}{14b^3d} - \frac{3(14A+11C)(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{154b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\left(17A\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)+11C\cos^4\right.}{187d}$$

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(17*A*Cos[c + d*x]^2*Hypergeometric
2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2] + 11*C*Cos[c + d*x]^4*Hypergeometric2F
1[1/2, 17/6, 23/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(187*d)
```

Maple [F]

$$\int \left(\cos^{2}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{\frac{2}{3}}\left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right)dx$$

[In] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2),x)

[Out] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2),x)

Fricas [F]

$$\int \cos^{2}(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^{2}(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^{2}\,dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm ="fricas")

[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(2/3), x)

Sympy [F(-1)]

Timed out.

$$\int \cos^{2}(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^{2}(c+dx)\right)\,dx=\text{Timed out}$$

[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Maxima [F]

$$\int \cos^{2}(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^{2}(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^{2}\,dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)

Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
[Out] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

3.147 $\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^2/d-3/88*(11*A+8*C)*(b*cos(d*x+c))
^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3093, 2722}

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11b^2d}-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{88b^2d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^2*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c +
 d*x])/(88*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{5/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b}\\
&= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}+\frac{(11A+8C)\int (b\cos(c+dx))^{5/3}\,dx}{11b}\\
&= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}\\
&\quad -\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^2d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3(b\cos(c+dx))^{5/3}\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)+4C\cos^2(c+dx)\,\text{Hyperge}\right.}{56bd}$$

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 4/3, 7/
3, Cos[c + d*x]^2] + 4*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/3, 10/3, C
os[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(56*b*d)
```

Maple [F]

$$\int \cos \left(dx+c\right) \left(\cos \left(dx+c\right) b\right)^{\frac{2}{3}} \left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right)dx$$

```
[In] int(cos(d*x+c)*(cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

3.148 $\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(5/
3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3093, 2722}

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{40bd\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos[c
 + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(40*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}+\frac{1}{8}(8A+5C)\int (b\cos(c+dx))^{2/3}\,dx\\ &= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}\\ &\quad -\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\left(11A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hyper}\right.}{55d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6, 1
1/6, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17/6
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*d)
```

Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{\frac{2}{3}} \left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2),x)
[Out] int((cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}} \, dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}} \, dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

3.149 $\int (b\cos(c+dx))^{2/3} (A + C\cos^2(c+dx)) \sec(c+dx)\, dx$

Optimal result

Integrand size = 31, antiderivative size = 89

$$\int (b\cos(c+dx))^{2/3} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\, dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/3)
*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\int (b\cos(c+dx))^{2/3} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\, dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5d} - \frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) - (3*(5*A + 2*C)*(b*Cos[c +
 d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(10*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx\\ &= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d}+\frac{1}{5}(b(5A+2C))\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx\\ &= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d}\\ &\quad -\frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$

$$-\frac{3b\cot(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\right.\right.}{8d\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (-3*b*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] +
C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int (\cos (dx+c) b)^{\frac{2}{3}} \left(A+C\left(\cos^2 (dx+c)\right)\right) \sec (dx+c) \, dx$$

```
[In] int((cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] int((cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec(c+dx) \, dx = \int \left(C\cos (dx+c)^2+A\right)(b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c) \, dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec(c+dx) \, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x), x)
```

3.150 $\int (b\cos(c+dx))^{2/3} \left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 91

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)+3/5*(2*A-C)*(b*cos(d*x+c))^(5/3)*hy
pergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}} + \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d
*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/
(5*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx \\
&= \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}+(-2A+C)\int (b\cos(c+dx))^{2/3}\,dx \\
&= \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} \\
&\quad +\frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$
\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =
$$

$$
-\frac{3b\csc(c+dx)\left(-5A\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\right.\right.}{5d\sqrt[3]{b\cos(c+dx)}}
$$

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b*Csc[c + d*x]*(-5*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]
+ C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[
Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{2}{3}} \left(A+C\left(\cos^2 (dx+c)\right)\right) \left(\sec^2 (dx+c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
[Out] int((cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec^2(c+dx)\, dx = \int \left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)})^{\frac{2}{3}} \sec{(dx+c)}^2\, dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec^2(c+dx)\, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec^2(c+dx)\, dx = \int \left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)})^{\frac{2}{3}} \sec{(dx+c)}^2\, dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^2,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^2, x)
```

3.151 $\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 90

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)-3/8*(A+4*C)*(b*cos(d*x+c))^(2/3
)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c
 + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x
])/(8*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx\\ &= \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{1}{4}(b(A+4C))\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx\\ &= \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}\\ &\quad -\frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$

$$-\frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hyper}\right.}{4d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1
/3, Cos[c + d*x]^2]) + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/3, 1/2, 4/3,
Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F]

$$\int (\cos (dx+c) b)^{\frac{2}{3}} \left(A+C\left(\cos ^2 (dx+c)\right)\right) \left(\sec ^3 (dx+c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
[Out] int((cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\, dx = \int (C\cos (dx+c)^2+A)(b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c)^3 \,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\, dx = \int (C\cos (dx+c)^2+A)(b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c)^3 \,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3\,dx$$

[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm ="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^3,x)

[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^3, x)

3.152 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right) dx = \frac{3C(b\cos(c+dx))^{13/3}\sin(c+dx)}{16b^3d} - \frac{3(16A+13C)(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{208b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/16*C*(b*cos(d*x+c))^(13/3)*sin(d*x+c)/b^3/d-3/208*(16*A+13*C)*(b*cos(d*x+
c))^(13/3)*hypergeom([1/2, 13/6],[19/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin
(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3093, 2722}

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right) dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{13/3}}{16b^3d} - \frac{3(16A+13C)\sin(c+dx)(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)}{208b^3d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(13/3)*Sin[c + d*x])/(16*b^3*d) - (3*(16*A + 13*C)*(b
*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Si
n[c + d*x])/(208*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (b\cos(c+dx))^{10/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}\\ &= \frac{3C(b\cos(c+dx))^{13/3}\sin(c+dx)}{16b^3d} + \frac{(16A+13C)\int (b\cos(c+dx))^{10/3}\,dx}{16b^2}\\ &= \frac{3C(b\cos(c+dx))^{13/3}\sin(c+dx)}{16b^3d}\\ &\quad - \frac{3(16A+13C)(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{208b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{3\cos^2(c+dx)(b\cos(c+dx))^{4/3}\cot(c+dx)\left(19A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)+13C\cos^2\right.}{247d}$$

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(19*A*Hypergeometric
2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2] + 13*C*Cos[c + d*x]^2*Hypergeometric2F
1[1/2, 19/6, 25/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(247*d)
```

Maple [F]

$$\int \left(\cos^2\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{\frac{4}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)dx$$

[In] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2),x)

[Out] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2),x)

Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^2\,dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm ="fricas")

[Out] integral((C*b*cos(d*x + c)^5 + A*b*cos(d*x + c)^3)*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^2\,dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)

Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,dx$$

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
[Out] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

3.153 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d}-\frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{130b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/13*C*(b*cos(d*x+c))^(10/3)*sin(d*x+c)/b^2/d-3/130*(13*A+10*C)*(b*cos(d*x+
c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d
*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3093, 2722}

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13b^2d}-\frac{3(13A+10C)\sin(c+dx)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{130b^2d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^2*d) - (3*(13*A + 10*C)*(b
*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(130*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{7/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b}\\
&= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d} + \frac{(13A+10C)\int (b\cos(c+dx))^{7/3}\,dx}{13b}\\
&= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d}\\
&\quad - \frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{130b^2d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3(b\cos(c+dx))^{7/3}\cot(c+dx)\left(8A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hyperge}\right.}{80bd}$$

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*(8*A*Hypergeometric2F1[1/2, 5/3, 8/
3, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 8/3, 11/3, C
os[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(80*b*d)
```

Maple [F]

$$\int \cos(dx+c)\left(\cos(dx+c)\,b\right)^{\frac{4}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)dx$$

[In] int(cos(d*x+c)*(cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2),x)

[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2),x)

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^4 + A*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A + C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,dx$$

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

3.154 $\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) \, dx = \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b/d-3/70*(10*A+7*C)*(b*cos(d*x+c))^(
7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3093, 2722}

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) \, dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd} - \frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{70bd\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*Cos
[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(70*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd} + \frac{1}{10}(10A+7C)\int (b\cos(c+dx))^{4/3}\,dx$$

$$= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\left(13A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)+7C\cos^2(c+dx)\,\text{Hyper}\right.}{91d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(13*A*Hypergeometric2F1[1/2, 7/6, 1
3/6, Cos[c + d*x]^2] + 7*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 13/6, 19/6
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*d)
```

Maple [F]

$$\int (\cos (dx + c) b)^{\frac{4}{3}} \left(A + C\left(\cos^2 (dx + c)\right)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2),x)
[Out] int((cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c + dx))^{4/3} \left(A + C\cos^2(c + dx)\right) \, dx = \int \left(C\cos (dx + c)^2 + A\right)(b\cos (dx + c))^{\frac{4}{3}} \, dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c + dx))^{4/3} \left(A + C\cos^2(c + dx)\right) \, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c + dx))^{4/3} \left(A + C\cos^2(c + dx)\right) \, dx = \int \left(C\cos (dx + c)^2 + A\right)(b\cos (dx + c))^{\frac{4}{3}} \, dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

3.155 $\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 89

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d}-\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(4/3)
*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3093, 2722}

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7d}-\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{28d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*d) - (3*(7*A + 4*C)*(b*Cos[c +
 d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(28*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx\\
&= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d}+\frac{1}{7}(b(7A+4C))\int \sqrt[3]{b\cos(c+dx)}\,dx\\
&= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d}\\
&\quad -\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$

$$-\frac{3b\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(5A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hyperge}\right.}{20d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3,
5/3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3,
Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*d)
```

Maple [F]

$$\int (\cos (dx+c) b)^{\frac{4}{3}} \left(A + C\left(\cos^2 (dx+c)\right)\right) \sec (dx+c)\, dx$$

```
[In] int((cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
[Out] int((cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\, dx = \int \left(C \cos (dx+c)^2 + A\right)(b\cos (dx+c))^{\frac{4}{3}} \sec (dx+c)\; dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\, dx = \int \left(C \cos (dx+c)^2 + A\right)(b\cos (dx+c))^{\frac{4}{3}} \sec (dx+c)\; dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x), x)
```

3.156 $\int (b\cos(c+dx))^{4/3} (A + C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 89

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{3bC\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{3b(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*b*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d-3/4*b*(4*A+C)*(b*cos(d*x+c))^(1/3
)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3093, 2722}

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{3bC\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4d} - \frac{3b(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*b*(4*A + C)*(b*Cos[c
 + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x
])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx \\
&= \frac{3bC\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{1}{4}\left(b^2(4A+C)\right)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx \\
&= \frac{3bC\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} \\
&\quad - \frac{3b(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$
\int (b\cos(c+dx))^{4/3}\left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =
$$

$$
-\frac{3b^2\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right) + C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6}\right.\right.}{7d(b\cos(c+dx))^{2/3}}
$$

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
+ C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[
Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{\frac{4}{3}} \left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right) \left(\sec^{2}\left(dx+c\right)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
[Out] int((cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^{2}(c+dx)\right)\sec^{2}(c+dx)\, dx = \int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^{2}\, dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="fricas")
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^{2}(c+dx)\right)\sec^{2}(c+dx)\, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^{2}(c+dx)\right)\sec^{2}(c+dx)\, dx = \int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^{2}\, dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^2,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^2, x)
```

3.157 $\int (b\cos(c+dx))^{4/3} (A + C\cos^2(c+dx)) \sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 90

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)+3/8*(A-2*C)*(b*cos(d*x+c))^(4/3
)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c
 + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x
])/(8*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^3 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx \\
&= \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} - \frac{1}{2}(b(A-2C)) \int \sqrt[3]{b\cos(c+dx)}\,dx \\
&= \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} \\
&\quad + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^{4/3}\left(A + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$

$$-\frac{3b^2\csc(c+dx)\left(-2A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right) + C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\right.}{4d(b\cos(c+dx))^{2/3}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (-3*b^2*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2
] + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int (\cos(dx+c)\,b)^{\frac{4}{3}} \left(A + C\left(\cos^2(dx+c)\right)\right) \left(\sec^3(dx+c)\right) dx$$

[In] int((cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

[Out] int((cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm ="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec (d*x + c)^3, x)

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)

[Out] Timed out

Maxima [F]

$$\int (b\cos(c+dx))^{4/3} \left(A + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^3,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^3, x)
```

3.158 $\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d} - \frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^3/d-3/88*(11*A+8*C)*(b*cos(d*x+c))
^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3093, 2722}

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11b^3d} - \frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{88b^3d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^3*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c +
 d*x])/(88*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (b\cos(c+dx))^{5/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}\\ &= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d}+\frac{(11A+8C)\int (b\cos(c+dx))^{5/3}\,dx}{11b^2}\\ &= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d}\\ &\quad-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cot(c+dx)\left(7A\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)+4C\cos^4(c+dx)\,\text{Hypergeor}\right.}{56d\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(7*A*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c
 + d*x]^2] + 4*C*Cos[c + d*x]^4*Hypergeometric2F1[1/2, 7/3, 10/3, Cos[c + d
*x]^2])*Sqrt[Sin[c + d*x]^2])/(56*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int \frac{\left(\cos^2\left(dx+c\right)\right)\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)}{\left(\cos\left(dx+c\right)b\right)^{\frac{1}{3}}}dx$$

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3)/b, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3), x)
```

3.159 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^2/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(
5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8b^2d} - \frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{40b^2d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^2*d) - (3*(8*A + 5*C)*(b*Cos
[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(40*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b}\\ &= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d}+\frac{(8A+5C)\int (b\cos(c+dx))^{2/3}\,dx}{8b}\\ &= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d}\\ &\quad-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\left(11A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hyp}\right.}{55bd}$$

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6, 1
1/6, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17/6
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*b*d)

Maple [F]

$$\int \frac{\cos\left(dx+c\right)\left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right)}{\left(\cos\left(dx+c\right)b\right)^{\frac{1}{3}}}dx$$

[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)

[Out] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)

Fricas [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^{2}(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="
fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/b, x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^{2}(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\text{Timed out}$$

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^{2}(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="
maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3),x)
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3), x)
```

3.160 $\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/
3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3093, 2722}

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd} - \frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*Cos[c
 + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x
])/(10*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{1}{5}(5A+2C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cot(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{8d\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + C*
Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int \frac{A + C(\cos^2(dx + c))}{(\cos(dx + c)\, b)^{\frac{1}{3}}} dx$$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
[Out] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{1}{3}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b*cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\, dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{1}{3}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(dx+c\right)^2 + A}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + A}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3), x)
```

3.161 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 90

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)+3/5*(2*A-C)*(b*cos(d*x+c))^(5/3)*hype
rgeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3091, 2722}

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}} + \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d*x
])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5
*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = b\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$$

$$= \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} - \frac{(2A-C)\int (b\cos(c+dx))^{2/3}\,dx}{b}$$

$$= \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \frac{(A + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx =$$

$$-\frac{3\left(-5A\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right) + C\cos(c+dx)\cot(c+dx)\,\text{Hypergeo}\right.}{5d\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(-5*A*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
C*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^
2])*Sqrt[Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int \frac{(A + C(\cos^2(dx+c)))\sec(dx+c)}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(1/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2 + A\right)\sec(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b*cos(
d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{(A + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)\,\left(b\,\cos\left(c + d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/3)), x)
```

3.162 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 91

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)-3/8*(A+4*C)*(b*cos(d*x+c))^(2/3)*
hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8bd\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c +
 d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(8*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx \\
&= \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{1}{4}(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx \\
&= \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} \\
&\quad - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx =$$
$$-\frac{3b\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hypergeometric2F1}}{4d(b\cos(c+dx))^{4/3}}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*b*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])
+ 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{(A+C(\cos^2(dx+c)))(\sec^2(dx+c))}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(1/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b*co
s(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{(A+C\cos^2{(c+dx)})\sec^2{(c+dx)}}{\sqrt[3]{b\cos{(c+dx)}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c + d*x))**(1/3), x
)
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2+A\right)\sec{(dx+c)}^2}{(b\cos{(dx+c)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2+A\right)\sec{(dx+c)}^2}{(b\cos{(dx+c)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{C\cos{(c+d\,x)}^2+A}{\cos{(c+d\,x)}^2\,(b\,\cos{(c+d\,x)})^{1/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)), x)
```

3.163 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 92

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/7*(4*A+7*C)*hypergeom([-1/6,
1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3(4A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*(4*A + 7*C)*Hyperg
eometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d
*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b^3\int\frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx\\ &= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{1}{7}(b(4A+7C))\int\frac{1}{(b\cos(c+dx))^{4/3}}\,dx\\ &= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int\frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\frac{3\left(7C\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)+A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c\right.\right.}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*(7*C*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x] + A*
Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sec[c + d*x]*Tan[c + d*x
]))/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Maple [F]

$$\int \frac{(A + C(\cos^2(dx + c)))(\sec^3(dx + c))}{(\cos(dx + c)\, b)^{\frac{1}{3}}}dx$$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(1/3),x)
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\, dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^3}{\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b*co
s(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\, dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\, dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^3}{\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^3}{\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)^3\left(b\,\cos\left(c + d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/3)), x)
```

3.164 $\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^3d} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b^3/d-3/70*(10*A+7*C)*(b*cos(d*x+c))
^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3093, 2722}

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10b^3d} - \frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{70b^3d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b^3*d) - (3*(10*A + 7*C)*(b*C
os[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(70*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$

$$= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^3d} + \frac{(10A+7C)\int (b\cos(c+dx))^{4/3}\,dx}{10b^2}$$

$$= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^3d} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx =$$

$$-\frac{3\cot(c+dx)\left(13A\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)+7C\cos^4(c+dx)\,\text{Hypergeon}\right.}{91d(b\cos(c+dx))^{2/3}}$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(13*A*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos
[c + d*x]^2] + 7*C*Cos[c + d*x]^4*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c
+ d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int \frac{(\cos^2(dx+c))\,(A+C(\cos^2(dx+c)))}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)
[Out] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="fricas")
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3),x)
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3), x)
```

3.165 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d}-\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/b^2/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(
4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3093, 2722}

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7b^2d}-\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{28b^2d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b^2*d) - (3*(7*A + 4*C)*(b*Cos
[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d
*x])/(28*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx}{b}\\ &= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d}+\frac{(7A+4C)\int \sqrt[3]{b\cos(c+dx)}\,dx}{7b}\\ &= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d}\\ &\quad -\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{\cos(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(5A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hypergeon}\right.}{20bd}$$

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3, 5/
3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3, Co
s[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*b*d)
```

Maple [F]

$$\int \frac{\cos \left(d x+c\right) \left(A+C \left(\cos^{2}\left(d x+c\right)\right)\right)}{\left(\cos \left(d x+c\right) b\right)^{\frac{2}{3}}}d x$$

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)
[Out] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx) \left(A+C \cos^2(c+dx)\right)}{(b \cos(c+dx))^{2/3}} \, dx = \int \frac{\left(C \cos \left(d x + c\right)^{2} + A\right) \cos \left(d x + c\right)}{\left(b \cos \left(d x + c\right)\right)^{\frac{2}{3}}} \,d x$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx) \left(A+C \cos^2(c+dx)\right)}{(b \cos(c+dx))^{2/3}} \, dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx) \left(A+C \cos^2(c+dx)\right)}{(b \cos(c+dx))^{2/3}} \, dx = \int \frac{\left(C \cos \left(d x + c\right)^{2} + A\right) \cos \left(d x + c\right)}{\left(b \cos \left(d x + c\right)\right)^{\frac{2}{3}}} \,d x$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3),x)
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3), x)
```

3.166 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4bd} - \frac{3(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d-3/4*(4*A+C)*(b*cos(d*x+c))^(1/3)*
hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3093, 2722}

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd} - \frac{3(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*(4*A + C)*(b*Cos[c +
 d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])
/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3C\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4bd} + \frac{1}{4}(4A+C)\int\frac{1}{(b\cos(c+dx))^{2/3}}\,dx\\ &= \frac{3C\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4bd}\\ &\quad -\frac{3(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int\frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx =$$

$$-\frac{3\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\right.\right.}{7d(b\cos(c+dx))^{2/3}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + C*
Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[Sin[
c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int \frac{A + C(\cos^2(dx + c))}{(\cos(dx + c)\, b)^{\frac{2}{3}}} dx$$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)
[Out] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{2/3}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{2}{3}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{2/3}}\, dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{2/3}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{2}{3}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(2/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(2/3), x)
```

3.167 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 90

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3A\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)+3/8*(A-2*C)*(b*cos(d*x+c))^(4/3)*hy
pergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3091, 2722}

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\right.}{8b^2d\sqrt{\sin^2(c+dx)}} + \frac{3A\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c + d
*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
8*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx\\
&= \frac{3A\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}-\frac{(A-2C)\int \sqrt[3]{b\cos(c+dx)}\,dx}{2b}\\
&= \frac{3A\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}\\
&\quad+\frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\left(-2A\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)+C\cos(c+dx)\cot(c+dx)\,\text{Hypergeome}\right.}{4d(b\cos(c+dx))^{2/3}}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(-2*A*Csc[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] +
C*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2
])*Sqrt[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int \frac{(A + C(\cos^2 (dx + c))) \sec (dx + c)}{(\cos (dx + c) b)^{\frac{2}{3}}} dx$$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(2/3),x)
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c + dx)) \sec(c + dx)}{(b\cos(c + dx))^{2/3}} dx = \int \frac{(C\cos (dx + c)^2 + A) \sec (dx + c)}{(b\cos (dx + c))^{\frac{2}{3}}} dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos(
d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c + dx)) \sec(c + dx)}{(b\cos(c + dx))^{2/3}} dx = \int \frac{(A + C\cos^2 (c + dx)) \sec (c + dx)}{(b\cos (c + dx))^{\frac{2}{3}}} dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(2/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx)) \sec(c + dx)}{(b\cos(c + dx))^{2/3}} dx = \int \frac{(C\cos (dx + c)^2 + A) \sec (dx + c)}{(b\cos (dx + c))^{\frac{2}{3}}} dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)\,\left(b\,\cos\left(c + d\,x\right)\right)^{2/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)), x)
```

3.168 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 93

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} - \frac{3(2A+5C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)-3/5*(2*A+5*C)*(b*cos(d*x+c))^(1/3
)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} - \frac{3(2A+5C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) - (3*(2*A + 5*C)*(b*Cos[c
 + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x
])/(5*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{8/3}}\,dx\\ &= \frac{3Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}}+\frac{1}{5}(2A+5C)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx\\ &= \frac{3Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}}\\ &\quad-\frac{3(2A+5C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3b\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6}\right.\right.}{5d(b\cos(c+dx))^{5/3}}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*b*Csc[c + d*x]*(-(A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2])
+ 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(5/3))
```

Maple [F]

$$\int \frac{(A + C(\cos^2(dx + c)))(\sec^2(dx + c))}{(\cos(dx + c)b)^{\frac{2}{3}}}dx$$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(2/3),x)
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{(C\cos(dx + c)^2 + A)\sec(dx + c)^2}{(b\cos(dx + c))^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm
="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*co
s(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c + d*x))**(2/3), x
)
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{(C\cos(dx + c)^2 + A)\sec(dx + c)^2}{(b\cos(dx + c))^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^2}{\left(b\cos\left(dx + c\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)^2\left(b\,\cos\left(c + d\,x\right)\right)^{2/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)), x)
```

3.169 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 92

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3Ab^2\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}} + \frac{3(5A+8C)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{16d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(8/3)+3/16*(5*A+8*C)*hypergeom([-1/3,
 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3Ab^2\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}} + \frac{3(5A+8C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{16d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(8*d*(b*Cos[c + d*x])^(8/3)) + (3*(5*A + 8*C)*Hyperg
eometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(16*d*(b*Cos[c +
d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{11/3}}\,dx\\ &= \frac{3Ab^2\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}}+\frac{1}{8}(b(5A+8C))\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx\\ &= \frac{3Ab^2\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}}+\frac{3(5A+8C)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{16d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\frac{3\left(4C\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)+\right.}{8d(b\cos(c+}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*(4*C*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x] + A*
Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sec[c + d*x]*Tan[c + d*x
]))/(8*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Maple [F]

$$\int \frac{(A + C(\cos^2(dx + c)))(\sec^3(dx + c))}{(\cos(dx + c)b)^{\frac{2}{3}}}dx$$

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(2/3),x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(2/3),x)

Fricas [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{(C\cos(dx + c)^2 + A)\sec(dx + c)^3}{(b\cos(dx + c))^{\frac{2}{3}}}\,dx$$

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm ="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \text{Timed out}$$

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(2/3),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{(C\cos(dx + c)^2 + A)\sec(dx + c)^3}{(b\cos(dx + c))^{\frac{2}{3}}}\,dx$$

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)), x)
```

3.170 $\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^3/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(
5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3093, 2722}

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8b^3d} - \frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{40b^3d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^3*d) - (3*(8*A + 5*C)*(b*Cos
[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(40*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}\\
&= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d} + \frac{(8A+5C)\int (b\cos(c+dx))^{2/3}\,dx}{8b^2}\\
&= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}\\
&\quad - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^3d\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$
\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx =
$$

$$
-\frac{3\cos^2(c+dx)\cot(c+dx)\left(11A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hypergeon}\right.}{55d(b\cos(c+dx))^{4/3}}
$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6, 11/6, Cos
[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c
+ d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{(\cos^2(dx+c))(A+C(\cos^2(dx+c)))}{(\cos(dx+c)b)^{\frac{4}{3}}}dx$$

[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)

[Out] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)

Fricas [F]

$$\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(C\cos(dx+c)^2+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm ="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/b^2, x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(C\cos(dx+c)^2+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}\,dx$$

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3), x)
```

3.171 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d}-\frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b^2/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(
2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3093, 2722}

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5b^2d}-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d) - (3*(5*A + 2*C)*(b*Cos
[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d
*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} + \frac{(5A+2C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{5b}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx =$$

$$-\frac{3\cot(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\right.\right.}{8bd\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + C*
Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(8*b*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int \frac{\cos (dx+c) (A+C(\cos ^2 (dx+c)))}{(\cos (dx+c) b)^{\frac{4}{3}}}dx$$

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx) (A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \int \frac{\left(C \cos (dx+c)^2+A\right) \cos (dx+c)}{(b\cos (dx+c))^{\frac{4}{3}}} dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx) (A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx) (A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \int \frac{\left(C \cos (dx+c)^2+A\right) \cos (dx+c)}{(b\cos (dx+c))^{\frac{4}{3}}} dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3), x)
```

3.172 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)+3/5*(2*A-C)*(b*cos(d*x+c))^(5/3)*hy
pergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3091, 2722}

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+d\right.}{5b^3d\sqrt{\sin^2(c+dx)}} + \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d
*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/
(5*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} - \frac{(2A-C)\int(b\cos(c+dx))^{2/3}\,dx}{b^2}$$

$$= \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx =$$

$$-\frac{3\cot(c+dx)\left(-5A\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{5d(b\cos(c+dx))^{4/3}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(-5*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[Si
n[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{A + C(\cos^2(dx + c))}{(\cos(dx + c)\, b)^{\frac{4}{3}}} dx$$

```
[In] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{4}{3}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\, dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{4}{3}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos{(dx + c)}^2 + A}{(b\cos{(dx + c)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos{(c + d\,x)}^2 + A}{{(b\,\cos{(c + d\,x)})}^{4/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3), x)
```

3.173 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 90

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)-3/8*(A+4*C)*(b*cos(d*x+c))^(2/3)*hy
pergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3091, 2722}

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c + d
*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
8*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx \\
&= \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{4b} \\
&= \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} \\
&\quad - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx =$$

$$-\frac{3\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3}\right.\right.}{4d(b\cos(c+dx))^{4/3}}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]) +
2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2])*Sqrt[S
in[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{(A + C(\cos^2(dx + c))) \sec(dx + c)}{(\cos(dx + c) b)^{\frac{4}{3}}} dx$$

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(4/3),x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(4/3),x)

Fricas [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}} dx = \int \frac{(C\cos(dx + c)^2 + A)\sec(dx + c)}{(b\cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*cos(d*x + c)^2), x)

Sympy [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}} dx = \int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{\frac{4}{3}}} dx$$

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)

[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d*x))**(4/3), x)

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}} dx = \int \frac{(C\cos(dx + c)^2 + A)\sec(dx + c)}{(b\cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)\,\left(b\,\cos\left(c + d\,x\right)\right)^{4/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)), x)
```

3.174 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 93

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/7*(4*A+7*C)*hypergeom([-1/6, 1/
2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3(4A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx\right.}{7bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}} + \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*(4*A + 7*C)*Hypergeo
metric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*(b*Cos[c + d
*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx \\
&= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{1}{7}(4A+7C)\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx \\
&= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b^2\cot(c+dx)\left(A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\right.}{7d}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*(A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2] +
 7*C*Cos[c + d*x]^2*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F]

$$\int \frac{(A + C(\cos^2(dx + c)))(\sec^2(dx + c))}{(\cos(dx + c) b)^{\frac{4}{3}}} dx$$

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(4/3),x)
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}} dx = \int \frac{(C\cos(dx + c)^2 + A)\sec(dx + c)^2}{(b\cos(dx + c))^{\frac{4}{3}}} dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm
="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*
cos(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}} dx = \text{Timed out}$$

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}} dx = \int \frac{(C\cos(dx + c)^2 + A)\sec(dx + c)^2}{(b\cos(dx + c))^{\frac{4}{3}}} dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^2}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)^2\left(b\,\cos\left(c + d\,x\right)\right)^{4/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)), x)
```

3.175 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 92

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3Ab^2\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}}+\frac{3(7A+10C)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{40d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/10*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(10/3)+3/40*(7*A+10*C)*hypergeom([-2
/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3Ab^2\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}}+\frac{3(7A+10C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{40d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(10*d*(b*Cos[c + d*x])^(10/3)) + (3*(7*A + 10*C)*Hyp
ergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(40*d*(b*Cos[c
 + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^3 \int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{13/3}}\,dx \\
&= \frac{3Ab^2\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}} + \frac{1}{10}(b(7A+10C)) \int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx \\
&= \frac{3Ab^2\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}} \\
&\quad + \frac{3(7A+10C)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{40d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b^2\csc(c+dx)\left(2A\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\cos^2(c+dx)\right)\right.}{20}$$

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
[Out] (3*b^2*Csc[c + d*x]*(2*A*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]
 + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])*Sq
rt[Sin[c + d*x]^2])/(20*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F]

$$\int \frac{(A + C(\cos^2(dx+c)))(\sec^3(dx+c))}{(\cos(dx+c)b)^{\frac{4}{3}}}dx$$

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(4/3),x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(4/3),x)

Fricas [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(C\cos(dx+c)^2 + A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm ="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(C\cos(dx+c)^2 + A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)), x)
```

3.176 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 148

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right) dx = \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(10+3m)} - \frac{3b(C(7+3m)+A(10+3m))\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m)\right.}{d(7+3m)(10+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*b*C*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(10+3*m)-3*b*(C*(7
+3*m)+A*(10+3*m))*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6
+1/2*m],[13/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+51*m+70)/(sin(d*x+c)
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right) dx = \frac{3bC\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)}{d(3m+10)} - \frac{3b\left(\frac{A}{3m+7}+\frac{C}{3m+10}\right)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+7),\frac{1}{6}(3m+1\right.}{d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*b*C*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(10 + 3
*m)) - (3*b*(A/(7 + 3*m) + C/(10 + 3*m))*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*
x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(10+3m)} + \frac{\left(b\left(C\left(\frac{7}{3}+m\right)+A\left(\frac{10}{3}+m\right)\right)\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)\,dx}{\left(\frac{10}{3}+m\right)\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(10+3m)} - \frac{3b(C(7+3m)+A(10+3m))\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m\right.}{d(7+3m)(10+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\left(A(13+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m)\right.\right.}{d(7}$$

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(A*(13 + 3*m)*
Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2] + C*(7 +
3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (13 + 3*m)/6, (19 + 3*m)/6, Cos[
c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(13 + 3*m))
```

Maple [F]

$$\int \left(\cos^m\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{\frac{4}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)dx$$

```
[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos
(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,dx$$

```
[In] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
[Out] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

3.177 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 33, antiderivative size = 146

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)}-\frac{3(C(5+3m)+A(8+3m))\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3\ldots}{d(5+3m)(8+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d/(8+3*m)-3*(C*(5+3*m)
+A*(8+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 5/6+1/2*m
],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+39*m+40)/(sin(d*x+c)^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)}{d(3m+8)}-\frac{3\left(\frac{A}{3m+5}+\frac{C}{3m+8}\right)\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+5),\frac{1}{6}(3m+\ldots}{d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(8 + 3*m)
) - (3*(A/(5 + 3*m) + C/(8 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2
/3)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{2}{3}+m}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$= \frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)} + \frac{\left(\left(C\left(\frac{5}{3}+m\right)+A\left(\frac{8}{3}+m\right)\right)(b\cos(c+dx))^{2/3}\right)\int \cos^{\frac{2}{3}+m}(c+dx)\,dx}{\left(\frac{8}{3}+m\right)\cos^{\frac{2}{3}}(c+dx)}$$

$$= \frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)} - \frac{3(C(5+3m)+A(8+3m))\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\right.}{d(5+3m)(8+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\left(A(11+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m\right.\right.}{d}$$

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]

[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(A*(11 + 3*m)*
Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + C*(5 +
3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (11 + 3*m)/6, (17 + 3*m)/6, Cos[
c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 3*m)*(11 + 3*m))

Maple [F]

$$\int \left(\cos^m\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{\frac{2}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)dx$$

[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2),x)

[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2),x)

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^m\,dx$$

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A + C\cos^2(c+dx)\right)\,dx = \int \cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

```
[In] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
[Out] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

3.178 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 146

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)}$$
$$-\frac{3(C(4+3m)+A(7+3m))\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m\ldots\right.}{d(4+3m)(7+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(7+3*m)-3*(C*(4+3*m)
+A*(7+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 2/3+1/2*m
],[5/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+33*m+28)/(sin(d*x+c)^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)}{d(3m+7)}$$
$$-\frac{3\left(\frac{A}{3m+4}+\frac{C}{3m+7}\right)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+4),\frac{1}{6}(3m+1\ldots\right.}{d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*m)
) - (3*(A/(4 + 3*m) + C/(7 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1
/3)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{b\cos(c+dx)}\int\cos^{\frac{1}{3}+m}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt[3]{\cos(c+dx)}}\\
&= \frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)}\\
&\quad+\frac{\left(\left(C\left(\frac{4}{3}+m\right)+A\left(\frac{7}{3}+m\right)\right)\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{1}{3}+m}(c+dx)\,dx}{\left(\frac{7}{3}+m\right)\sqrt[3]{\cos(c+dx)}}\\
&= \frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)}\\
&\quad-\frac{3(C(4+3m)+A(7+3m))\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\right.}{d(4+3m)(7+3m)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(C(4+3m)\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2},\frac{8}{3}+\right.\right.}{d(4+3m}$$

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(C*(4 + 3*m)*C
os[c + d*x]^2*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2]
+ A*(10 + 3*m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^
2])*Sqrt[Sin[c + d*x]^2])/(d*(4 + 3*m)*(10 + 3*m))
```

Maple [F]

$$\int \left(\cos^m(dx+c)\right)\left(\cos(dx+c)\,b\right)^{\frac{1}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+A\right)\left(b\cos(dx+c)\right)^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Sympy [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{b\cos\left(c+dx\right)}\left(A+C\cos^2\left(c+dx\right)\right)\cos^m\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
[Out] Integral((b*cos(c + d*x))**(1/3)*(A + C*cos(c + d*x)**2)*cos(c + d*x)**m, x
)
```

Maxima [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

```
[In] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
[Out] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

3.179 $\int \frac{\cos^m(c+dx)(A+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 146

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3(C(2+3m)+A(5+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx\right.}{d(2+3m)(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(2+3*m)
+A*(5+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x
+c)^2)*sin(d*x+c)/d/(9*m^2+21*m+10)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C\sin(c+dx)\cos^{m+1}(c+dx)}{d(3m+5)\sqrt[3]{b\cos(c+dx)}} - \frac{3\left(\frac{A}{3m+2}+\frac{C}{3m+5}\right)\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+2),\frac{1}{6}(3m+8),\cos^2(c+dx\right.}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)
) - (3*(A/(2 + 3*m) + C/(5 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1
```

```
/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d
*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{1}{3}+m}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt[3]{b\cos(c+dx)}} \\
&= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} \\
&\quad+\frac{\left(\left(C\left(\frac{2}{3}+m\right)+A\left(\frac{5}{3}+m\right)\right)\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{1}{3}+m}(c+dx)\,dx}{\left(\frac{5}{3}+m\right)\sqrt[3]{b\cos(c+dx)}} \\
&= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} \\
&\quad-\frac{3(C(2+3m)+A(5+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(\right.}{d(2+3m)(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(8+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\right.}{d(2+3m)(8+3m)}$$

```
[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(8 + 3*m)*Hypergeometric2F1[1/2, (
2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + C*(2 + 3*m)*Cos[c + d*x]^2*Hyper
geometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(2 + 3*m)*(8 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int \frac{(\cos^m(dx+c))\left(A+C(\cos^2(dx+c))\right)}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

```
[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*co
s(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(A+C\cos^2\left(c+dx\right)\right)\cos^m\left(c+dx\right)}{\sqrt[3]{b\cos\left(c+dx\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x
)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^m\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

```
[In] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3),x)
[Out] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3), x)
```

3.180 $\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 144

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} - \frac{3(C+3Cm+A(4+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c+dx)\right)\text{si}}{d(1+3m)(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)-3*(C+3*C*m+A
*(4+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/6+1/2*m],[7/6+1/2*m],cos(d*x+c
)^2)*sin(d*x+c)/d/(9*m^2+15*m+4)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sin(c+dx)\cos^{m+1}(c+dx)}{d(3m+4)(b\cos(c+dx))^{2/3}} - \frac{3(A(3m+4)+3Cm+C)\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+1),\frac{1}{6}(3m+7),\cos^2\right.}{d(3m+1)(3m+4)\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

```
[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)
) - (3*(C + 3*C*m + A*(4 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2
, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(4 +
 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{-\frac{2}{3}+m}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{(b\cos(c+dx))^{2/3}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} + \frac{\left(\left(C\left(\frac{1}{3}+m\right)+A\left(\frac{4}{3}+m\right)\right)\cos^{\frac{2}{3}}(c+dx)\right)\int \cos^{-\frac{2}{3}+m}(c+dx)\,dx}{\left(\frac{4}{3}+m\right)(b\cos(c+dx))^{2/3}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} - \frac{3(C+3Cm+A(4+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c\right.}{d(1+3m)(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(7+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c+dx)\right)+\ldots\right.}{d(1+3m)(7+3m)(\ldots}$$

```
[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(7 + 3*m)*Hypergeometric2F1[1/2, (
1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2] + C*(1 + 3*m)*Cos[c + d*x]^2*Hyper
geometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(1 + 3*m)*(7 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int \frac{\left(\cos^m\left(dx+c\right)\right)\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)}{\left(\cos\left(dx+c\right)b\right)^{\frac{2}{3}}}dx$$

```
[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*co
s(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(A+C\cos^2(c+dx))\cos^m(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x
)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(c+d\,x)^m\,\left(C\cos(c+d\,x)^2+A\right)}{(b\,\cos(c+d\,x))^{2/3}}\,dx$$

```
[In] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3),x)
[Out] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3), x)
```

3.181 $\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 149

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3(C(1-3m)-A(2+3m))\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(c+dx)\right)\,\mathrm{s}}{bd(1-3m)(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^m*sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(1-3*m)-A
*(2+3*m))*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m],[5/6+1/2*m],cos(d*x+c)^2
)*sin(d*x+c)/b/d/(-9*m^2-3*m+2)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3\left(\frac{A}{1-3m}-\frac{C}{3m+2}\right)\sin(c+dx)\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\right.}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+}} + \frac{3C\sin(c+dx)\cos^m(c+dx)}{bd(3m+2)\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^m*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)) +
(3*(A/(1 - 3*m) - C/(2 + 3*m))*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 +
3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1
/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{4}{3}+m}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(\left(C\left(-\frac{1}{3}+m\right)+A\left(\frac{2}{3}+m\right)\right)\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{4}{3}+m}(c+dx)\,dx}{b\left(\frac{2}{3}+m\right)\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3(C(1-3m)-A(2+3m))\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(\ldots}{bd(1-3m)(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(5+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(c+dx)\right)\right.}{d(-1+3m)(5+3m}$$

[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]

[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(5 + 3*m)*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2] + C*(-1 + 3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(4/3))

Maple [F]

$$\int \frac{(\cos^m(dx+c))\left(A+C(\cos^2(dx+c))\right)}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)

[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*cos(d*x + c)^2), x)

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(A+C\cos^2{\left(c+dx\right)}\right)\cos^m{\left(c+dx\right)}}{\left(b\cos{\left(c+dx\right)}\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(4/3), x
)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^m\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

```
[In] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3),x)
[Out] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3), x)
```

3.182 $\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 33, antiderivative size = 144

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)}$$
$$-\frac{(C(1+m+n)+A(2+m+n))(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+\right.}{ad(1+m+n)(2+m+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*sin(d*x+c)/a/d/(2+m+n)-(C*(1+m+n)+A
*(2+m+n))*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2+1/2*m+1
/2*n],[3/2+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a/d/(1+m+n)/(2+m+n)/(sin(d
*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{C\sin(c+dx)(a\cos(c+dx))^{m+1}(b\cos(c+dx))^n}{ad(m+n+2)}$$
$$-\frac{(A(m+n+2)+C(m+n+1))\sin(c+dx)(a\cos(c+dx))^{m+1}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{ad(m+n+1)(m+n+2)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(a*d*(2 + m +
n)) - ((C*(1 + m + n) + A*(2 + m + n))*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c +
d*x])^n*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2
]*Sin[c + d*x])/(a*d*(1 + m + n)*(2 + m + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left((a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int (a\cos(c+dx))^{m+n}\left(A+C\cos^2(c+dx)\right)dx$$

$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)} + \left(\left(A + \frac{C(1+m+n)}{2+m+n}\right)(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int (a\cos(c+dx))^{m+n}\,dx$$

$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)} - \frac{\left(A+\frac{C(1+m+n)}{2+m+n}\right)(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+\right.}{ad(1+m+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{(a\cos(c+dx))^m(b\cos(c+dx))^n\cot(c+dx)\left(A(3+m+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3\right.\right.}{}$$

```
[In] Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] -(((a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + m + n)*Hyperg
eometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2] + C*(1 + m +
 n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + m + n)/2, (5 + m + n)/2, Cos
[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m + n)*(3 + m + n)))
```

Maple [F]

$$\int \left(\cos\left(dx+c\right)a\right)^{m}\left(\cos\left(dx+c\right)b\right)^{n}\left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right)dx$$

```
[In] int((cos(d*x+c)*a)^m*(cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int((cos(d*x+c)*a)^m*(cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(a\cos\left(dx+c\right)\right)^{m}\left(b\cos\left(dx+c\right)\right)^{n}\,dx$$

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Sympy [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int (a\cos{(c+dx)})^m\,(b\cos{(c+dx)})^n\left(A+C\cos^2{(c+dx)}\right)\,dx$$

```
[In] integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
[Out] Integral((a*cos(c + d*x))**m*(b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2), x
)
```

Maxima [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos{(dx+c)}^2+A\right)(a\cos{(dx+c)})^m\,(b\cos{(dx+c)})^n\,dx$$

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Giac [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos{(dx+c)}^2+A\right)(a\cos{(dx+c)})^m\,(b\cos{(dx+c)})^n\,dx$$

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(a\,\cos\left(c+d\,x\right)\right)^m\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(a*cos(c + d*x))^m*(b*cos(c + d*x))^n,x)
[Out] int((A + C*cos(c + d*x)^2)*(a*cos(c + d*x))^m*(b*cos(c + d*x))^n, x)
```

3.183 $\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 31, antiderivative size = 117

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)} - \frac{(C(3+n)+A(4+n))(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)(4+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(3+n)*sin(d*x+c)/b^3/d/(4+n)-(C*(3+n)+A*(4+n))*(b*cos(d*x+
c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b
^3/d/(3+n)/(4+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3093, 2722}

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{b^3d(n+4)} - \frac{(A(n+4)+C(n+3))\sin(c+dx)(b\cos(c+dx))^{n+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+5}{2},\cos^2(c+dx)\right)}{b^3d(n+3)(n+4)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(3 + n)*Sin[c + d*x])/(b^3*d*(4 + n)) - ((C*(3 + n) + A
*(4 + n))*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*(4 + n)*Sqrt[Sin[c + d*x]
^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{2+n}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$
$$= \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)} + \frac{\left(A+\frac{C(3+n)}{4+n}\right)\int (b\cos(c+dx))^{2+n}\,dx}{b^2}$$
$$= \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)} - \frac{\left(A+\frac{C(3+n)}{4+n}\right)(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{\cos^2(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(A(5+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)+C\right.}{d(3+n)(5+n)}$$

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(5 + n)*Hypergeometric
2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + C*(3 + n)*Cos[c + d*x]^2*H
ypergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(3 + n)*(5 + n)))
```

Maple [F]

$$\int \left(\cos^2\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^n\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)dx$$

[In] int(cos(d*x+c)^2*(cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)

[Out] int(cos(d*x+c)^2*(cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)

Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^2\,dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^2\,dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)

Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^2 dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int \cos\left(c+d\,x\right)^2 \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n dx$$

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.184 $\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 117

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)} - \frac{(C(2+n)+A(3+n))(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)(3+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(2+n)*sin(d*x+c)/b^2/d/(3+n)-(C*(2+n)+A*(3+n))*(b*cos(d*x+
c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d
/(2+n)/(3+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 3093, 2722}

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{b^2d(n+3)} - \frac{(A(n+3)+C(n+2))\sin(c+dx)(b\cos(c+dx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{b^2d(n+2)(n+3)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(2 + n)*Sin[c + d*x])/(b^2*d*(3 + n)) - ((C*(2 + n) + A
*(3 + n))*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*(3 + n)*Sqrt[Sin[c + d*x]
^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{1+n}\left(A+C\cos^2(c+dx)\right)\,dx}{b}$$
$$= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)} + \frac{\left(A+\frac{C(2+n)}{3+n}\right)\int (b\cos(c+dx))^{1+n}\,dx}{b}$$
$$= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)} - \frac{\left(A+\frac{C(2+n)}{3+n}\right)(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \cos(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{\cos(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(A(4+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)+C\right.}{d(2+n)(4+n)}$$

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(4 + n)*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + C*(2 + n)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(2 + n)*(4 + n)))
```

Maple [F]

$$\int \cos(dx+c)(\cos(dx+c)b)^n\left(A+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)*(cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\cos(dx+c)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fric
as")
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\cos(dx+c)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="maxi
ma")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)})^n\cos{(dx+c)}\ dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac
")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int \cos{(c+d\,x)}\ \left(C\cos{(c+d\,x)}^2+A\right)\ (b\,\cos{(c+d\,x)})^n\,dx$$

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.185 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} - \frac{(C(1+n)+A(2+n))(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)(2+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(1+n)*sin(d*x+c)/b/d/(2+n)-(C*(1+n)+A*(2+n))*(b*cos(d*x+c)
)^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b/d
/(1+n)/(2+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3093, 2722}

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)} - \frac{(A(n+2)+C(n+1))\sin(c+dx)(b\cos(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{bd(n+1)(n+2)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(2 + n)) - ((C*(1 + n) + A*(
2 + n))*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} + \left(A+\frac{C(1+n)}{2+n}\right)\int(b\cos(c+dx))^n\,dx$$

$$= \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} - \frac{\left(A+\frac{C(1+n)}{2+n}\right)(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{(b\cos(c+dx))^n\cot(c+dx)\left(A(3+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)+C(1+n)\cos^2\right.}{d(1+n)(3+n)}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + n)*Hypergeometric2F1[1/2, (1 + n
)/2, (3 + n)/2, Cos[c + d*x]^2] + C*(1 + n)*Cos[c + d*x]^2*Hypergeometric2F
1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 +
 n)*(3 + n)))
```

Maple [F]

$$\int (\cos (dx+c) b)^n \left(A+C\left(\cos^2 (dx+c)\right)\right) dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)
[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)(b\cos\left(dx+c\right))^n \, dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx = \int (b\cos\left(c+dx\right))^n \left(A+C\cos^2\left(c+dx\right)\right) \, dx$$

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
[Out] Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)(b\cos\left(dx+c\right))^n \, dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx=\int\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx=\int\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.186 $\int (b\cos(c+dx))^n (A+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 100

$$\int (b\cos(c+dx))^n (A+C\cos^2(c+dx))\sec(c+dx)\,dx = \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)} - \frac{(A+An+Cn)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn(1+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+n)-(A*n+C*n+A)*(b*cos(d*x+c))^n*hypergeo
m([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(1+n)/(sin(d*x+c)^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3093, 2722}

$$\int (b\cos(c+dx))^n (A+C\cos^2(c+dx))\sec(c+dx)\,dx = \frac{C\sin(c+dx)(b\cos(c+dx))^n}{d(n+1)} - \frac{(An+A+Cn)\sin(c+dx)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(c+dx)\right)}{dn(n+1)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)) - ((A + A*n + C*n)*(b*Cos[c
 + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d
*x])/(d*n*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b\int (b\cos(c+dx))^{-1+n}\left(A+C\cos^2(c+dx)\right)\,dx\\ &= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)}+\frac{(b(A+An+Cn))\int (b\cos(c+dx))^{-1+n}\,dx}{1+n}\\ &= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)}\\ &\quad-\frac{(A+An+Cn)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn(1+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\int (b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$

$$-\frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\left(A(2+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)+Cn\cos^2(c+\right.}{dn(2+n)}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(A*(2 + n)*Hypergeometric2F1[1/
2, n/2, (2 + n)/2, Cos[c + d*x]^2] + C*n*Cos[c + d*x]^2*Hypergeometric2F1[1
/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(2 +
n)))
```

Maple [F]

$$\int (\cos (dx+c)\, b)^{n} \left(A+C\left(\cos^{2}(dx+c)\right)\right) \sec (dx+c)\, dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\sec(dx+c)\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fric
as")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Sympy [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
[Out] Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2)*sec(c + d*x), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\sec(dx+c)\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxi
ma")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac
")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x), x)
```

3.187 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn} - \frac{b(C(1-n)-An)(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)n\sqrt{\sin^2(c+dx)}}$$

```
[Out] b*C*(b*cos(d*x+c))^(-1+n)*sin(d*x+c)/d/n-b*(C*(1-n)-A*n)*(b*cos(d*x+c))^(-1
+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n
)/n/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3093, 2722}

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{bC\sin(c+dx)(b\cos(c+dx))^{n-1}}{dn} - \frac{b(C(1-n)-An)\sin(c+dx)(b\cos(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\cos^2(c+dx)\right)}{d(1-n)n\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (b*C*(b*Cos[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*n) - (b*(C*(1 - n) - A*n)*(
b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*n*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b^2\int (b\cos(c+dx))^{-2+n}\left(A+C\cos^2(c+dx)\right)\,dx\\ &= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn}-\frac{\left(b^2(C(1-n)-An)\right)\int (b\cos(c+dx))^{-2+n}\,dx}{n}\\ &= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn}\\ &\quad-\frac{b(C(1-n)-An)(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin}{d(1-n)n\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int (b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{b(b\cos(c+dx))^{-1+n}\csc(c+dx)\left(A(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)+C(}{d(-1+n)(1+n)}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(A*(1 + n)*Hypergeometric2F1[1/
2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + C*(-1 + n)*Cos[c + d*x]^2*Hyper
geometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^
2])/(d*(-1 + n)*(1 + n)))
```

Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{n} \left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right) \left(\sec^{2}\left(dx+c\right)\right) dx$$

[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

Fricas [F]

$$\int (b\cos(c+dx))^{n} \left(A+C\cos^{2}(c+dx)\right)\sec^{2}(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{2}\,dx$$

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{n} \left(A+C\cos^{2}(c+dx)\right)\sec^{2}(c+dx)\,dx=\text{Timed out}$$

[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)

[Out] Timed out

Maxima [F]

$$\int (b\cos(c+dx))^{n} \left(A+C\cos^{2}(c+dx)\right)\sec^{2}(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^{2}+A\right)\left(b\cos\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{2}\,dx$$

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="gi
ac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^2,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^2, x)
```

3.188 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 125

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)} + \frac{b^2(A(1-n)+C(2-n))(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b^2*C*(b*cos(d*x+c))^(-2+n)*sin(d*x+c)/d/(1-n)+b^2*(A*(1-n)+C*(2-n))*(b*co
s(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+c)
/d/(n^2-3*n+2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3093, 2722}

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \frac{b^2(A(1-n)+C(2-n))\sin(c+dx)(b\cos(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\cos^2(c+dx)\right)}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}} - \frac{b^2C\sin(c+dx)(b\cos(c+dx))^{n-2}}{d(1-n)}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] -((b^2*C*(b*Cos[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(1 - n))) + (b^2*(A*(1
- n) + C*(2 - n))*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)
/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*(2 - n)*Sqrt[Sin[c + d*x]
^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = b^3\int (b\cos(c+dx))^{-3+n}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)}+\left(b^3\left(A+\frac{C(2-n)}{1-n}\right)\right)\int (b\cos(c+dx))^{-3+n}\,dx$$
$$= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)} + \frac{b^2\left(A+\frac{C(2-n)}{1-n}\right)(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.91

$$\int (b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\left(An\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)+C(-2+n)\cos^2}{d(-2+n)n}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*n*Hypergeometric2F1[1/2, (-2 + n)/2,
n/2, Cos[c + d*x]^2] + C*(-2 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, n/2
, (2 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(d*(-2 +
 n)*n))
```

Maple [F]

$$\int (\cos (dx+c) b)^n \left(A+C\left(\cos^2 (dx+c)\right)\right) \left(\sec^3 (dx+c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)})^n\sec{(dx+c)}^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\sec(dx+c)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="ma
xima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\sec(dx+c)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="gi
ac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \frac{\left(C\cos(c+d\,x)^2+A\right)\,\left(b\,\cos(c+d\,x)\right)^n}{\cos(c+d\,x)^3}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^3,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^3, x)
```

3.189 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 127

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)} + \frac{b^3(A(2-n)+C(3-n))(b\cos(c+dx))^{-3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b^3*C*(b*cos(d*x+c))^(-3+n)*sin(d*x+c)/d/(2-n)+b^3*(A*(2-n)+C*(3-n))*(b*co
s(d*x+c))^(-3+n)*hypergeom([1/2, -3/2+1/2*n],[-1/2+1/2*n],cos(d*x+c)^2)*sin
(d*x+c)/d/(n^2-5*n+6)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 3093, 2722}

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \frac{b^3(A(2-n)+C(3-n))\sin(c+dx)(b\cos(c+dx))^{n-3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-3}{2},\frac{n-1}{2},\cos^2(c+dx)\right)}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}} - \frac{b^3C\sin(c+dx)(b\cos(c+dx))^{n-3}}{d(2-n)}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] -((b^3*C*(b*Cos[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(2 - n))) + (b^3*(A*(2
- n) + C*(3 - n))*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)
/2, (-1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*(3 - n)*Sqrt[Sin[c
 + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b^4\int (b\cos(c+dx))^{-4+n}\left(A+C\cos^2(c+dx)\right)\,dx\\ &= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)}+\left(b^4\left(A+\frac{C(3-n)}{2-n}\right)\right)\int (b\cos(c+dx))^{-4+n}\,dx\\ &= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)}\\ &\quad+\frac{b^3\left(A+\frac{C(3-n)}{2-n}\right)(b\cos(c+dx))^{-3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+dx\right.}{d(3-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int (b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=$$
$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\left(A(-1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+dx)\right)\right.}{d(-3+}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + n)*Hypergeometric2F1[1/2, (-3 +
 n)/2, (-1 + n)/2, Cos[c + d*x]^2] + C*(-3 + n)*Cos[c + d*x]^2*Hypergeometr
ic2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^3*Sqrt[Sin[
c + d*x]^2])/(d*(-3 + n)*(-1 + n)))
```

Maple [F]

$$\int (\cos (dx+c)\, b)^{n} \left(A+C \left(\cos^{2}(dx+c)\right)\right) \left(\sec^{4}(dx+c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="ma
xima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="gi
ac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^4}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^4,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^4, x)
```

3.190 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \frac{2C\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(9+2n)}$$
$$-\frac{2(C(7+2n)+A(9+2n))\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),c\right.}{d(7+2n)(9+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*C*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(9+2*n)-2*(C*(7+2*n)+A*(
9+2*n))*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+
1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2+32*n+63)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \frac{2C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n}{d(2n+9)}$$
$$-\frac{2\left(\frac{A}{2n+7}+\frac{C}{2n+9}\right)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+7),\frac{1}{4}(2n+11),c\right.}{d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*C*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(9 + 2*n)) - (2
*(A/(7 + 2*n) + C/(9 + 2*n))*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{5}{2}+n}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$= \frac{2C\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(9+2n)} + \frac{\left(\left(C\left(\frac{7}{2}+n\right)+A\left(\frac{9}{2}+n\right)\right)\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int\cos^{\frac{5}{2}+n}(c+dx)\,dx}{\frac{9}{2}+n}$$

$$= \frac{2C\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(9+2n)} - \frac{2(C(7+2n)+A(9+2n))\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(1\right.}{d(7+2n)(9+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(A(11+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\text{co}\right.\right.}{d(7}$$

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(11 + 2*n)*Hyperg
eometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + C*(7 + 2*n)*C
os[c + d*x]^2*Hypergeometric2F1[1/2, (11 + 2*n)/4, (15 + 2*n)/4, Cos[c + d*
x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 2*n)*(11 + 2*n))
```

Maple [F]

$$\int \left(\cos^{\frac{5}{2}}(dx+c)\right)(\cos(dx+c)\,b)^n\left(A+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}}\,dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos(
d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{5}{2}} \, dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{5}{2}} \, dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx = \int \cos\left(c+d\,x\right)^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n \,dx$$

```
[In] int(cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.191 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)} - \frac{2(C(5+2n)+A(7+2n))\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),c\right.}{d(5+2n)(7+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*C*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(7+2*n)-2*(C*(5+2*n)+A*(
7+2*n))*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1
/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2+24*n+35)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \frac{2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n}{d(2n+7)} - \frac{2\left(\frac{A}{2n+5}+\frac{C}{2n+7}\right)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),c\right.}{d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) - (2
*(A/(5 + 2*n) + C/(7 + 2*n))*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*S
qrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)} + \frac{\left(\left(C\left(\frac{5}{2}+n\right)+A\left(\frac{7}{2}+n\right)\right)\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int\cos^{\frac{3}{2}+n}(c+dx)\,dx}{\frac{7}{2}+n}$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)} - \frac{2(C(5+2n)+A(7+2n))\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+}{d(5+2n)(7+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = -\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \csc(c+dx) \left(A(9+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos\right.\right.}{d(5}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]

[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(9 + 2*n)*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + C*(5 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 2*n)*(9 + 2*n))

Maple [F]

$$\int \left(\cos^{\frac{3}{2}}(dx+c)\right)(\cos(dx+c)\,b)^n \left(A+C\left(\cos^2(dx+c)\right)\right) dx$$

[In] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)

[Out] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2),x)

Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n \cos(dx+c)^{\frac{3}{2}}\, dx$$

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)(b\cos\left(dx+c\right))^n\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)(b\cos\left(dx+c\right))^n\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int \cos\left(c+d\,x\right)^{3/2} \left(C\cos\left(c+d\,x\right)^2+A\right) \left(b\,\cos\left(c+d\,x\right)\right)^n dx$$

```
[In] int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.192 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)}$$
$$-\frac{2(C(3+2n)+A(5+2n))\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\text{co}\right.}{d(3+2n)(5+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(5+2*n)-2*(C*(3+2*n)+A*(
5+2*n))*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1
/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2+16*n+15)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n}{d(2n+5)}$$
$$-\frac{2\left(\frac{A}{2n+3}+\frac{C}{2n+5}\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\text{co}\right.}{d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) - (2
*(A/(3 + 2*n) + C/(5 + 2*n))*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*S
qrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)} + \frac{\left(\left(C\left(\frac{3}{2}+n\right)+A\left(\frac{5}{2}+n\right)\right)\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int\cos^{\frac{1}{2}+n}(c+dx)\,dx}{\frac{5}{2}+n}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)} - \frac{2(C(3+2n)+A(5+2n))\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7}{d(3+2n)(5+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(A(7+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2\right.\right.}{d(3 -}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(7 + 2*n)*Hyperge
ometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + C*(3 + 2*n)*Cos
[c + d*x]^2*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^
2])*Sqrt[Sin[c + d*x]^2])/(d*(3 + 2*n)*(7 + 2*n))
```

Maple [F]

$$\int \left(\cos\left(dx+c\right)b\right)^n \left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sqrt{\cos}(dx+c)\right)dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\sqrt{\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\sqrt{\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\sqrt{\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt{\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,dx$$

```
[In] int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.193 $\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 140

$$\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)} - \frac{2(C+2Cn+A(3+2n))\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),c\right.}{d(1+2n)(3+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*C*(b*cos(d*x+c))^n*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(3+2*n)-2*(C+2*C*n+A*(3+
2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(4*n^2+8*n+3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n}{d(2n+3)} - \frac{2(A(2n+3)+2Cn+C)\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+1),\right.}{d(2n+1)(2n+3)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2
*(C + 2*C*n + A*(3 + 2*n))*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeome
```

```
tric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1
+ 2*n)*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int\cos^{-\frac{1}{2}+n}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx\\
&= \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)}\\
&\quad+\frac{\left(\left(C\left(\frac{1}{2}+n\right)+A\left(\frac{3}{2}+n\right)\right)\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int\cos^{-\frac{1}{2}+n}(c+dx)\,dx}{\frac{3}{2}+n}\\
&= \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)}\\
&\quad-\frac{2(C+2Cn+A(3+2n))\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+\right.}{d(1+2n)(3+2n)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx =$$

$$-\frac{2\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\csc(c+dx)\left(A(5+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\text{co}\right.\right.}{d(1}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(5 + 2*n)*Hyperge
ometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + C*(1 + 2*n)*Cos
[c + d*x]^2*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2
])*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(5 + 2*n))
```

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n\left(A+C\left(\cos^2(dx+c)\right)\right)}{\sqrt{\cos(dx+c)}}\,dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n (A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos{(c+dx)})^n (A+C\cos^2{(c+dx)})}{\sqrt{\cos{(c+dx)}}}\,dx$$

[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)

[Out] Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2)/sqrt(cos(c + d*x)), x)

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n (A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)})^n}{\sqrt{\cos{(dx+c)}}}\,dx$$

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \frac{(b\cos(c+dx))^n (A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)})^n}{\sqrt{\cos{(dx+c)}}}\,dx$$

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm ="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n (A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos{(c+d\,x)}^2+A\right)\,{\left(b\,\cos{(c+d\,x)}\right)}^n}{\sqrt{\cos{(c+d\,x)}}}\,dx$$

[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(1/2),x)

[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(1/2), x)

3.194 $\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 136

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}}+\frac{2(A-C(1-2n)+2An)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d\left(1-4n^2\right)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+2*n)/cos(d*x+c)^(1/2)+2*(A-C*(1-2*n)+2
*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(-4*n^2+1)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{2(2An+A-C(1-2n))\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\cos^2(c+dx)\right)}{d\left(1-4n^2\right)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(2n+1)\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Cos[c + d*x]]) + (2
*(A - C*(1 - 2*n) + 2*A*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 +
2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 4*n^2)*Sqrt[Cos[
c + d*x]]*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \frac{\left(\left(C\left(-\frac{1}{2}+n\right)+A\left(\frac{1}{2}+n\right)\right)\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\,dx}{\frac{1}{2}+n}$$

$$= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \frac{2(A-C(1-2n)+2An)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c}{d\left(1-4n^2\right)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(3+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\right.}{d(-1+2n)(3+}$$

[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]

[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(3 + 2*n)*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + C*(-1 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + 2*n)*(3 + 2*n)*Sqrt[Cos[c + d*x]])

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n\left(A+C(\cos^2(dx+c))\right)}{\cos(dx+c)^{\frac{3}{2}}}dx$$

[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)

[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm ="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\cos{(c+dx)})^n\left(A+C\cos^2{(c+dx)}\right)}{\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
[Out] Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2)/cos(c + d*x)**(3/2), x
)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)})^n}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)})^n}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos{\left(c+d\,x\right)}^2+A\right)\,{\left(b\,\cos{\left(c+d\,x\right)}\right)}^n}{{\cos{\left(c+d\,x\right)}}^{3/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(3/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(3/2), x)
```

3.195 $\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 140

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)} + \frac{2(A+C(3-2n)-2An)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)}{d(1-2n)(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(3/2)+2*(A+C*(3-2*n)-
2*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/d/(4*n^2-8*n+3)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(\frac{A}{3-2n}+\frac{C}{1-2n}\right)\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)} - \frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Cos[c + d*x]^(3/2)) + (
2*(C/(1 - 2*n) + A/(3 - 2*n))*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3
 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)
*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx\\ &= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}\\ &\quad+\frac{\left(\left(C\left(-\frac{3}{2}+n\right)+A\left(-\frac{1}{2}+n\right)\right)\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int\cos^{-\frac{5}{2}+n}(c+dx)\,dx}{-\frac{1}{2}+n}\\ &= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}\\ &\quad+\frac{2(A(1-2n)+C(3-2n))(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c\right.}{d(1-2n)(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx =$$
$$-\frac{2(b\cos(c+dx))^n \csc(c+dx)\left(A(1+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\right.}{d(-3+2n)(1+}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-3
 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + C*(-3 + 2*n)*Cos[c + d*x]^2*Hyper
geometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(-3 + 2*n)*(1 + 2*n)*Cos[c + d*x]^(3/2))
```

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n \left(A+C(\cos^2(dx+c))\right)}{\cos(dx+c)^{\frac{5}{2}}}dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(5/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(5/2), x)
```

3.196 $\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} + \frac{2(A(3-2n)+C(5-2n))(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+d$$
$$\overline{d(3-2n)(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(5/2)+2*(A*(3-2*n)+C*
(5-2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+
c)^2)*sin(d*x+c)/d/(4*n^2-16*n+15)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(\frac{A}{5-2n}+\frac{C}{3-2n}\right)\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-5),\frac{1}{4}(2n-1),\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)} - \frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(5/2)) + (
2*(C/(3 - 2*n) + A/(5 - 2*n))*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5
 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2
)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(\left(C\left(-\frac{5}{2}+n\right)+A\left(-\frac{3}{2}+n\right)\right)\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int\cos^{-\frac{7}{2}+n}(c+dx)\,dx}{-\frac{3}{2}+n}$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} + \frac{2(A(3-2n)+C(5-2n))(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2\right.}{d(3-2n)(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2(b\cos(c+dx))^n \csc(c+dx) \left(A(-1+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+d\right.\right.}{d(-5+2n)(-}$$

[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]

[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + C*(-5 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-5 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(5/2))

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n \left(A+C(\cos^2(dx+c))\right)}{\cos(dx+c)^{\frac{7}{2}}}dx$$

[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)

[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm ="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(c+d\,x)^2+A\right)\,\left(b\,\cos(c+d\,x)\right)^n}{\cos(c+d\,x)^{7/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(7/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(7/2), x)
```

3.197 $\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=-\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(5-2n)\cos^{\frac{7}{2}}(c+dx)}+\frac{2(A(5-2n)+C(7-2n))(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-7+2n),\frac{1}{4}(-3+2n),\cos^2(c+d\dots}{d(5-2n)(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(7/2)+2*(A*(5-2*n)+C*
(7-2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, -7/4+1/2*n],[-3/4+1/2*n],cos(d*x+
c)^2)*sin(d*x+c)/d/(4*n^2-24*n+35)/cos(d*x+c)^(7/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {20, 3093, 2722}

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\frac{2\left(\frac{A}{7-2n}+\frac{C}{5-2n}\right)\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-7),\frac{1}{4}(2n-3),\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)}-\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(5-2n)\cos^{\frac{7}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(7/2)) + (
2*(C/(5 - 2*n) + A/(7 - 2*n))*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-7
 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Cos[c + d*x]^(7/2
)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{9}{2}+n}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(5-2n)\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(\left(C\left(-\frac{7}{2}+n\right)+A\left(-\frac{5}{2}+n\right)\right)\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int\cos^{-\frac{9}{2}+n}(c+dx)\,dx}{-\frac{5}{2}+n}$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(5-2n)\cos^{\frac{7}{2}}(c+dx)} + \frac{2(A(5-2n)+C(7-2n))(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-7+2n),\frac{1}{4}(-3+2n),\cos^2\right.}{d(5-2n)(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = -\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(-3+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-7+2n),\frac{1}{4}(-3+2n),\cos^2(c+d\right.\right.}{d(-7+2n)(-}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-3 + 2*n)*Hypergeometric2F1[1/2, (-
7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2] + C*(-7 + 2*n)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c
+ d*x]^2])/(d*(-7 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(7/2))
```

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n\left(A+C(\cos^2(dx+c))\right)}{\cos(dx+c)^{\frac{9}{2}}}dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(9/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(9/2), x)
```

3.198 $\int (a+a\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 170

$$\int (a+a\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx$$
$$= -\frac{C(a+a\cos(e+fx))^m\sin(e+fx)}{f\left(2+3m+m^2\right)}+\frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}$$
$$+\frac{2^{\frac{1}{2}+m}(C(1+m+m^2)+A(2+3m+m^2))\,(1+\cos(e+fx))^{-\frac{1}{2}-m}(a+a\cos(e+fx))^m\,\text{Hypergeometri}}{f(1+m)(2+m)}$$

```
[Out] -C*(a+a*cos(f*x+e))^m*sin(f*x+e)/f/(m^2+3*m+2)+C*(a+a*cos(f*x+e))^(1+m)*sin
(f*x+e)/a/f/(2+m)+2^(1/2+m)*(C*(m^2+m+1)+A*(m^2+3*m+2))*(1+cos(f*x+e))^(-1/
2-m)*(a+a*cos(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*cos(f*x+e))*si
n(f*x+e)/f/(m^2+3*m+2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3103, 2830, 2731, 2730}

$$\int (a+a\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx$$
$$= \frac{2^{m+\frac{1}{2}}(A(m^2+3m+2)+C(m^2+m+1))\sin(e+fx)(\cos(e+fx)+1)^{-m-\frac{1}{2}}(a\cos(e+fx)+a)^m\,\text{Hyper}}{f(m+1)(m+2)}$$
$$-\frac{C\sin(e+fx)(a\cos(e+fx)+a)^m}{f\left(m^2+3m+2\right)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

```
[In] Int[(a + a*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
```

```
[Out] -((C*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(2 + 3*m + m^2))) + (C*(a + a*
Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 + m)*(C*(1 + m
+ m^2) + A*(2 + 3*m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Cos[e + f*
x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Sin[e + f*
x])/(f*(1 + m)*(2 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} \\
&\quad+\frac{\int (a+a\cos(e+fx))^m(a(C(1+m)+A(2+m))-aC\cos(e+fx))\,dx}{a(2+m)} \\
&= -\frac{C(a+a\cos(e+fx))^m\sin(e+fx)}{f(2+3m+m^2)}+\frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} \\
&\quad+\frac{(C(1+m+m^2)+A(2+3m+m^2))\int (a+a\cos(e+fx))^m\,dx}{(1+m)(2+m)}
\end{aligned}
$$

$$= -\frac{C(a+a\cos(e+fx))^m\sin(e+fx)}{f(2+3m+m^2)} + \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{\left((C(1+m+m^2)+A(2+3m+m^2))(1+\cos(e+fx))^{-m}(a+a\cos(e+fx))^m\right)\int(1+\cos(e+}{(1+m)(2+m)}$$

$$= -\frac{C(a+a\cos(e+fx))^m\sin(e+fx)}{f(2+3m+m^2)} + \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{2^{\frac{1}{2}+m}(C(1+m+m^2)+A(2+3m+m^2))(1+\cos(e+fx))^{-\frac{1}{2}-m}(a+a\cos(e+fx))^m \text{Hyperg}}{f(1+m)(2+m)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.42

$$\int (a+a\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx = \frac{i4^{-1-m}e^{-i(2+m)(e+fx)}\left(1+e^{i(e+fx)}\right)\left(e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\right)^{2m}\cos^{-2m}\left(\frac{1}{2}(e+fx)\right)(a(1+\cos(e+fx)))^m}{}$$

```
[In] Integrate[(a + a*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
```

```
[Out] (I*4^(-1 - m)*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*
x)))^(2*m)*(a*(1 + Cos[e + f*x]))^m*(C*E^(I*m*(e + f*x))*(-2 + m)*m*Hyperge
ometric2F1[1, -1 + m, -1 - m, -E^(I*(e + f*x))] + E^(I*(2 + m)*(e + f*x))*(
2 + m)*(2*(2*A + C)*(-2 + m)*Hypergeometric2F1[1, 1 + m, 1 - m, -E^(I*(e +
f*x))] + C*E^((2*I)*(e + f*x))*m*Hypergeometric2F1[1, 3 + m, 3 - m, -E^(I*(
e + f*x))])))/(E^(I*(2 + m)*(e + f*x))*f*(-2 + m)*m*(2 + m)*Cos[(e + f*x)/2
]^(2*m))
```

Maple [F]

$$\int (a+\cos(fx+e)\,a)^m \left(A+C\left(\cos^2(fx+e)\right)\right) dx$$

```
[In] int((a+cos(f*x+e)*a)^m*(A+C*cos(f*x+e)^2),x)
```

```
[Out] int((a+cos(f*x+e)*a)^m*(A+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a + a\cos(e + fx))^m \left(A + C\cos^2(e + fx)\right) dx$$
$$= \int (C\cos(fx + e)^2 + A)(a\cos(fx + e) + a)^m dx$$

```
[In] integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="fricas")
[Out] integral((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a + a\cos(e + fx))^m \left(A + C\cos^2(e + fx)\right) dx$$
$$= \int (a(\cos(e + fx) + 1))^m \left(A + C\cos^2(e + fx)\right) dx$$

```
[In] integrate((a+a*cos(f*x+e))**m*(A+C*cos(f*x+e)**2),x)
[Out] Integral((a*(cos(e + f*x) + 1))**m*(A + C*cos(e + f*x)**2), x)
```

Maxima [F]

$$\int (a + a\cos(e + fx))^m \left(A + C\cos^2(e + fx)\right) dx$$
$$= \int (C\cos(fx + e)^2 + A)(a\cos(fx + e) + a)^m dx$$

```
[In] integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="maxima")
[Out] integrate((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a + a\cos(e + fx))^m \left(A + C\cos^2(e + fx)\right) dx$$
$$= \int (C\cos(fx + e)^2 + A)(a\cos(fx + e) + a)^m dx$$

```
[In] integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="giac")
[Out] integrate((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(e + fx))^m \left(A + C\cos^2(e + fx)\right) dx$$
$$= \int \left(C\cos\left(e + f\,x\right)^2 + A\right) \left(a + a\,\cos\left(e + f\,x\right)\right)^m dx$$

```
[In] int((A + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m,x)
[Out] int((A + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m, x)
```

3.199 $\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 135

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{9C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d}+\frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad}+\frac{(40A+19C)(a+a\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{10\ 2^{5/6}d(1+\cos(c+dx))^{7/6}}$$

```
[Out] -9/40*C*(a+a*cos(d*x+c))^(2/3)*sin(d*x+c)/d+3/8*C*(a+a*cos(d*x+c))^(5/3)*si
n(d*x+c)/a/d+1/20*(40*A+19*C)*(a+a*cos(d*x+c))^(2/3)*hypergeom([-1/6, 1/2],
[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(7/6)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3103, 2830, 2731, 2730}

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(40A+19C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{10\ 2^{5/6}d(\cos(c+dx)+1)^{7/6}}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}-\frac{9C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{40d}$$

```
[In] Int[(a + a*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-9*C*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(40*d) + (3*C*(a + a*Cos[c +
 d*x])^(5/3)*Sin[c + d*x])/(8*a*d) + ((40*A + 19*C)*(a + a*Cos[c + d*x])^(2
```

```
/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(
10*2^(5/6)*d*(1 + Cos[c + d*x])^(7/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} \\
&\quad+\frac{3\int(a+a\cos(c+dx))^{2/3}\left(\frac{1}{3}a(8A+5C)-aC\cos(c+dx)\right)\,dx}{8a} \\
&= -\frac{9C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d}+\frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} \\
&\quad+\frac{1}{40}(40A+19C)\int(a+a\cos(c+dx))^{2/3}\,dx \\
&= -\frac{9C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d}+\frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} \\
&\quad+\frac{\left((40A+19C)(a+a\cos(c+dx))^{2/3}\right)\int(1+\cos(c+dx))^{2/3}\,dx}{40(1+\cos(c+dx))^{2/3}}
\end{aligned}$$

$$= -\frac{9C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d} + \frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} + \frac{(40A+19C)(a+a\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{10\ 2^{5/6}d(1+\cos(c+dx))^{7/6}}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.30

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)dx = \frac{(a(1+\cos(c+dx)))^{2/3}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(6\ 2^{5/6}(40A+28C+14C\cos(c+dx)+5C\cos \ldots\right.}{\ldots}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*(6*2^(5/6)*(40*A + 28*C +
14*C*Cos[c + d*x] + 5*C*Cos[2*(c + d*x)])*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]
])^(1/6)*Sin[c + d*x] - 4*(40*A + 19*C)*Hypergeometric2F1[1/2, 5/6, 3/2, Co
s[(d*x)/2 - ArcTan[Cot[c/2]]]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]]))/(320*2^(5/
6)*d*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6))
```

Maple [F]

$$\int (a+\cos(dx+c)\,a)^{\frac{2}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int((a+cos(d*x+c)*a)^(2/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+cos(d*x+c)*a)^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)dx = \int \left(C\cos(dx+c)^2+A\right)(a\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx=\text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+a\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}} \, dx$$

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}} \, dx$$

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{2/3} \,d x$$

```
[In] int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(2/3),x)
[Out] int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(2/3), x)
```

3.200 $\int \sqrt[3]{a+a\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 135

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{9C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{28d}+\frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad}+\frac{(28A+13C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{14\sqrt[6]{2}d(1+\cos(c+dx))^{5/6}}$$

```
[Out] -9/28*C*(a+a*cos(d*x+c))^(1/3)*sin(d*x+c)/d+3/7*C*(a+a*cos(d*x+c))^(4/3)*si
n(d*x+c)/a/d+1/28*(28*A+13*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[
3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/d/(1+cos(d*x+c))^(5/6)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3103, 2830, 2731, 2730}

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(28A+13C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{14\sqrt[6]{2}d(\cos(c+dx)+1)^{5/6}}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}-\frac{9C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{28d}$$

```
[In] Int[(a + a*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-9*C*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(28*d) + (3*C*(a + a*Cos[c +
 d*x])^(4/3)*Sin[c + d*x])/(7*a*d) + ((28*A + 13*C)*(a + a*Cos[c + d*x])^(1
/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(1
4*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad} \\
&\quad + \frac{3\int \sqrt[3]{a+a\cos(c+dx)}\left(\frac{1}{3}a(7A+4C)-aC\cos(c+dx)\right)\,dx}{7a} \\
&= -\frac{9C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{28d}+\frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad} \\
&\quad + \frac{1}{28}(28A+13C)\int \sqrt[3]{a+a\cos(c+dx)}\,dx
\end{aligned}
$$

$$= -\frac{9C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{28d} + \frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad} + \frac{\left((28A+13C)\sqrt[3]{a+a\cos(c+dx)}\right)\int\sqrt[3]{1+\cos(c+dx)}\,dx}{28\sqrt[3]{1+\cos(c+dx)}}$$

$$= -\frac{9C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{28d} + \frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad} + \frac{(28A+13C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{14\sqrt[6]{2}d(1+\cos(c+dx))^{5/6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 289 vs. 2(135) = 270.

Time = 2.82 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.14

$$\int\sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt[3]{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(-2(28A+13C)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\cos^2\left(\frac{dx}{2}+\arctan\left(\tan\left(\frac{c}{2}\right)\right)\right)\right)\sec\left(\frac{c}{2}\right.\right.}{}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(1/3)*Sec[(c + d*x)/2]*(-2*(28*A + 13*C)*Hypergeome
tricPFQ[{-1/2, -1/6}, {5/6}, Cos[(d*x)/2 + ArcTan[Tan[c/2]]]^2]*Sec[c/2]*Si
n[(d*x)/2 + ArcTan[Tan[c/2]]] + ((5*(28*A + 13*C)*Cos[(c - d*x - 2*ArcTan[T
an[c/2]])/2]*Csc[c/2]*Sec[c/2] + (28*A + 13*C)*Cos[(c + d*x + 2*ArcTan[Tan[
c/2]])/2]*Csc[c/2]*Sec[c/2] + 6*Cos[(c + d*x)/2]*Sqrt[Sec[c/2]^2]*(-((28*A
+ 13*C)*Cot[c/2]) + C*(Sin[c + d*x] + 2*Sin[2*(c + d*x)])))*Sqrt[Sin[(d*x)/
2 + ArcTan[Tan[c/2]]]^2])/2))/(28*d*Sqrt[Sec[c/2]^2]*Sqrt[Sin[(d*x)/2 + Arc
Tan[Tan[c/2]]]^2])
```

Maple [F]

$$\int (a+\cos(dx+c)\,a)^{\frac{1}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int((a+cos(d*x+c)*a)^(1/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+cos(d*x+c)*a)^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{a\left(\cos\left(c+dx\right)+1\right)}\left(A+C\cos^2\left(c+dx\right)\right)\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
[Out] Integral((a*(cos(c + d*x) + 1))**(1/3)*(A + C*cos(c + d*x)**2), x)
```

Maxima [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(1/3),x)
[Out] int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(1/3), x)
```

3.201 $\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 135

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = -\frac{9C\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}}+\frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{(10A+7C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{5\ 2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}$$

```
[Out] -9/10*C*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/3)+3/5*C*(a+a*cos(d*x+c))^(2/3)*si
n(d*x+c)/a/d+1/10*(10*A+7*C)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*cos(d*x+c))
*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(1/6)/(a+a*cos(d*x+c))^(1/3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3103, 2830, 2731, 2730}

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{(10A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{5\ 2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\sqrt[3]{a\cos(c+dx)+a}} + \frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad} - \frac{9C\sin(c+dx)}{10d\sqrt[3]{a\cos(c+dx)+a}}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-9*C*Sin[c + d*x])/(10*d*(a + a*Cos[c + d*x])^(1/3)) + (3*C*(a + a*Cos[c +
 d*x])^(2/3)*Sin[c + d*x])/(5*a*d) + ((10*A + 7*C)*Hypergeometric2F1[1/2, 5
/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(5*2^(5/6)*d*(1 + Cos[c + d*x]
)^(1/6)*(a + a*Cos[c + d*x])^(1/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad}+\frac{3\int\frac{\frac{1}{3}a(5A+2C)-aC\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx}{5a}\\
&= -\frac{9C\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}}+\frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad}\\
&\quad+\frac{1}{10}(10A+7C)\int\frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx
\end{aligned}$$

$$= -\frac{9C\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}} + \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{\left((10A+7C)\sqrt[3]{1+\cos(c+dx)}\right)\int\frac{1}{\sqrt[3]{1+\cos(c+dx)}}\,dx}{10\sqrt[3]{a+a\cos(c+dx)}}$$

$$= -\frac{9C\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}} + \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{(10A+7C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{5\ 2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int\frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = -\frac{3\ 2^{5/6}C\sqrt[6]{1-\cos\left(dx-2\arctan\left(\cot\left(\frac{c}{2}\right)\right)\right)}(\sin(c+dx)-\sin(2(c+dx)))+2(10A+7C)\,\text{Hypergeor}}{20d\sqrt[3]{a(1+\cos(c+dx))}\sqrt[6]{\sin^2\left(\frac{dx}{2}-\text{arc}\right.}}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] -1/20*(3*2^(5/6)*C*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*(Sin[c + d*x]
- Sin[2*(c + d*x)]) + 2*(10*A + 7*C)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(
d*x)/2 - ArcTan[Cot[c/2]]]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]])/(d*(a*(1 + Cos
[c + d*x]))^(1/3)*(Sin[(d*x)/2 - ArcTan[Cot[c/2]]]^2)^(1/6))
```

Maple [F]

$$\int\frac{A+C(\cos^2(dx+c))}{(a+\cos(dx+c)\,a)^{\frac{1}{3}}}dx$$

```
[In] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(1/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{a + a\cos(c + dx)}} \, dx = \int \frac{C \cos (dx + c)^2 + A}{(a \cos (dx + c) + a)^{\frac{1}{3}}} \, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{a + a\cos(c + dx)}} \, dx = \int \frac{A + C\cos^2{(c + dx)}}{\sqrt[3]{a (\cos{(c + dx)} + 1)}} \, dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**(1/3), x)
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{a + a\cos(c + dx)}} \, dx = \int \frac{C \cos (dx + c)^2 + A}{(a \cos (dx + c) + a)^{\frac{1}{3}}} \, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{a + a\cos(c + dx)}} \, dx = \int \frac{C \cos (dx + c)^2 + A}{(a \cos (dx + c) + a)^{\frac{1}{3}}} \, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{C\cos{(c+d\,x)}^2+A}{{(a+a\,\cos{(c+d\,x)})}^{1/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3), x)
```

3.202 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 138

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx=\frac{3(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}}+\frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad}-\frac{(4A+7C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2\sqrt[6]{2}ad(1+\cos(c+dx))^{5/6}}$$

```
[Out] 3*(A+C)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(2/3)+3/4*C*(a+a*cos(d*x+c))^(1/3)*si
n(d*x+c)/a/d-1/4*(4*A+7*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[3/2
],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/a/d/(1+cos(d*x+c))^(5/6)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3103, 2829, 2731, 2730}

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx=-\frac{(4A+7C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}ad(\cos(c+dx)+1)^{5/6}}+\frac{3(A+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*(A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) + (3*C*(a + a*Cos[c
 + d*x])^(1/3)*Sin[c + d*x])/(4*a*d) - ((4*A + 7*C)*(a + a*Cos[c + d*x])^(1
```

```
/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2
*2^(1/6)*a*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad}+\frac{3\int\frac{\frac{1}{3}a(4A+C)-aC\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx}{4a}\\
&=\frac{3(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}}+\frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad}\\
&\quad-\frac{(4A+7C)\int\sqrt[3]{a+a\cos(c+dx)}\,dx}{4a}\\
&=\frac{3(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}}+\frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad}\\
&\quad-\frac{\left((4A+7C)\sqrt[3]{a+a\cos(c+dx)}\right)\int\sqrt[3]{1+\cos(c+dx)}\,dx}{4a\sqrt[3]{1+\cos(c+dx)}}
\end{aligned}$$

$$= \frac{3(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad} - \frac{(4A+7C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2\sqrt[6]{2}ad(1+\cos(c+dx))^{5/6}}$$

Mathematica [F]

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3),x]
[Out] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3), x]
```

Maple [F]

$$\int \frac{A+C\left(\cos^2\left(dx+c\right)\right)}{\left(a+\cos\left(dx+c\right)a\right)^{\frac{2}{3}}}dx$$

```
[In] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(2/3),x)
[Out] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{A+C\cos^2\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(2/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**(2/3), x)
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(a + a\cos(c + dx))^{2/3}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(a\cos\left(dx + c\right) + a\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(a + a\cos(c + dx))^{2/3}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(a\cos\left(dx + c\right) + a\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(a + a\cos(c + dx))^{2/3}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\left(a + a\,\cos\left(c + d\,x\right)\right)^{2/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3), x)
```

3.203 $\int (a+b\cos(c+dx))^{2/3} (A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\int (a+b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) dx = \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} - \frac{3a(a+b)C\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{(3a^2C+b^2(8A+5C))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] 3/8*C*(a+b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d-3/8*a*(a+b)*C*AppellF1(1/2,-5/3
,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*
sin(d*x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2
)+1/8*(3*a^2*C+b^2*(8*A+5*C))*AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a
+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b^2/d/((a+b*cos(d
*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

$= \{3103, 2835, 2744, 144, 143\}$

$$\int (a+b\cos(c+dx))^{2/3} \left(A + C\cos^2(c+dx)\right) dx = \frac{(3a^2C+b^2(8A+5C))\sin(c+dx)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\right.}{4\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} - \frac{3aC(a+b)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{4\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

```
[In] Int[(a + b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*a*(a + b)*C*Appe
llF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a +
 b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos
[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + ((3*a^2*C + b^2*(8*A + 5
*C))*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*
x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqr
t[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{3\int(a+b\cos(c+dx))^{2/3}\left(\frac{1}{3}b(8A+5C)-aC\cos(c+dx)\right)\,dx}{8b} \\ &= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} - \frac{(3aC)\int(a+b\cos(c+dx))^{5/3}\,dx}{8b^2} \\ &\quad + \frac{1}{8}\left(8A+\left(5+\frac{3a^2}{b^2}\right)C\right)\int(a+b\cos(c+dx))^{2/3}\,dx \\ &= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} \\ &\quad + \frac{(3aC\sin(c+dx))\text{Subst}\left(\int\frac{(a+bx)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{8b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} \\ &\quad + \frac{\left(\left(-8A-\left(5+\frac{3a^2}{b^2}\right)C\right)\sin(c+dx)\right)\text{Subst}\left(\int\frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{8d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\end{aligned}$$

$$= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} - \frac{(3a(-a-b)C(a+b\cos(c+dx))^{2/3}\sin(c+dx))\,\mathrm{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{8b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}} + \frac{\left(\left(-8A-\left(5+\frac{3a^2}{b^2}\right)C\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos\right.}{8d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}}$$

$$= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} - \frac{3a(a+b)C\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{\left(8A+\left(5+\frac{3a^2}{b^2}\right)C\right)\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx}{4\sqrt{2}d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.01

$$\int (a+b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(60a(a^2-b^2)C\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-}{}}\right.}{}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(60*a*(a^2 - b^2)*C*AppellF1[2/
3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b
)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(
a - b))] + 4*(40*A*b^2 - 6*a^2*C + 25*b^2*C)*AppellF1[5/3, 1/2, 1/2, 8/3, (
a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 +
Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c
 + d*x]) - 20*b^2*C*(2*a + 5*b*Cos[c + d*x])*Sin[c + d*x]^2))/(800*b^3*d)
```

Maple [F]

$$\int (a+\cos (dx+c)\, b)^{\frac{2}{3}} \left(A+C\left(\cos^2 (dx+c)\right)\right) dx$$

```
[In] int((a+cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2),x)
[Out] int((a+cos(d*x+c)*b)^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{2/3} \left(A + C\cos^2(c+dx)\right)\, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\, dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\, dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{2/3} \left(A + C\cos^2(c+dx)\right)\, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\, dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{2/3} (A + C\cos^2(c+dx)) \, dx = \int (C\cos(dx+c)^2 + A)(b\cos(dx+c) + a)^{\frac{2}{3}} \, dx$$

[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3} (A + C\cos^2(c+dx)) \, dx = \int (C\cos(c+dx)^2 + A) (a+b\cos(c+dx))^{2/3} \, dx$$

[In] int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3),x)

[Out] int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3), x)

3.204 $\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right) dx = \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{3\sqrt{2}a(a+b)C\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{2}(3a^2C+b^2(7A+4C))\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 3/7*C*(a+b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d-3/7*a*(a+b)*C*AppellF1(1/2,-4/3
,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*
sin(d*x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2
)+1/7*(3*a^2*C+b^2*(7*A+4*C))*AppellF1(1/2,-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a
+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/b^2/d/((a
+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

$= \{3103, 2835, 2744, 144, 143\}$

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$= \frac{\sqrt{2}(3a^2C+b^2(7A+4C))\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\text{co}}{}\right.}{7b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$- \frac{3\sqrt{2}aC(a+b)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{7b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$+ \frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

```
[In] Int[(a + b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*Sqrt[2]*a*(a + b
)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x
]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos
[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(3*a^2*C + b^2*
(7*A + 4*C))*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Co
s[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqr
t[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} \\
&\quad + \frac{3\int \sqrt[3]{a+b\cos(c+dx)}\left(\frac{1}{3}b(7A+4C)-aC\cos(c+dx)\right)\,dx}{7b} \\
&= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{(3aC)\int (a+b\cos(c+dx))^{4/3}\,dx}{7b^2} \\
&\quad + \frac{1}{7}\left(7A+\left(4+\frac{3a^2}{b^2}\right)C\right)\int \sqrt[3]{a+b\cos(c+dx)}\,dx \\
&= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} \\
&\quad + \frac{(3aC\sin(c+dx))\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{7b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} \\
&\quad + \frac{\left(\left(-7A-\left(4+\frac{3a^2}{b^2}\right)C\right)\sin(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{7d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}
\end{aligned}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{\left(3a(-a-b)C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int\frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}dx,x,\cos(c+dx)\right)}{7b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}} + \frac{\left(\left(-7A-\left(4+\frac{3a^2}{b^2}\right)C\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}}dx,\right.}{7d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{3\sqrt{2}a(a+b)C\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{2}\left(7A+\left(4+\frac{3a^2}{b^2}\right)C\right)\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c}{7d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00

$$\int\sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(12a(a^2-b^2)C\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(}{}}\right.}{}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(12*a*(a^2 - b^2)*C*AppellF1[1/
3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b
)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(
a - b))] + (28*A*b^2 - 3*a^2*C + 16*b^2*C)*AppellF1[4/3, 1/2, 1/2, 7/3, (a
+ b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Co
s[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c +
 d*x]) - 4*b^2*C*(a + 4*b*Cos[c + d*x])*Sin[c + d*x]^2))/(112*b^3*d)
```

Maple [F]

$$\int (a+\cos (dx+c)\, b)^{\frac{1}{3}} \left(A+C\left(\cos ^2(dx+c)\right)\right) dx$$

```
[In] int((a+cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2),x)
[Out] int((a+cos(d*x+c)*b)^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\, dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\, dx = \int \left(A+C\cos^2\left(c+dx\right)\right)\sqrt[3]{a+b\cos\left(c+dx\right)}\, dx$$

```
[In] integrate((a+b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
[Out] Integral((A + C*cos(c + d*x)**2)*(a + b*cos(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\, dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3),x)
[Out] int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3), x)
```

3.205 $\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 274

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3\sqrt{2}aC\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{\sqrt{2}(3a^2C+b^2(5A+2C))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\sin}{5b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

```
[Out] 3/5*C*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d-3/5*a*C*AppellF1(1/2,-2/3,1/2,3
/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*
x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^(1/2)+1/5*
(3*a^2*C+b^2*(5*A+2*C))*AppellF1(1/2,1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2
-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1/2)/b^2/d/(a
+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3103, 2835, 2744, 144, 143}

$$
\begin{aligned}
&\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx \\
&= \frac{\sqrt{2}(3a^2C+b^2(5A+2C))\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos}{a+}\right.}{5b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}} \\
&\quad - \frac{3\sqrt{2}aC\sin(c+dx)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{5b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} \\
&\quad + \frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}
\end{aligned}
$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*Sqrt[2]*a*C*Appe
llF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a +
 b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x
]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(3*a^2*C + b^2*(5*A + 2
*C))*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x
]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(5*b^2*d*Sq
rt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}+\frac{3\int \frac{\frac{1}{3}b(5A+2C)-aC\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx}{5b}\\
&= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}-\frac{(3aC)\int (a+b\cos(c+dx))^{2/3}\,dx}{5b^2}\\
&\quad+\frac{1}{5}\left(5A+\left(2+\frac{3a^2}{b^2}\right)C\right)\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx\\
&= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}\\
&\quad+\frac{(3aC\sin(c+dx))\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{5b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\\
&\quad+\frac{\left(\left(-5A-\left(2+\frac{3a^2}{b^2}\right)C\right)\sin(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+bx}}\,dx,x,\cos(c+dx)\right)}{5d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}
\end{aligned}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{(3aC(a+b\cos(c+dx))^{2/3}\sin(c+dx))\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}dx, x, \cos(c+dx)\right)}{5b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}} + \frac{\left(\left(-5A-\left(2+\frac{3a^2}{b^2}\right)C\right)\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}\sin(c+dx)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{-\frac{a}{-a-b}-}}\right.}{5d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3\sqrt{2}aC\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{\sqrt{2}\left(5A+\left(2+\frac{3a^2}{b^2}\right)C\right)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c+}{a+b}}}{5d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.93

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}dx = -\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(5Ab^2+3a^2C+2b^2C)\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+}{a+b}\right.\right.}{}$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(5*A*b^2 + 3*a^2*C + 2*b^2*C
)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
 d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
 + d*x]))/(-a + b)] - 6*a*C*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x
])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a
 + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 10*b^2
*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Maple [F]

$$\int \frac{A+C(\cos^2(dx+c))}{(a+\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

[In] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(1/3),x)

[Out] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(1/3),x)

Fricas [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{C\cos(dx+c)^2+A}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$$

[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)

[Out] Integral((A + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(1/3), x)

Maxima [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{C\cos(dx+c)^2+A}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{a + b\cos(c + dx)}}\, dx = \int \frac{C\cos{(dx + c)}^2 + A}{(b\cos{(dx + c)} + a)^{\frac{1}{3}}}\, dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{a + b\cos(c + dx)}}\, dx = \int \frac{C\cos{(c + d\,x)}^2 + A}{{(a + b\,\cos{(c + d\,x)})}^{1/3}}\, dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3), x)
```

3.206 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 272

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}$$
$$-\frac{3aC\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{(3a^2C+b^2(4A+C))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\sin(c+dx)}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

```
[Out] 3/4*C*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d-3/4*a*C*AppellF1(1/2,-1/3,1/2,3
/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*
x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)+1/4*
(3*a^2*C+b^2*(4*A+C))*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1
/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c)/b^2/d/(a+b*cos(d*x
+c))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

$= \{3103, 2835, 2744, 144, 143\}$

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{(3a^2C+b^2(4A+C))\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\right.}{2\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}$$
$$-\frac{3aC\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{2\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*a*C*AppellF1[1/2
, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a
+ b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x
]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + ((3*a^2*C + b^2*(4*A + C))*Appel
lF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b
)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt
[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
```

```
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}+\frac{3\int\frac{\frac{1}{3}b(4A+C)-aC\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx}{4b}\\
&= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}-\frac{(3aC)\int\sqrt[3]{a+b\cos(c+dx)}\,dx}{4b^2}\\
&\quad+\frac{1}{4}\left(4A+C+\frac{3a^2C}{b^2}\right)\int\frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx\\
&= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}\\
&\quad+\frac{(3aC\sin(c+dx))\text{Subst}\left(\int\frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{4b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\\
&\quad+\frac{\left(\left(-4A-C-\frac{3a^2C}{b^2}\right)\sin(c+dx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{2/3}}\,dx,x,\cos(c+dx)\right)}{4d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd} \\
&+ \frac{\left(3aC\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{4b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}} \\
&+ \frac{\left(\left(-4A-C-\frac{3a^2C}{b^2}\right)\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}\,dx, x\right.}{4d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}} \\
&= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd} \\
&- \frac{3aC\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+ \frac{\left(4A+\left(1+\frac{3a^2}{b^2}\right)C\right)\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\text{si}}{2\sqrt{2}d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 1.46 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.94

$$
\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx =
$$

$$
-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(4Ab^2+(3a^2+b^2)C)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\right.}{}
$$

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(4*A*b^2 + (3*a^2 + b^2)*C)*
AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d
*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c +
 d*x]))/(-a + b)] + C*(-3*a*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x
])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a
 + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 4*b^2*
Sin[c + d*x]^2)))/(16*b^3*d)
```

Maple [F]

$$\int \frac{A + C(\cos^2(dx + c))}{(a + \cos(dx + c)\, b)^{\frac{2}{3}}} dx$$

```
[In] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(2/3),x)
[Out] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{A + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{2/3}} dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c) + a)^{\frac{2}{3}}} dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F]

$$\int \frac{A + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{2/3}} dx = \int \frac{A + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{\frac{2}{3}}} dx$$

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{2/3}} dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c) + a)^{\frac{2}{3}}} dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos(dx+c)^2+A}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos(c+dx)^2+A}{(a+b\cos(c+dx))^{2/3}}\,dx$$

```
[In] int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3), x)
```

3.207 $\int (a+b\cos(e+fx))^m \left(A - A\cos^2(e+fx)\right) dx$

Optimal result

Integrand size = 26, antiderivative size = 211

$$\int (a+b\cos(e+fx))^m \left(A - A\cos^2(e+fx)\right) dx =$$

$$-\frac{4\sqrt{2}A\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}}{f\sqrt{1+\cos(e+fx)}}$$

$$+\frac{4\sqrt{2}A\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}}{f\sqrt{1+\cos(e+fx)}}$$

```
[Out] -4*A*AppellF1(1/2,-m,-3/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a
+b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/f/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f
*x+e))^(1/2)+4*A*AppellF1(1/2,-m,-1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*co
s(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/f/(((a+b*cos(f*x+e))/(a+b))
^m)/(1+cos(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3097, 2834, 144, 143, 2863}

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right) dx$$
$$= \frac{4\sqrt{2}A\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1}{}\right.}{f\sqrt{\cos(e+fx)+1}}$$
$$-\frac{4\sqrt{2}A\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\right.}{f\sqrt{\cos(e+fx)+1}}$$

```
[In] Int[(a + b*Cos[e + f*x])^m*(A - A*Cos[e + f*x]^2),x]
```

```
[Out] (-4*Sqrt[2]*A*AppellF1[1/2, -3/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Co
s[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[
e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (4*Sqrt[2]*A*AppellF1[1/2, -1
/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*C
os[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x]
)/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
```

```
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 3097

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A - C, Int[(a + b*Sin[e + f*x])^m*(1 + S
in[e + f*x]), x], x] + Dist[C, Int[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]
)^2, x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A + C, 0] &&  !Intege
rQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(A\int (1+\cos(e+fx))^2(a+b\cos(e+fx))^m\,dx\right) \\
&\quad + (2A)\int (1+\cos(e+fx))(a+b\cos(e+fx))^m\,dx \\
&= \frac{(A\sin(e+fx))\text{Subst}\left(\int \frac{(1+x)^{3/2}(a+bx)^m}{\sqrt{1-x}}\,dx, x, \cos(e+fx)\right)}{f\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} \\
&\quad - \frac{(2A\sin(e+fx))\text{Subst}\left(\int \frac{\sqrt{1+x}(a+bx)^m}{\sqrt{1-x}}\,dx, x, \cos(e+fx)\right)}{f\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} \\
&= \frac{\left(A(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{3/2}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^m}{\sqrt{1-x}}\,dx, x, \cos\right.}{f\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} \\
&\quad - \frac{\left(2A(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^m}{\sqrt{1-x}}\,dx, x, \mathrm{c}\right.}{f\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}}
\end{aligned}$$

$$= -\frac{4\sqrt{2}A\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos}{a-}\right.}{f\sqrt{1+\cos(e+fx)}}$$
$$+\frac{4\sqrt{2}A\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos}{a+}\right.}{f\sqrt{1+\cos(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.56

$$\int (a+b\cos(e+fx))^m\left(A-A\cos^2(e+fx)\right)\,dx$$
$$=\frac{4A\,\text{AppellF1}\left(\frac{3}{2},-\frac{1}{2},-m,\frac{5}{2},\sin^2\left(\frac{1}{2}(e+fx)\right),\frac{2b\sin^2\left(\frac{1}{2}(e+fx)\right)}{a+b}\right)\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)}(a+b\cos(e+fx))^m\left(\frac{a}{}\right.}{3f}$$

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(A - A*Cos[e + f*x]^2),x]
[Out] (4*A*AppellF1[3/2, -1/2, -m, 5/2, Sin[(e + f*x)/2]^2, (2*b*Sin[(e + f*x)/2]
^2)/(a + b)]*Sqrt[Cos[(e + f*x)/2]^2]*(a + b*Cos[e + f*x])^m*Sin[e + f*x]*T
an[(e + f*x)/2]^2)/(3*f*((a + b*Cos[e + f*x])/(a + b))^m)
```

Maple [F]

$$\int (a+b\cos(fx+e))^m\left(A-A\left(\cos^2(fx+e)\right)\right)dx$$

```
[In] int((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x)
[Out] int((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^m\left(A-A\cos^2(e+fx)\right)\,dx$$
$$=\int -\left(A\cos(fx+e)^2-A\right)(b\cos(fx+e)+a)^m\,dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="fricas")
[Out] integral(-(A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(f*x+e))**m*(A-A*cos(f*x+e)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right)\,dx$$
$$= \int -\left(A\cos\left(fx+e\right)^2-A\right)\left(b\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="maxima")
[Out] -integrate((A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right)\,dx$$
$$= \int -\left(A\cos\left(fx+e\right)^2-A\right)\left(b\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="giac")
[Out] integrate(-(A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right)\,dx$$
$$= \int \left(A-A\cos\left(e+f\,x\right)^2\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^m\,dx$$

```
[In] int((A - A*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m,x)
[Out] int((A - A*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m, x)
```

3.208 $\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 285

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx = \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$
$$-\frac{\sqrt{2}a(a+b)C\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos}{\ldots}\right.}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$
$$+\frac{\sqrt{2}(a^2C+b^2(C(1+m)+A(2+m)))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\ldots}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$

```
[Out] C*(a+b*cos(f*x+e))^(1+m)*sin(f*x+e)/b/f/(2+m)-a*(a+b)*C*AppellF1(1/2,-1-m,1
/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*
x+e)*2^(1/2)/b^2/f/(2+m)/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*x+e))^(1/2)+
(a^2*C+b^2*(C*(1+m)+A*(2+m)))*AppellF1(1/2,-m,1/2,3/2,b*(1-cos(f*x+e))/(a+b
),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/b^2/f/(2+m)/(((
a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{3103, 2835, 2744, 144, 143\}$

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx$$

$$= \frac{\sqrt{2}\sin(e+fx)\left(a^2C+b^2(A(m+2)+C(m+1))\right)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\right.}{b^2f(m+2)\sqrt{\cos(e+fx)+1}}$$

$$- \frac{\sqrt{2}aC(a+b)\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m-1,\frac{3}{2},\frac{1}{2}(1-\cos(e\right.}{b^2f(m+2)\sqrt{\cos(e+fx)+1}}$$

$$+ \frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

```
[In] Int[(a + b*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*a*(a
 + b)*C*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e
 + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt
[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqrt[2]*(a^2*C + b^
2*(C*(1 + m) + A*(2 + m)))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Cos[e + f*x])/2
, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2
*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
```

```
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} \\
&\quad + \frac{\int (a+b\cos(e+fx))^m(b(C(1+m)+A(2+m))-aC\cos(e+fx))\,dx}{b(2+m)} \\
&= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \frac{(aC)\int (a+b\cos(e+fx))^{1+m}\,dx}{b^2(2+m)} \\
&\quad + \frac{\left(a^2C+b^2(C(1+m)+A(2+m))\right)\int (a+b\cos(e+fx))^m\,dx}{b^2(2+m)} \\
&= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} \\
&\quad + \frac{(aC\sin(e+fx))\text{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(e+fx)\right)}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} \\
&\quad - \frac{\left(\left(a^2C+b^2(C(1+m)+A(2+m))\right)\sin(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(e+fx)\right)}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}}
\end{aligned}
$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \frac{\left(a(-a-b)C(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+fx)\right)\text{Subst}\left(\int\frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{1+m}}{\sqrt{1-x}\sqrt{1+x}}d\right.}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} - \frac{\left(\left(a^2C+b^2(C(1+m)+A(2+m))\right)(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+fx)\right)\text{Subs}}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \frac{\sqrt{2}a(a+b)C\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m}{b^2f(2+m)\sqrt{1+\cos(e+fx)}} + \frac{\sqrt{2}\left(a^2C+b^2(C(1+m)+A(2+m))\right)\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10805 vs. $2(285) = 570$.

Time = 25.94 (sec) , antiderivative size = 10805, normalized size of antiderivative = 37.91

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
[Out] Result too large to show
```

Maple [F]

$$\int (a+b\cos(fx+e))^m \left(A+C\left(\cos^2(fx+e)\right)\right) dx$$

```
[In] int((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
[Out] int((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos\left(fx+e\right)^2+A\right)\left(b\cos\left(fx+e\right)+a\right)^m dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="fricas")
[Out] integral((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(f*x+e))**m*(A+C*cos(f*x+e)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos\left(fx+e\right)^2+A\right)\left(b\cos\left(fx+e\right)+a\right)^m dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="maxima")
[Out] integrate((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos\left(fx+e\right)^2+A\right)\left(b\cos\left(fx+e\right)+a\right)^m dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="giac")
[Out] integrate((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(e+f\,x\right)^2+A\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^m\,dx$$

```
[In] int((A + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m,x)
[Out] int((A + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m, x)
```

3.209 $\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)$

Optimal result

Integrand size = 30, antiderivative size = 141

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = -\frac{B(a\cos(e+fx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{a^2f(2+m)\sqrt{\sin^2(e+fx)}} - \frac{C(a\cos(e+fx))^{3+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{a^3f(3+m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -B*(a*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],cos(f*x+e)^2)*si
n(f*x+e)/a^2/f/(2+m)/(sin(f*x+e)^2)^(1/2)-C*(a*cos(f*x+e))^(3+m)*hypergeom(
[1/2, 3/2+1/2*m],[5/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/a^3/f/(3+m)/(sin(f*x+
e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3089, 2827, 2722}

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = -\frac{C\sin(e+fx)(a\cos(e+fx))^{m+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},\cos^2(e+fx)\right)}{a^3f(m+3)\sqrt{\sin^2(e+fx)}} - \frac{B\sin(e+fx)(a\cos(e+fx))^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(e+fx)\right)}{a^2f(m+2)\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[(a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] -((B*(a*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(a^2*f*(2 + m)*Sqrt[Sin[e + f*x]^2])) - (C*(a
*Cos[e + f*x])^(3 + m)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e +
 f*x]^2]*Sin[e + f*x])/(a^3*f*(3 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (a\cos(e+fx))^{1+m}(B+C\cos(e+fx))\,dx}{a}\\ &= \frac{B\int (a\cos(e+fx))^{1+m}\,dx}{a}+\frac{C\int (a\cos(e+fx))^{2+m}\,dx}{a^2}\\ &= -\frac{B(a\cos(e+fx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{a^2f(2+m)\sqrt{\sin^2(e+fx)}}\\ &\quad -\frac{C(a\cos(e+fx))^{3+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{a^3f(3+m)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int (a\cos(e+fx))^m\left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx =$$

$$-\frac{\cos(e+fx)(a\cos(e+fx))^m\cot(e+fx)\left(B(3+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx)\right)+\right.}{f(2+m)(3+m)}$$

```
[In] Integrate[(a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] -((Cos[e + f*x]*(a*Cos[e + f*x])^m*Cot[e + f*x]*(B*(3 + m)*Hypergeometric2F
1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2] + C*(2 + m)*Cos[e + f*x]*Hyper
geometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^
2])/(f*(2 + m)*(3 + m)))
```

Maple [F]

$$\int (\cos (fx+e)\, a)^{m} \left(\cos (fx+e)\, B+C\left(\cos ^{2}(fx+e)\right)\right) dx$$

```
[In] int((cos(f*x+e)*a)^m*(cos(f*x+e)*B+C*cos(f*x+e)^2),x)
```

```
[Out] int((cos(f*x+e)*a)^m*(cos(f*x+e)*B+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\, dx = \int \left(C\cos(fx+e)^2+B\cos(fx+e)\right)(a\cos(fx+e))^m\, dx$$

```
[In] integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

Sympy [F]

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\, dx = \int (a\cos(e+fx))^m (B+C\cos(e+fx))\cos(e+fx)\, dx$$

```
[In] integrate((a*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Integral((a*cos(e + f*x))**m*(B + C*cos(e + f*x))*cos(e + f*x), x)
```

Maxima [F]

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos(fx+e)^2 + B\cos(fx+e)\right)(a\cos(fx+e))^m dx$$

```
[In] integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="maxi
ma")
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos(fx+e)^2 + B\cos(fx+e)\right)(a\cos(fx+e))^m dx$$

```
[In] integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="giac
")
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int (a\cos(e+fx))^m \left(C\cos(e+fx)^2 + B\cos(e+fx)\right) dx$$

```
[In] int((a*cos(e + f*x))^m*(B*cos(e + f*x) + C*cos(e + f*x)^2),x)
[Out] int((a*cos(e + f*x))^m*(B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

3.210 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(B\cos(c+dx)+C\cos$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3B\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(c}{d(7+3m)\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+dx)\right)\sin(c}{d(10+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[13/6
+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*x
+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 5/3+1/2*m],[8/3+1/2*m],cos(d
*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+7),\frac{1}{6}(3m+13),\cos^2(c+}{d(3m+7)\sqrt{\sin^2(c+dx)}} - \frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+10),\frac{1}{6}(3m+16),\cos^2(c-}{d(3m+10)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7
 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[Si
n[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeom
etric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
(10 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{\frac{1}{3}+m}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{\frac{4}{3}+m}(c+dx)(B+C\cos(c+dx))\,dx}{\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{\left(B\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}} + \frac{\left(C\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{7}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}}$$

$$= -\frac{3B\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx\right.}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3C\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+dx\right.}{d(10+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(C(7+3m)\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2},\frac{8}{3}\right.\right.}{d(7+}$$

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(C*(7 + 3*m)*C
os[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] +
B*(10 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]
^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(10 + 3*m))
```

Maple [F]

$$\int \left(\cos^m(dx+c)\right)\left(\cos(dx+c)\,b\right)^{\frac{1}{3}}\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)\left(b\cos(dx+c)\right)^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*x
 + c)^m, x)
```

Sympy [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{b\cos(c+dx)}(B+C\cos(c+dx))\cos(c+dx)\cos^m(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*(B + C*cos(c + d*x))*cos(c + d*x)*cos(c +
d*x)**m, x)
```

Maxima [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*
x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*
x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^m\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.211 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos\right.$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3B\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{1}{6}(14+3m),\cos^2(c+dx)\right)\sin(c+}{d(8+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3C\cos^{3+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(11+3m),\frac{1}{6}(17+3m),\cos^2(c+dx)\right)\sin(c+}{d(11+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 4/3+1/2*m],[7/3+
1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(8+3*m)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*x+
c)^(3+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 11/6+1/2*m],[17/6+1/2*m],cos(
d*x+c)^2)*sin(d*x+c)/d/(11+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+8),\frac{1}{6}(3m+14),\cos^2(c+d\right.}{d(3m+8)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+11),\frac{1}{6}(3m+17),\cos^2(c+\right.}{d(3m+11)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (8
 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(8 + 3*m)*Sqrt[Si
n[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeom
etric2F1[1/2, (11 + 3*m)/6, (17 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
(11 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{2}{3}+m}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\cos^{\frac{2}{3}}(c+dx)}\\
&= \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{5}{3}+m}(c+dx)(B+C\cos(c+dx))\,dx}{\cos^{\frac{2}{3}}(c+dx)}\\
&= \frac{\left(B(b\cos(c+dx))^{2/3}\right)\int \cos^{\frac{5}{3}+m}(c+dx)\,dx}{\cos^{\frac{2}{3}}(c+dx)}+\frac{\left(C(b\cos(c+dx))^{2/3}\right)\int \cos^{\frac{8}{3}+m}(c+dx)\,dx}{\cos^{\frac{2}{3}}(c+dx)}
\end{aligned}$$

$$= -\frac{3B\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{1}{6}(14+3m),\cos^2(c+dx\right.}{d(8+3m)\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{3+m}(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(11+3m),\frac{1}{6}(17+3m),\cos^2(c+d\right.}{d(11+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\left(B(11+3m)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{7}{3}+\frac{m}{2},\cos^2\right.\right.}{d(8+}$$

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(B*(11 + 3*m)*
Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2] + C*(8 + 3*m
)*Cos[c + d*x]*Hypergeometric2F1[1/2, (11 + 3*m)/6, (17 + 3*m)/6, Cos[c + d
*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(8 + 3*m)*(11 + 3*m))
```

Maple [F]

$$\int \left(\cos^m(dx+c)\right)\left(\cos(dx+c)\,b\right)^{\frac{2}{3}}\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d*x
 + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d*
x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d*
x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^m\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.212 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos\right.$

Optimal result

Integrand size = 40, antiderivative size = 169

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3bB\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+dx)\right)\sin(c-}{d(10+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3bC\cos^{4+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(13+3m),\frac{1}{6}(19+3m),\cos^2(c+dx)\right)\sin(c-}{d(13+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*b*B*cos(d*x+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 5/3+1/2*m],[8/
3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^(1/2)-3*b*C*cos
(d*x+c)^(4+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 13/6+1/2*m],[19/6+1/2*m]
,cos(d*x+c)^2)*sin(d*x+c)/d/(13+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+10),\frac{1}{6}(3m+16),\cos^2(c+\right.}{d(3m+10)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3bC\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+4}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+13),\frac{1}{6}(3m+19),\cos^2(c+\right.}{d(3m+13)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-3*b*B*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2,
(10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(10 + 3*m)*Sqr
t[Sin[c + d*x]^2]) - (3*b*C*Cos[c + d*x]^(4 + m)*(b*Cos[c + d*x])^(1/3)*Hyp
ergeometric2F1[1/2, (13 + 3*m)/6, (19 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x
])/(d*(13 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt[3]{\cos(c+dx)}} \\
&= \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{7}{3}+m}(c+dx)(B+C\cos(c+dx))\,dx}{\sqrt[3]{\cos(c+dx)}} \\
&= \frac{\left(bB\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{7}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}}+\frac{\left(bC\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{10}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}}
\end{aligned}$$

$$= -\frac{3bB\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+d\right.}{d(10+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3bC\cos^{4+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(13+3m),\frac{1}{6}(19+3m),\cos^2(c+d\right.}{d(13+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.83

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{2+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\left(B(13+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2},\frac{8}{3}+\frac{m}{2},\cos^2(c\right.\right.}{d(10+}$$

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(B*(13 + 3*m)*
Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] + C*(10 + 3*m)
*Cos[c + d*x]*Hypergeometric2F1[1/2, (13 + 3*m)/6, (19 + 3*m)/6, Cos[c + d*
x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(10 + 3*m)*(13 + 3*m))
```

Maple [F]

$$\int \left(\cos^m(dx+c)\right)\left(\cos(dx+c)\,b\right)^{\frac{4}{3}}\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)\right.$$
$$\left.+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)\left(b\cos(dx+c)\right)^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3)*c
os(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(4/3)*cos(d*
x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(4/3)*cos(d*
x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.213 $\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{3+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{1}{6}(14+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(8+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x+c)^2)
*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*x
+c)^(3+m)*hypergeom([1/2, 4/3+1/2*m],[7/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d
/(8+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+5),\frac{1}{6}(3m+11),\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}} - \frac{3C\sin(c+dx)\cos^{m+3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+8),\frac{1}{6}(3m+14),\cos^2(c+dx)\right)}{d(3m+8)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(
1/3),x]
```

```
[Out] (-3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6
, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Si
n[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*Hypergeometric2F1[1/2, (8 + 3*m)
/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(8 + 3*m)*(b*Cos[c + d*x
])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{1}{3}+m}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt[3]{b\cos(c+dx)}}\\
&= \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{\frac{2}{3}+m}(c+dx)(B+C\cos(c+dx))\,dx}{\sqrt[3]{b\cos(c+dx)}}\\
&= \frac{\left(B\sqrt[3]{\cos(c+dx)}\right)\int \cos^{\frac{2}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\cos(c+dx)}}+\frac{\left(C\sqrt[3]{\cos(c+dx)}\right)\int \cos^{\frac{5}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\cos(c+dx)}}
\end{aligned}$$

$$= -\frac{3B\cos^{2+m}(c+dx)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{3+m}(c+dx)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{1}{6}(14+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(8+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cos^{2+m}(c+dx)\csc(c+dx)\left(B(8+3m)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\right.}{d(5+3m)(8+3m)}$$

```
[In] Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d
*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(8 + 3*m)*Hypergeometric2F1[1/2, (
5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + C*(5 + 3*m)*Cos[c + d*x]*Hyperg
eometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]
^2])/(d*(5 + 3*m)*(8 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int \frac{(\cos^m(dx+c))\left(B\cos(dx+c)+C(\cos^2(dx+c))\right)}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

```
[In] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/b, x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \int \frac{(B+C\cos(c+dx))\cos(c+dx)\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

```
[In] integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3
),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x))
**(1/3), x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
1/3),x)
```

```
[Out] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
1/3), x)
```

3.214 $\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{3+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 2/3+1/2*m],[5/3+1/2*m],cos(d*x+c)^2)*
sin(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*x+
c)^(3+m)*hypergeom([1/2, 7/6+1/2*m],[13/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d
/(7+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+4),\frac{1}{6}(3m+10),\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}} - \frac{3C\sin(c+dx)\cos^{m+3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+7),\frac{1}{6}(3m+13),\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

```
[In] Int[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(
2/3),x]
```

```
[Out] (-3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6
, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Si
n[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*Hypergeometric2F1[1/2, (7 + 3*m)
/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*(b*Cos[c + d*x
])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\cos^{\frac{2}{3}}(c+dx)\int\cos^{-\frac{2}{3}+m}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{(b\cos(c+dx))^{2/3}}\\
&= \frac{\cos^{\frac{2}{3}}(c+dx)\int\cos^{\frac{1}{3}+m}(c+dx)(B+C\cos(c+dx))\,dx}{(b\cos(c+dx))^{2/3}}\\
&= \frac{\left(B\cos^{\frac{2}{3}}(c+dx)\right)\int\cos^{\frac{1}{3}+m}(c+dx)\,dx}{(b\cos(c+dx))^{2/3}}+\frac{\left(C\cos^{\frac{2}{3}}(c+dx)\right)\int\cos^{\frac{4}{3}+m}(c+dx)\,dx}{(b\cos(c+dx))^{2/3}}
\end{aligned}
$$

$$= -\frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{3+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\cos^{2+m}(c+dx)\csc(c+dx)\left(B(7+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{5}{3}+\frac{m}{2},\cos^2(c+dx)\right)+C(4\right.}{d(4+3m)(7+3m)(b\,c}$$

```
[In] Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d
*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(7 + 3*m)*Hypergeometric2F1[1/2, (
4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] + C*(4 + 3*m)*Cos[c + d*x]*Hypergeom
etric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]
^2])/(d*(4 + 3*m)*(7 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int \frac{(\cos^m(dx+c))\left(B\cos(dx+c)+C(\cos^2(dx+c))\right)}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

```
[In] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)
```

```
[Out] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}$$

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/b, x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(B+C\cos(c+dx))\cos(c+dx)\cos^m(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3
),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x))
**(2/3), x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,d$$

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(2/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,d$$

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}$$

```
[In] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
2/3),x)
```

```
[Out] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
2/3), x)
```

3.215 $\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 173

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3B\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*B*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)*
sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*
x+c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)
/b/d/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3B\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+2),\frac{1}{6}(3m+8),\cos^2(c+dx)\right)}{bd(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}} - \frac{3C\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+5),\frac{1}{6}(3m+11),\cos^2(c+dx)\right)}{bd(3m+5)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(
4/3),x]
```

```
[Out] (-3*B*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6,
 Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[S
in[c + d*x]^2]) - (3*C*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (5 + 3*m
)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(5 + 3*m)*(b*Cos[c +
d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{\cos(c+dx)}\int\cos^{-\frac{4}{3}+m}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b\sqrt[3]{b\cos(c+dx)}}\\
&= \frac{\sqrt[3]{\cos(c+dx)}\int\cos^{-\frac{1}{3}+m}(c+dx)(B+C\cos(c+dx))\,dx}{b\sqrt[3]{b\cos(c+dx)}}\\
&= \frac{\left(B\sqrt[3]{\cos(c+dx)}\right)\int\cos^{-\frac{1}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\cos(c+dx)}}+\frac{\left(C\sqrt[3]{\cos(c+dx)}\right)\int\cos^{\frac{2}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\cos(c+dx)}}
\end{aligned}$$

$$= -\frac{3B\cos^{1+m}(c+dx)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{2+m}(c+dx)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\cos^{2+m}(c+dx)\csc(c+dx)\left(B(5+3m)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)+C\right.}{d(2+3m)(5+3m)(b}$$

```
[In] Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d
*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(5 + 3*m)*Hypergeometric2F1[1/2, (
2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + C*(2 + 3*m)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{(\cos^m(dx+c))\left(B\cos(dx+c)+C(\cos^2(dx+c))\right)}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

```
[In] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,d$$

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*co
s(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(B+C\cos(c+dx))\cos(c+dx)\cos^m(c+dx)}{(b\cos(c+dx))^{\frac{4}{3}}}\,d$$

```
[In] integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3
),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x))
**(4/3), x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}\,dx$$

```
[In] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
4/3),x)
[Out] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
4/3), x)
```

3.216 $\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = -\frac{B(a\cos(c+dx))^{2+m}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2+m+n),\frac{1}{2}(4+m+n),\cos^2(c+dx)\right)}{a^2d(2+m+n)\sqrt{\sin^2(c+dx)}} - \frac{C(a\cos(c+dx))^{3+m}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(3+m+n),\frac{1}{2}(5+m+n),\cos^2(c+dx)\right)}{a^3d(3+m+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -B*(a*cos(d*x+c))^(2+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1+1/2*m+1/2*n],[2+
1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a^2/d/(2+m+n)/(sin(d*x+c)^2)^(1/2)-C*
(a*cos(d*x+c))^(3+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/2+1/2*m+1/2*n],[5/2
+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a^3/d/(3+m+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = -\frac{C\sin(c+dx)(a\cos(c+dx))^{m+3}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m+n+3),\frac{1}{2}(m+n+5),\cos^2(c+dx)\right)}{a^3d(m+n+3)\sqrt{\sin^2(c+dx)}} - \frac{B\sin(c+dx)(a\cos(c+dx))^{m+2}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m+n+2),\frac{1}{2}(m+n+4),\cos^2(c+dx)\right)}{a^2d(m+n+2)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] -((B*(a*Cos[c + d*x])^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (2
+ m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a^2*d*(2 + m + n)
*Sqrt[Sin[c + d*x]^2])) - (C*(a*Cos[c + d*x])^(3 + m)*(b*Cos[c + d*x])^n*Hy
pergeometric2F1[1/2, (3 + m + n)/2, (5 + m + n)/2, Cos[c + d*x]^2]*Sin[c +
d*x])/(a^3*d*(3 + m + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int(a\cos(c+dx))^{m+n}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx\\
&= \frac{\left((a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int(a\cos(c+dx))^{1+m+n}(B+C\cos(c+dx))\,dx}{a}\\
&= \frac{\left(B(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int(a\cos(c+dx))^{1+m+n}\,dx}{a}\\
&\quad+\frac{\left(C(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int(a\cos(c+dx))^{2+m+n}\,dx}{a^2}
\end{aligned}
$$

$$= -\frac{B(a\cos(c+dx))^{2+m}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2+m+n),\frac{1}{2}(4+m+n),\cos^2\right.}{a^2d(2+m+n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{C(a\cos(c+dx))^{3+m}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(3+m+n),\frac{1}{2}(5+m+n),\cos^2\right.}{a^3d(3+m+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.81

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx =$$
$$-\frac{\cos(c+dx)(a\cos(c+dx))^m(b\cos(c+dx))^n\cot(c+dx)\left(B(3+m+n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2+\right.\right.}{}$$

```
[In] Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] -((Cos[c + d*x]*(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(3 +
m + n)*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]
 + C*(2 + m + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m + n)/2, (5 + m
+ n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + m + n)*(3 + m + n)))
```

Maple [F]

$$\int (\cos(dx+c)\,a)^m (\cos(dx+c)\,b)^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right) dx$$

```
[In] int((cos(d*x+c)*a)^m*(cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((cos(d*x+c)*a)^m*(cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(a\cos(dx+c))^m (b\cos(dx+c))^n \, dx$$

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*x
+ c))^n, x)
```

Sympy [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (a\cos(c+dx))^m(b\cos(c+dx))^n(B+C\cos(c+dx))\cos(c+dx)\,dx$$

```
[In] integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Integral((a*cos(c + d*x))**m*(b*cos(c + d*x))**n*(B + C*cos(c + d*x))*cos(c
 + d*x), x)
```

Maxima [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(a\cos(dx+c))^m(b\cos(dx+c))^n\,dx$$

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*x
 + c))^n, x)
```

Giac [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(a\cos(dx+c))^m(b\cos(dx+c))^n\,dx$$

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*x
 + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (a\,\cos\left(c+d\,x\right))^m\,(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

[In] int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),x)

[Out] int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2), x)

3.217 $\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(\right.$

Optimal result

Integrand size = 38, antiderivative size = 141

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{B(b\cos(c+dx))^{4+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}} - \frac{C(b\cos(c+dx))^{5+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+n}{2},\frac{7+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^5d(5+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -B*(b*cos(d*x+c))^(4+n)*hypergeom([1/2, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*si
n(d*x+c)/b^4/d/(4+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(5+n)*hypergeom(
[1/2, 5/2+1/2*n],[7/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^5/d/(5+n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 3089, 2827, 2722}

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{C\sin(c+dx)(b\cos(c+dx))^{n+5}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+5}{2},\frac{n+7}{2},\cos^2(c+dx)\right)}{b^5d(n+5)\sqrt{\sin^2(c+dx)}} - \frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+4}{2},\frac{n+6}{2},\cos^2(c+dx)\right)}{b^4d(n+4)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x
]
```

```
[Out] -((B*(b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(b
*Cos[c + d*x])^(5 + n)*Hypergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^5*d*(5 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{2+n}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^2} \\
&= \frac{\int (b\cos(c+dx))^{3+n}(B+C\cos(c+dx))\,dx}{b^3} \\
&= \frac{B\int (b\cos(c+dx))^{3+n}\,dx}{b^3}+\frac{C\int (b\cos(c+dx))^{4+n}\,dx}{b^4} \\
&= -\frac{B(b\cos(c+dx))^{4+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}} \\
&\quad -\frac{C(b\cos(c+dx))^{5+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+n}{2},\frac{7+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^5d(5+n)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{\cos^3(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(B(5+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)+C(\right.}{d(4+n)(5+n)}$$

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] -((Cos[c + d*x]^3*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(5 + n)*Hypergeometric
2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2] + C*(4 + n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(4 + n)*(5 + n)))
```

Maple [F]

$$\int \left(\cos^2(dx+c)\right)(\cos(dx+c)\,b)^n\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)^2*(cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\cos(dx+c)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3)*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \int \cos\left(c+d\,x\right)^2\left(b\,\cos\left(c+d\,x\right)\right)^n\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),x
)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),
x)
```

3.218 $\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx$

Optimal result

Integrand size = 36, antiderivative size = 141

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{B(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}} - \frac{C(b\cos(c+dx))^{4+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -B*(b*cos(d*x+c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(4+n)*hyperg
eom([1/2, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(4+n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {16, 3089, 2827, 2722}

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{C\sin(c+dx)(b\cos(c+dx))^{n+4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+4}{2},\frac{n+6}{2},\cos^2(c+dx)\right)}{b^4d(n+4)\sqrt{\sin^2(c+dx)}} - \frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+5}{2},\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] -((B*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(b
*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{1+n}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b} \\
&= \frac{\int (b\cos(c+dx))^{2+n}(B+C\cos(c+dx))\,dx}{b^2} \\
&= \frac{B\int (b\cos(c+dx))^{2+n}\,dx}{b^2}+\frac{C\int (b\cos(c+dx))^{3+n}\,dx}{b^3} \\
&= -\frac{B(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}} \\
&\quad -\frac{C(b\cos(c+dx))^{4+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{\cos^2(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(B(4+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)+C\right.}{d(3+n)(4+n)}$$

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]

[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(4 + n)*Hypergeometric
2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + C*(3 + n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(3 + n)*(4 + n)))

Maple [F]

$$\int \cos\left(dx+c\right)\left(\cos\left(dx+c\right)b\right)^{n}\left(B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)dx$$

[In] int(cos(d*x+c)*(cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)

[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)\,dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.219 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 141

$$\begin{aligned}&\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx\\&=-\frac{B(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}\\&\quad-\frac{C(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

```
[Out] -B*(b*cos(d*x+c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*si
n(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(3+n)*hypergeom(
[1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3089, 2827, 2722}

$$\begin{aligned}&\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx\\&=-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+5}{2},\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}\\&\quad-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

```
[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] -((B*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(b
*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{1+n}(B+C\cos(c+dx))\,dx}{b}\\
&= \frac{B\int (b\cos(c+dx))^{1+n}\,dx}{b}+\frac{C\int (b\cos(c+dx))^{2+n}\,dx}{b^2}\\
&= -\frac{B(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}\\
&\quad-\frac{C(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int (b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{\cos(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(B(3+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)+C\right.}{d(2+n)(3+n)}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(3 + n)*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + C*(2 + n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^
2])/(d*(2 + n)*(3 + n)))
```

Maple [F]

$$\int (\cos (dx+c)\, b)^{n} \left(B\cos (dx+c)+C\left(\cos^{2}(dx+c)\right)\right) dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{n} \left(B\cos(c+dx)+C\cos^{2}(c+dx)\right)\, dx = \int \left(C\cos (dx+c)^{2}+B\cos (dx+c)\right)(b\cos (dx+c))^{n}\, dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\cos(c+dx))^{n} \left(B\cos(c+dx)+C\cos^{2}(c+dx)\right)\, dx = \int (b\cos (c+dx))^{n} (B+C\cos (c+dx))\cos (c+dx)\, dx$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))*cos(c + d*x), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos{(dx+c)}^2 + B\cos{(dx+c)}\right)(b\cos{(dx+c)})^n \, dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxi
ma")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos{(dx+c)}^2 + B\cos{(dx+c)}\right)(b\cos{(dx+c)})^n \, dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac
")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int (b\cos{(c+d\,x)})^n \left(C\cos{(c+d\,x)}^2 + B\cos{(c+d\,x)}\right) dx$$

```
[In] int((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.220 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 36, antiderivative size = 141

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = -\frac{B(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}} - \frac{C(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -B*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(2+n)*hypergeo
m([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {16, 3089, 2827, 2722}

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = -\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}} - \frac{B\sin(c+dx)(b\cos(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] -((B*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(b*C
os[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d
*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b\int (b\cos(c+dx))^{-1+n}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\
&= \int (b\cos(c+dx))^{n}(B+C\cos(c+dx))\,dx\\
&= B\int (b\cos(c+dx))^{n}\,dx+\frac{C\int (b\cos(c+dx))^{1+n}\,dx}{b}\\
&= -\frac{B(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}\\
&\quad-\frac{C(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$
$$-\frac{(b\cos(c+dx))^n\cot(c+dx)\left(B(2+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)+C(1+n)\cos(c\right.}{d(1+n)(2+n)}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x],x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(2 + n)*Hypergeometric2F1[1/2, (1 + n
)/2, (3 + n)/2, Cos[c + d*x]^2] + C*(1 + n)*Cos[c + d*x]*Hypergeometric2F1[
1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + n
)*(2 + n)))
```

Maple [F]

$$\int (\cos(dx+c)\,b)^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\sec(dx+c)\,dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sec(dx+c)\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c
), x)
```

Sympy [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int (b\cos(c+dx))^n\,(B+C\cos(c+dx))\cos(c+dx)\sec(c+dx)\,dx$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
[Out] Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))*cos(c + d*x)*sec(c + d*x)
, x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sec(dx+c)\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sec(dx+c)\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \frac{(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x),x
)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x),
x)
```

3.221 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 132

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = -\frac{B(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}} - \frac{C(b\cos(c+dx))^{1+n}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -B*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+
c)/d/n/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*
n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 3089, 2827, 2722}

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = -\frac{B\sin(c+dx)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}} - \frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x
]
```

```
[Out] -((B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]
^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])) - (C*(b*Cos[c + d*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int (b\cos(c+dx))^{-2+n} \left(B\cos(c+dx) + C\cos^2(c+dx)\right) \, dx \\
&= b \int (b\cos(c+dx))^{-1+n} (B + C\cos(c+dx)) \, dx \\
&= (bB) \int (b\cos(c+dx))^{-1+n} \, dx + C \int (b\cos(c+dx))^n \, dx \\
&= -\frac{B(b\cos(c+dx))^n \, \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{C(b\cos(c+dx))^{1+n} \, \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\left(B(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)+Cn\cos(c+\right.}{dn(1+n)}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^2,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(B*(1 + n)*Hypergeometric2F1[1/
2, n/2, (2 + n)/2, Cos[c + d*x]^2] + C*n*Cos[c + d*x]*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(1 + n)
))
```

Maple [F]

$$\int (\cos(dx+c)\,b)^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^2(dx+c)\right)dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sec(dx+c)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c
)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \frac{(b\,\cos{(c+d\,x)})^n\,\left(C\cos{(c+d\,x)}^2+B\,\cos{(c+d\,x)}\right)}{\cos{(c+d\,x)}^2}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^2
,x)
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^2
, x)
```

3.222 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 131

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{bB(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}} - \frac{C(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}$$

```
[Out] b*B*(b*cos(d*x+c))^(-1+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c
)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^n*hypergeom([
1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 3089, 2827, 2722}

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{bB\sin(c+dx)(b\cos(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}} - \frac{C\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x
]
```

```
[Out] (b*B*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]) - (C*(b*Co
s[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^3\int (b\cos(c+dx))^{-3+n}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\
&= b^2\int (b\cos(c+dx))^{-2+n}(B+C\cos(c+dx))\,dx\\
&= \left(b^2B\right)\int (b\cos(c+dx))^{-2+n}\,dx+(bC)\int (b\cos(c+dx))^{-1+n}\,dx\\
&= \frac{bB(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}\\
&\quad-\frac{C(b\cos(c+dx))^{n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{b(b\cos(c+dx))^{-1+n}\csc(c+dx)\left(Bn\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)+C(-1+n\right.}{d(-1+n)n}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^3,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(B*n*Hypergeometric2F1[1/2, (-1
 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + C*(-1 + n)*Cos[c + d*x]*Hypergeometri
c2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + n
)*n))
```

Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{n} \left(B\cos \left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right) \left(\sec^{3}\left(dx+c\right)\right) dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{3}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c
)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \frac{(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)^3}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^3
,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^3
, x)
```

3.223 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 139

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{b^2B(b\cos(c+dx))^{-2+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}} + \frac{bC(b\cos(c+dx))^{-1+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] b^2*B*(b*cos(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)+b*C*(b*cos(d*x+c))^(-1+n)*hypergeo
m([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 3089, 2827, 2722}

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{b^2B\sin(c+dx)(b\cos(c+dx))^{n-2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}} + \frac{bC\sin(c+dx)(b\cos(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x
]
```

```
[Out] (b^2*B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]) + (b*C*(b*Cos[
c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*
x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^4 \int (b\cos(c+dx))^{-4+n} \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx \\
&= b^3 \int (b\cos(c+dx))^{-3+n}(B+C\cos(c+dx))\,dx \\
&= \left(b^3B\right)\int (b\cos(c+dx))^{-3+n}\,dx+\left(b^2C\right)\int (b\cos(c+dx))^{-2+n}\,dx \\
&= \frac{b^2B(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}} \\
&\quad+\frac{bC(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.85

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$
$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\left(B(-1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)+C(-2\right.}{d(-2+n)(}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^4,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-1 + n)*Hypergeometric2F1[1/2, (-2 +
 n)/2, n/2, Cos[c + d*x]^2] + C*(-2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2
, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^
2])/(d*(-2 + n)*(-1 + n)))
```

Maple [F]

$$\int (\cos(dx+c)\,b)^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^4(dx+c)\right)dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c
)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \frac{(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)^4}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^4
,x)
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^4
, x)
```

3.224 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(\right.$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{2B\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(9+2n),\frac{1}{4}(13+2n),\cos^2(c+dx)\right)\sin(c+dx}{d(9+2n)\sqrt{\sin^2(c+dx)}} - \frac{2C\cos^{\frac{11}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(11+2n),\frac{1}{4}(15+2n),\cos^2(c+dx)\right)\sin(c+}{d(11+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*B*cos(d*x+c)^(9/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 9/4+1/2*n],[13/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(9+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)^
(11/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 11/4+1/2*n],[15/4+1/2*n],cos(d*x+c)
^2)*sin(d*x+c)/d/(11+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{2B\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+9),\frac{1}{4}(2n+13),\cos^2(c+dx)\right.}{d(2n+9)\sqrt{\sin^2(c+dx)}} - \frac{2C\sin(c+dx)\cos^{\frac{11}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+11),\frac{1}{4}(2n+15),\cos^2(c+dx\right.}{d(2n+11)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-2*B*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (9 + 2*n
)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqrt[Sin[c +
d*x]^2]) - (2*C*Cos[c + d*x]^(11/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/
2, (11 + 2*n)/4, (15 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(11 + 2*n)*
Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{5}{2}+n}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{7}{2}+n}(c+dx)(B+C\cos(c+dx))\,dx$$

$$= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{7}{2}+n}(c+dx)\,dx + \left(C\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{9}{2}+n}(c+dx)\,dx$$

$$= -\frac{2B\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(9+2n),\frac{1}{4}(13+2n),\cos^2(c+dx)\right)\sin}{d(9+2n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{2C\cos^{\frac{11}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(11+2n),\frac{1}{4}(15+2n),\cos^2(c+dx)\right)\text{s}}{d(11+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{2\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(B(11+2n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(9+2n),\frac{1}{4}(13+2n),\text{c}\right.\right.}{d(9}$$

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
[Out] (-2*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(11 + 2*n)*Hyperg
eometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2] + C*(9 + 2*n)*C
os[c + d*x]*Hypergeometric2F1[1/2, (11 + 2*n)/4, (15 + 2*n)/4, Cos[c + d*x]
^2])*Sqrt[Sin[c + d*x]^2])/(d*(9 + 2*n)*(11 + 2*n))
```

Maple [F]

$$\int \left(\cos^{\frac{5}{2}}(dx+c)\right)(\cos(dx+c)\,b)^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}}\,dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3)*(b*cos(d*x + c))^n*sqrt(cos(
d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^{5/2}\left(b\,\cos\left(c+d\,x\right)\right)^n\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.225 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2\right.$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2B\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin(c+d}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

$$-\frac{2C\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(9+2n),\frac{1}{4}(13+2n),\cos^2(c+dx)\right)\sin(c+d}{d(9+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*B*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)^
(9/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 9/4+1/2*n],[13/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(9+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2B\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+7),\frac{1}{4}(2n+11),\cos^2(c+dx\right.}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$

$$-\frac{2C\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+9),\frac{1}{4}(2n+13),\cos^2(c+dx\right.}{d(2n+9)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n
)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c +
d*x]^2]) - (2*C*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2
, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqr
t[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx \\
&= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{5}{2}+n}(c+dx)(B+C\cos(c+dx))\,dx \\
&= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{5}{2}+n}(c+dx)\,dx \\
&\quad + \left(C\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{7}{2}+n}(c+dx)\,dx
\end{aligned}$$

$$= -\frac{2B\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\text{s}}{d(7+2n)\sqrt{\sin^2(c+dx)}} - \frac{2C\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(9+2n),\frac{1}{4}(13+2n),\cos^2(c+dx)\right)\text{si}}{d(9+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \csc(c+dx)\left(B(9+2n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\text{c}\right.\right.}{d(7}$$

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(9 + 2*n)*Hyperge
ometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + C*(7 + 2*n)*Co
s[c + d*x]*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2
])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 2*n)*(9 + 2*n))
```

Maple [F]

$$\int \left(\cos^{\frac{3}{2}}(dx+c)\right)(\cos(dx+c)\,b)^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos(
d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^{3/2}\left(b\,\cos\left(c+d\,x\right)\right)^n\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
[Out] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.226 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx) + C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = -\frac{2B\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+2n)\sqrt{\sin^2(c+dx)}} - \frac{2C\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*B*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*
n],cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)^(
7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = -\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}} - \frac{2C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+7),\frac{1}{4}(2n+11),\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-2*B*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n
)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*C*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)(B+C\cos(c+dx))\,dx$$

$$= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\,dx + \left(C\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{5}{2}+n}(c+dx)\,dx$$

$$= -\frac{2B\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{2C\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \csc(c+dx)\left(B(7+2n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2\right.\right.}{d(5+}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(7 + 2*n)*Hyperge
ometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + C*(5 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]
)*Sqrt[Sin[c + d*x]^2])/(d*(5 + 2*n)*(7 + 2*n))
```

Maple [F]

$$\int (\cos(dx+c)\,b)^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sqrt{\cos}(dx+c)\right)dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*
x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2
),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sqrt{\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d
*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sqrt{\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d
*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.227 $\int \frac{(b\cos(c+dx))^n(B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2B\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3+2n)\sqrt{\sin^2(c+dx)}} - \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*B*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*
n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)^(
5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}} - \frac{2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d
*x]],x]
```

```
[Out] (-2*B*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n
)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)\left(B\cos(c+dx)\right.\\
&\qquad\left.+C\cos^2(c+dx)\right)\,dx\\
&= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+dx)(B+C\cos(c+dx))\,dx\\
&= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+dx)\,dx\\
&\quad+\left(C\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\,dx
\end{aligned}$$

$$= -\frac{2B\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx =$$
$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(B(5+2n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\text{co}\right.\right.}{d(3}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos
[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(5 + 2*n)*Hyperge
ometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + C*(3 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2])
*Sqrt[Sin[c + d*x]^2])/(d*(3 + 2*n)*(5 + 2*n))
```

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n\left(B\cos(dx+c)+C(\cos^2(dx+c))\right)}{\sqrt{\cos(dx+c)}}dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int (b\cos(c+dx))^n\,(B+C\cos(c+dx))\sqrt{\cos(c+dx)}\,dx$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2
),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/sqrt(cos(d
*x + c)), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/sqrt(cos(d
*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
1/2),x)
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
1/2), x)
```

3.228 $\int \frac{(b\cos(c+dx))^n(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2B\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+2n)\sqrt{\sin^2(c+dx)}} - \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*
n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*B*(b*cos(d*x+c
))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*cos(d*
x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}} - \frac{2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(
3/2),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n
)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\left(B\cos(c+dx) + C\cos^2(c+dx)\right)\,dx \\
&= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)(B+C\cos(c+dx))\,dx \\
&= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)\,dx \\
&\quad + \left(C\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+dx)\,dx
\end{aligned}$$

$$= -\frac{2B\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{2\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\csc(c+dx)\left(B(3+2n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos\right.\right.}{d(1+}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(3 + 2*n)*Hyperge
ometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + C*(1 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2])
*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(3 + 2*n))
```

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n\,(B\cos(dx+c)+C(\cos^2(dx+c)))}{\cos(dx+c)^{\frac{3}{2}}}dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(c+dx))^n \left(B+C\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2
),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(3/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
3/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
3/2), x)
```

3.229 $\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)-2*C*(b*cos(d*x+c
))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*cos(d*
x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2B\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}} - \frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(
5/2),x]
```

```
[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d
*x]^2]) - (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\left(B\cos(c+dx)\right.\\
&\qquad\left.+C\cos^2(c+dx)\right)\,dx\\
&= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)(B+C\cos(c+dx))\,dx\\
&= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\,dx\\
&\quad+\left(C\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)\,dx
\end{aligned}$$

$$= \frac{2B(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-1+2n), \frac{1}{4}(3+2n), \cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

$$- \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(1+2n), \frac{1}{4}(5+2n), \cos^2(c+dx)\right)\sin}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx) + C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)} dx =$$

$$-\frac{2(b\cos(c+dx))^n \csc(c+dx)\left(B(1+2n)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-1+2n), \frac{1}{4}(3+2n), \cos^2(c+dx)\right)\right.}{d\left(-1+4n^2\right)}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(1 + 2*n)*Hypergeometric2F1[1/2, (-1
 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + C*(-1 + 2*n)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(-1 + 4*n^2)*Sqrt[Cos[c + d*x]])
```

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n \left(B\cos(dx+c) + C(\cos^2(dx+c))\right)}{\cos(dx+c)^{\frac{5}{2}}} dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx) + C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)} dx$$
$$= \int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(c+dx))^n\,(B+C\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2
),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(5/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\int \frac{(b\ \cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\ \cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
5/2),x)
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
5/2), x)
```

3.230 $\int \frac{(b\cos(c+dx))^n(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{2C(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)+2*C*(b*cos(d*x+c
))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-
2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$\int \frac{(b\cos(c+dx))^n(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2B\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$
$$+\frac{2C\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(
7/2),x]
```

```
[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d
*x]^2]) + (2*C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 +
 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx)\left(B\cos(c+dx)\right.\\
&\qquad\left.+C\cos^2(c+dx)\right)\,dx\\
&= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)(B+C\cos(c+dx))\,dx\\
&= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\,dx\\
&\quad+\left(C\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\,dx
\end{aligned}$$

$$= \frac{2B(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-3+2n), \frac{1}{4}(1+2n), \cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}} + \frac{2C(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-1+2n), \frac{1}{4}(3+2n), \cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n (B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2(b\cos(c+dx))^n \csc(c+dx)\left(B(-1+2n)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-3+2n), \frac{1}{4}(1+2n), \cos^2(c+dx)\right)\right.}{d(-3+2n)(-1}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(7/2),x]
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-1 + 2*n)*Hypergeometric2F1[1/2, (-
3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + C*(-3 + 2*n)*Cos[c + d*x]*Hyperg
eometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2))
```

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n (B\cos(dx+c)+C(\cos^2(dx+c)))}{\cos(dx+c)^{\frac{7}{2}}}dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n (B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
, algorithm="fricas")
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2
),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(7/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
7/2), x)
```

3.231 $\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$
\begin{aligned}
&\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx \\
&= \frac{2B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}} \\
&\quad+\frac{2C(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

```
[Out] 2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)+2*C*(b*cos(d*x+
c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(3
-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {20, 3089, 2827, 2722}

$$
\begin{aligned}
&\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx \\
&= \frac{2B\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-5),\frac{1}{4}(2n-1),\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)} \\
&\quad+\frac{2C\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

```
[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(
9/2),x]
```

```
[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c +
d*x]^2]) + (2*C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1
+ 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqr
t[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{9}{2}+n}(c+dx)\left(B\cos(c+dx)\right.\\
&\qquad \left.+C\cos^2(c+dx)\right)\,dx\\
&= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx)(B+C\cos(c+dx))\,dx\\
&= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx)\,dx\\
&\quad +\left(C\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\,dx
\end{aligned}
$$

$$= \frac{2B(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-5+2n), \frac{1}{4}(-1+2n), \cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}} + \frac{2C(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-3+2n), \frac{1}{4}(1+2n), \cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx) + C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{2(b\cos(c+dx))^n \csc(c+dx)\left(B(-3+2n)\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-5+2n), \frac{1}{4}(-1+2n), \cos^2(c+a\right.\right.}{d(-5+2n)(-}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(9/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-3 + 2*n)*Hypergeometric2F1[1/2, (-
5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + C*(-5 + 2*n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2))
```

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n \left(B\cos(dx+c) + C\left(\cos^2(dx+c)\right)\right)}{\cos(dx+c)^{\frac{9}{2}}}dx$$

```
[In] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx) + C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2
),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(9/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
9/2),x)
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
9/2), x)
```

3.232 $\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right.$

Optimal result

Integrand size = 32, antiderivative size = 173

$$\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)}$$
$$+\frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}$$
$$+\frac{2^{\frac{1}{2}+m}(Bm(2+m)+C(1+m+m^2))\,(1+\cos(e+fx))^{-\frac{1}{2}-m}(a+a\cos(e+fx))^m\,\text{Hypergeometric2F1}\,(}{f(1+m)(2+m)}$$

```
[Out] -(C-B*(2+m))*(a+a*cos(f*x+e))^m*sin(f*x+e)/f/(1+m)/(2+m)+C*(a+a*cos(f*x+e))
^(1+m)*sin(f*x+e)/a/f/(2+m)+2^(1/2+m)*(B*m*(2+m)+C*(m^2+m+1))*(1+cos(f*x+e)
)^(-1/2-m)*(a+a*cos(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*cos(f*x+
e))*sin(f*x+e)/f/(m^2+3*m+2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{3102, 2830, 2731, 2730\}$

$$\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= \frac{2^{m+\frac{1}{2}}(Bm(m+2)+C(m^2+m+1))\sin(e+fx)(\cos(e+fx)+1)^{-m-\frac{1}{2}}(a\cos(e+fx)+a)^m \operatorname{Hypergeon}}{f(m+1)(m+2)}$$
$$- \frac{(C-B(m+2))\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)(m+2)}$$
$$+ \frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

```
[In] Int[(a + a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] -(((C - B*(2 + m))*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m)*(2 + m))
) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 +
 m)*(B*m*(2 + m) + C*(1 + m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Co
s[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Si
n[e + f*x])/(f*(1 + m)*(2 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{\int (a+a\cos(e+fx))^m(aC(1+m)-a(C-B(2+m))\cos(e+fx))\,dx}{a(2+m)}$$

$$= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)} + \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{(Bm(2+m)+C(1+m+m^2))\int (a+a\cos(e+fx))^m\,dx}{(1+m)(2+m)}$$

$$= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)} + \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{((Bm(2+m)+C(1+m+m^2))(1+\cos(e+fx))^{-m}(a+a\cos(e+fx))^m)\int (1+\cos(e+fx))}{(1+m)(2+m)}$$

$$= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)} + \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{2^{\frac{1}{2}+m}(Bm(2+m)+C(1+m+m^2))(1+\cos(e+fx))^{-\frac{1}{2}-m}(a+a\cos(e+fx))^m\,\text{Hypergeomet}}{f(1+m)(2+m)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.97 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.06

$$\int (a+a\cos(e+fx))^m\left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$

$$= \frac{i4^{-1-m}e^{-2i(e+fx)}\left(1+e^{i(e+fx)}\right)^{-2m}\left(e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\right)^{2m}\cos^{-2m}\left(\frac{1}{2}(e+fx)\right)(a(1+\cos(e+fx)))^m\,(}{}$$

```
[In] Integrate[(a + a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (I*4^(-1 - m)*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m)*(a*(1 + Cos
[e + f*x]))^m*(C*m*(2 - m - 2*m^2 + m^3)*Hypergeometric2F1[-2 - m, -2*m, -1
 - m, -E^(I*(e + f*x))] + E^(I*(e + f*x))*(2 + m)*(2*B*m*(2 - 3*m + m^2)*Hy
```

```
pergeometric2F1[-1 - m, -2*m, -m, -E^(I*(e + f*x))] + E^(I*(e + f*x))*(1 +
m)*(2*B*E^(I*(e + f*x))*(-2 + m)*m*Hypergeometric2F1[1 - m, -2*m, 2 - m, -E
^(I*(e + f*x))] + C*(-1 + m)*(E^((2*I)*(e + f*x))*m*Hypergeometric2F1[2 - m
, -2*m, 3 - m, -E^(I*(e + f*x))] + 2*(-2 + m)*Hypergeometric2F1[-2*m, -m, 1
 - m, -E^(I*(e + f*x))])))))/(E^((2*I)*(e + f*x))*(1 + E^(I*(e + f*x)))^(2*
m)*f*(-2 + m)*(-1 + m)*m*(1 + m)*(2 + m)*Cos[(e + f*x)/2]^(2*m))
```

Maple [F]

$$\int \left(a+\cos \left(f x+e\right) a\right)^{m} \left(\cos \left(f x+e\right) B+C\left(\cos^{2}\left(f x+e\right)\right)\right) dx$$

```
[In] int((a+cos(f*x+e)*a)^m*(cos(f*x+e)*B+C*cos(f*x+e)^2),x)
```

```
[Out] int((a+cos(f*x+e)*a)^m*(cos(f*x+e)*B+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)\right)\left(a\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(a(\cos\left(e+fx\right)+1)\right)^m\left(B+C\cos\left(e+fx\right)\right)\cos\left(e+fx\right)dx$$

```
[In] integrate((a+a*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Integral((a*(cos(e + f*x) + 1))**m*(B + C*cos(e + f*x))*cos(e + f*x), x)
```

Maxima [F]

$$\int (a+a\cos(e+fx))^m\left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)\right)\left(a\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+a\cos(e+fx))^m\left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)\right)\left(a\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(e+fx))^m\left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(e+f\,x\right)^2+B\,\cos\left(e+f\,x\right)\right)\,\left(a+a\,\cos\left(e+f\,x\right)\right)^m\,dx$$

```
[In] int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m,x)
```

```
[Out] int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m, x)
```

3.233 $\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx\right.$

Optimal result

Integrand size = 32, antiderivative size = 295

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$
$$-\frac{\sqrt{2}(a+b)(aC-bB(2+m))\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$
$$+\frac{\sqrt{2}(a^2C+b^2C(1+m)-abB(2+m))\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$

```
[Out] C*(a+b*cos(f*x+e))^(1+m)*sin(f*x+e)/b/f/(2+m)-(a+b)*(a*C-b*B*(2+m))*AppellF
1(1/2,-1-m,1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+
e))^m*sin(f*x+e)*2^(1/2)/b^2/f/(2+m)/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*
x+e))^(1/2)+(a^2*C+b^2*C*(1+m)-a*b*B*(2+m))*AppellF1(1/2,-m,1/2,3/2,b*(1-co
s(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/b
^2/f/(2+m)/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used

$= \{3102, 2835, 2744, 144, 143\}$

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \frac{\sqrt{2}\sin(e+fx)\left(a^2C-abB(m+2)+b^2C(m+1)\right)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\right.}{b^2f(m+2)\sqrt{\cos(e+fx)+1}}$$
$$- \frac{\sqrt{2}(a+b)\sin(e+fx)(aC-bB(m+2))(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m-1\right.}{b^2f(m+2)\sqrt{\cos(e+fx)+1}}$$
$$+ \frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

```
[In] Int[(a + b*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*(a +
 b)*(a*C - b*B*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/
2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^
2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqr
t[2]*(a^2*C + b^2*C*(1 + m) - a*b*B*(2 + m))*AppellF1[1/2, 1/2, -m, 3/2, (1
 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*
Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(
a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} \\
&\quad + \frac{\int (a+b\cos(e+fx))^m(bC(1+m)-(aC-bB(2+m))\cos(e+fx))\,dx}{b(2+m)} \\
&= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} \\
&\quad + \frac{(-aC+bB(2+m))\int (a+b\cos(e+fx))^{1+m}\,dx}{b^2(2+m)} \\
&\quad + \frac{(a^2C+b^2C(1+m)-abB(2+m))\int (a+b\cos(e+fx))^m\,dx}{b^2(2+m)} \\
&= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} \\
&\quad - \frac{((-aC+bB(2+m))\sin(e+fx))\text{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(e+fx)\right)}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} \\
&\quad - \frac{((a^2C+b^2C(1+m)-abB(2+m))\sin(e+fx))\,\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(e+fx)\right)}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}}
\end{aligned}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} + \frac{\left((-a-b)(-aC+bB(2+m))(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+fx)\right)\text{Subst}\left(\int \frac{(}{}\right.}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} - \frac{\left((a^2C+b^2C(1+m)-abB(2+m))(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+fx)\right)\text{Sub}}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \frac{\sqrt{2}(a+b)(aC-bB(2+m))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+}{b^2f(2+m)\sqrt{1+\cos(e+fx)}} + \frac{\sqrt{2}(a^2C+b^2C(1+m)-abB(2+m))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 13441 vs. $2(295) = 590$.

Time = 26.64 (sec) , antiderivative size = 13441, normalized size of antiderivative = 45.56

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
[Out] Result too large to show
```

Maple [F]

$$\int (a+b\cos(fx+e))^m \left(\cos(fx+e)\,B+C\left(\cos^2(fx+e)\right)\right)dx$$

```
[In] int((a+b*cos(f*x+e))^m*(cos(f*x+e)*B+C*cos(f*x+e)^2),x)
[Out] int((a+b*cos(f*x+e))^m*(cos(f*x+e)*B+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)\right)\left(b\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fr
icas")
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)\right)\left(b\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="ma
xima")
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)\right)\left(b\cos\left(fx+e\right)+a\right)^m dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos\left(e+f\,x\right)^2+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^m dx$$

```
[In] int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m,x)
```

```
[Out] int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m, x)
```

3.234 $\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx)+C\cos^2(c+dx\right.$

Optimal result

Integrand size = 34, antiderivative size = 284

$$\int (a+b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}$$
$$+\frac{(a+b)(8bB-3aC)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$
$$-\frac{(8abB-3a^2C-5b^2C)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] 3/8*C*(a+b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d+1/8*(a+b)*(8*B*b-3*C*a)*AppellF
1(1/2,-5/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+
c))^(2/3)*sin(d*x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*
x+c))^(1/2)-1/8*(8*B*a*b-3*C*a^2-5*C*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-co
s(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b^2/d
/((a+b*cos(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3102, 2835, 2744, 144, 143}

$$\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx =$$

$$-\frac{(-3a^2C+8abB-5b^2C)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\text{co}}{a}\right.}{4\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

$$+\frac{(a+b)(8bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx}{a+b}\right.}{4\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

$$+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

```
[In] Int[(a + b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((a + b)*(8*b*B - 3
*a*C)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) - ((8*a*b*B - 3*
a^2*C - 5*b^2*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1
- Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[
2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} \\
&\quad + \frac{3\int (a+b\cos(c+dx))^{2/3}\left(\frac{5bC}{3}+\frac{1}{3}(8bB-3aC)\cos(c+dx)\right)\,dx}{8b} \\
&= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{(8bB-3aC)\int (a+b\cos(c+dx))^{5/3}\,dx}{8b^2} \\
&\quad - \frac{(8abB-3a^2C-5b^2C)\int (a+b\cos(c+dx))^{2/3}\,dx}{8b^2} \\
&= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} \\
&\quad - \frac{((8bB-3aC)\sin(c+dx))\text{Subst}\left(\int \frac{(a+bx)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{8b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} \\
&\quad + \frac{((8abB-3a^2C-5b^2C)\sin(c+dx))\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{8b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}
\end{aligned}$$

$$= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{((-a-b)(8bB-3aC)(a+b\cos(c+dx))^{2/3}\sin(c+dx))\,\text{Subst}\left(\int\frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}dx,x,\cos(c-}{8b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}} + \frac{((8abB-3a^2C-5b^2C)(a+b\cos(c+dx))^{2/3}\sin(c+dx))\,\text{Subst}\left(\int\frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}dx,x,\cos(c}{8b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}}$$

$$= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{(a+b)(8bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{}}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} - \frac{(8abB-3a^2C-5b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+d}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.02

$$\int(a+b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(-a^2+b^2)(8bB-3aC)\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right.\right.}{}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(-a^2 + b^2)*(8*b*B - 3*a*C)
*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[
c + d*x]))/(a - b))] + (16*a*b*B - 6*a^2*C + 25*b^2*C)*AppellF1[5/3, 1/2, 1
/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-
((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*
(a + b*Cos[c + d*x]) - 5*b^2*(8*b*B + 2*a*C + 5*b*C*Cos[c + d*x])*Sin[c + d
*x]^2))/(200*b^3*d)
```

Maple [F]

$$\int (a+\cos(dx+c)\,b)^{\frac{2}{3}}\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int((a+cos(d*x+c)*b)^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int((a+cos(d*x+c)*b)^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3), x
)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx) + C\cos^2(c+dx)\right) dx = \int \left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{2}{3}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3), x
)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx) + C\cos^2(c+dx)\right) dx = \int \left(C\cos(c+d\,x)^2 + B\,\cos(c+d\,x)\right)\,(a+b\,\cos(c+d\,x))^{2/3} dx$$

```
[In] int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3), x)
```

3.235 $\int \sqrt[3]{a+b\cos(c+dx)}(B\cos(c+dx)+C\cos^2(c+d$

Optimal result

Integrand size = 34, antiderivative size = 284

$$\int \sqrt[3]{a+b\cos(c+dx)}(B\cos(c+dx)+C\cos^2(c+dx))\,dx$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd}$$

$$+ \frac{\sqrt{2}(a+b)(7bB-3aC)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$- \frac{\sqrt{2}(7abB-3a^2C-4b^2C)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 3/7*C*(a+b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d+1/7*(a+b)*(7*B*b-3*C*a)*AppellF
1(1/2,-4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+
c))^(1/3)*sin(d*x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*
x+c))^(1/2)-1/7*(7*B*a*b-3*C*a^2-4*C*b^2)*AppellF1(1/2,-1/3,1/2,3/2,b*(1-co
s(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/
2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {3102, 2835, 2744, 144, 143}

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{\sqrt{2}(-3a^2C+7abB-4b^2C)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(}{}\right.}{7b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{2}(a+b)(7bB-3aC)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(}{a+}\right.}{7b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

```
[In] Int[(a + b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + (Sqrt[2]*(a + b)*(7
*b*B - 3*a*C)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - C
os[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*(7*a*
b*B - 3*a^2*C - 4*b^2*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2
, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/
(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} \\
&\quad + \frac{3\int \sqrt[3]{a+b\cos(c+dx)}\left(\frac{4bC}{3}+\frac{1}{3}(7bB-3aC)\cos(c+dx)\right)\,dx}{7b} \\
&= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{(7bB-3aC)\int (a+b\cos(c+dx))^{4/3}\,dx}{7b^2} \\
&\quad - \frac{(7abB-3a^2C-4b^2C)\int \sqrt[3]{a+b\cos(c+dx)}\,dx}{7b^2} \\
&= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} \\
&\quad - \frac{((7bB-3aC)\sin(c+dx))\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{7b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} \\
&\quad + \frac{((7abB-3a^2C-4b^2C)\sin(c+dx))\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{7b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{\left((-a-b)(7bB-3aC)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int\frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}dx,x,\cos(c+\right.}{7b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}} + \frac{\left((7abB-3a^2C-4b^2C)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}}dx,x\right.}{7b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{\sqrt{2}(a+b)(7bB-3aC)\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+}}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{\sqrt{2}(7abB-3a^2C-4b^2C)\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c}}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] (verified)

Time = 3.43 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.02

$$\int\sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(-a^2+b^2)(7bB-3aC)\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx}{a+b}\right.\right.}{}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*(7*b*B - 3*a*C)
*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[
c + d*x]))/(a - b))] + (7*a*b*B - 3*a^2*C + 16*b^2*C)*AppellF1[4/3, 1/2, 1/
2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-(
(b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*(
a + b*Cos[c + d*x]) - 4*b^2*(7*b*B + a*C + 4*b*C*Cos[c + d*x])*Sin[c + d*x]
^2))/(112*b^3*d)
```

Maple [F]

$$\int (a+\cos(dx+c)\,b)^{\frac{1}{3}}\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

[In] int((a+cos(d*x+c)*b)^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)

[Out] int((a+cos(d*x+c)*b)^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)

Fricas [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

[In] integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm ="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3), x)

Sympy [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (B+C\cos(c+dx))\sqrt[3]{a+b\cos(c+dx)}\cos(c+dx)\,dx$$

[In] integrate((a+b*cos(d*x+c))**(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Integral((B + C*cos(c + d*x))*(a + b*cos(c + d*x))**(1/3)*cos(c + d*x), x)

Maxima [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

[In] integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3), x)

Giac [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

```
[In] int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3), x)
```

3.236 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 281

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}$$
$$+\frac{\sqrt{2}(5bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$
$$-\frac{\sqrt{2}(5abB-3a^2C-2b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\,\text{s}}{5b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

```
[Out] 3/5*C*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d+1/5*(5*B*b-3*C*a)*AppellF1(1/2,
-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2
/3)*sin(d*x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^
(1/2)-1/5*(5*B*a*b-3*C*a^2-2*C*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-cos(d*x+c
))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1
/2)/b^2/d/(a+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3102, 2835, 2744, 144, 143}

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx =$$
$$-\frac{\sqrt{2}(-3a^2C+5abB-2b^2C)\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{5b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}$$
$$+\frac{\sqrt{2}(5bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{5b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$
$$+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + (Sqrt[2]*(5*b*B - 3
*a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*(5*a*b*B - 3*
a^2*C - 2*b^2*C)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 -
 Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])
/(5*b^2*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{3\int \frac{\frac{2bC}{3}+\frac{1}{3}(5bB-3aC)\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx}{5b}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{(5bB-3aC)\int(a+b\cos(c+dx))^{2/3}\,dx}{5b^2} - \frac{(5abB-3a^2C-2b^2C)\int\frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx}{5b^2}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{((5bB-3aC)\sin(c+dx))\text{Subst}\left(\int\frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{5b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} + \frac{((5abB-3a^2C-2b^2C)\sin(c+dx))\,\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+bx}}\,dx,x,\cos(c+dx)\right)}{5b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{((5bB-3aC)(a+b\cos(c+dx))^{2/3}\sin(c+dx))\,\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}dx, x, \cos(c+dx)\right)}{5b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}} + \frac{\left((5abB-3a^2C-2b^2C)\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}\sin(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{-\frac{a}{-a-b}-\cdots}}\right.}{5b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{\sqrt{2}(5bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\,\text{s}\cdots}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} - \frac{\sqrt{2}(5abB-3a^2C-2b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c\cdots}{a+b}}}{5b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.36 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.94

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = -\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(-5abB+3a^2C+2b^2C)\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c\cdots}{a+b}\right.\right.}{}$$

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(-5*a*b*B + 3*a^2*C + 2*b^2*
C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
+ d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Co
s[c + d*x]))/(a - b))] + 2*(5*b*B - 3*a*C)*AppellF1[5/3, 1/2, 1/2, 8/3, (a
+ b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Co
s[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c +
 d*x]) - 10*b^2*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Maple [F]

$$\int \frac{B\cos(dx+c)+C(\cos^2(dx+c))}{(a+\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(1/3),x)

[Out] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(1/3),x)

Fricas [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm ="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3), x)

Sympy [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{(B+C\cos(c+dx))\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$$

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)

[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)/(a + b*cos(c + d*x))**(1/3), x)

Maxima [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3), x)

Giac [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm ="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

[In] int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3),x)

[Out] int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3), x)

3.237 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 281

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}+\frac{(4bB-3aC)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(4abB-3a^2C-b^2C)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\sin(c+dx)}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

```
[Out] 3/4*C*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d+1/4*(4*B*b-3*C*a)*AppellF1(1/2,
-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1
/3)*sin(d*x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)*2^(1/2)/(1+cos(d*x+c))^
(1/2)-1/4*(4*B*a*b-3*C*a^2-C*b^2)*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x+c))
/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c)/b^2/d/
(a+b*cos(d*x+c))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used

$= \{3102, 2835, 2744, 144, 143\}$

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx =$$

$$-\frac{(-3a^2C+4abB-b^2C)\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{2\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}$$
$$+\frac{(4bB-3aC)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{2\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((4*b*B - 3*a*C)*Ap
pellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a
 + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) - ((4*a*b*B - 3*a^2*C -
b^2*C)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(2*Sqrt[2
]*b^2*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd} + \frac{3\int \frac{\frac{bC}{3}+\frac{1}{3}(4bB-3aC)\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx}{4b} \\
&= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd} + \frac{(4bB-3aC)\int \sqrt[3]{a+b\cos(c+dx)}\,dx}{4b^2} \\
&\quad - \frac{(4abB-3a^2C-b^2C)\int \frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx}{4b^2} \\
&= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd} \\
&\quad - \frac{((4bB-3aC)\sin(c+dx))\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{4b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} \\
&\quad + \frac{((4abB-3a^2C-b^2C)\sin(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{2/3}}\,dx, x, \cos(c+dx)\right)}{4b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}
\end{aligned}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd} - \frac{\left((4bB-3aC)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}}dx, x, \cos(c+d\right.}{4b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}} + \frac{\left((4abB-3a^2C-b^2C)\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}dx, x\right.}{4b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd} + \frac{(4bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c-}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(4abB-3a^2C-b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\text{si}}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.93

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = -\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(-4abB+3a^2C+b^2C)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\right.}{}$$

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-4*a*b*B + 3*a^2*C + b^2*C)
*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[
c + d*x]))/(a - b))] + (4*b*B - 3*a*C)*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*
Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c
+ d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x
]) - 4*b^2*C*Sin[c + d*x]^2))/(16*b^3*d)
```

Maple [F]

$$\int \frac{B\cos(dx+c)+C(\cos^2(dx+c))}{(a+\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(2/3),x)
```

```
[Out] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{(B+C\cos(c+dx))\cos(c+dx)}{(a+b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)/(a + b*cos(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3), x
)
```

Giac [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm ="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

[In] int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3),x)

[Out] int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3), x)

3.238 $\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+f$

Optimal result

Integrand size = 31, antiderivative size = 187

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \frac{C(a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}$$
$$-\frac{(C(1+m)+A(2+m))(a\cos(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+fx}{af(1+m)(2+m)\sqrt{\sin^2(e+fx)}}$$
$$-\frac{B(a\cos(e+fx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{a^2f(2+m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] C*(a*cos(f*x+e))^(1+m)*sin(f*x+e)/a/f/(2+m)-(C*(1+m)+A*(2+m))*(a*cos(f*x+e)
)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/a/f
/(1+m)/(2+m)/(sin(f*x+e)^2)^(1/2)-B*(a*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+
1/2*m],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/a^2/f/(2+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3102, 2827, 2722}

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= -\frac{B\sin(e+fx)(a\cos(e+fx))^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(e+fx)\right)}{a^2f(m+2)\sqrt{\sin^2(e+fx)}}$$
$$-\frac{(A(m+2)+C(m+1))\sin(e+fx)(a\cos(e+fx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx}{af(m+1)(m+2)\sqrt{\sin^2(e+fx)}}$$
$$+\frac{C\sin(e+fx)(a\cos(e+fx))^{m+1}}{af(m+2)}$$

```
[In] Int[(a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (C*(a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) - ((C*(1 + m) + A*(
2 + m))*(a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, Cos[e + f*x]^2]*Sin[e + f*x])/(a*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x]^2])
 - (B*(a*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
 Cos[e + f*x]^2]*Sin[e + f*x])/(a^2*f*(2 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{C(a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} \\
&\quad + \frac{\int (a\cos(e+fx))^m(a(C(1+m)+A(2+m))+aB(2+m)\cos(e+fx))\,dx}{a(2+m)} \\
&= \frac{C(a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{B\int (a\cos(e+fx))^{1+m}\,dx}{a} \\
&\quad + \left(A+\frac{C(1+m)}{2+m}\right)\int (a\cos(e+fx))^m\,dx
\end{aligned}
$$

$$= \frac{C(a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} - \frac{\left(A+\frac{C(1+m)}{2+m}\right)(a\cos(e+fx))^{1+m}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{af(1+m)\sqrt{\sin^2(e+fx)}} - \frac{B(a\cos(e+fx))^{2+m}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{a^2f(2+m)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx = \frac{(a\cos(e+fx))^m\cot(e+fx)\left(-\left((C(1+m)+A(2+m))\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx\right.\right.\right.}{}$$

```
[In] Integrate[(a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] ((a*Cos[e + f*x])^m*Cot[e + f*x]*(-((C*(1 + m) + A*(2 + m))*Hypergeometric2
F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]) + (1 +
m)*(C*Sin[e + f*x]^2 - B*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4
+ m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])))/(f*(1 + m)*(2 + m))
```

Maple [F]

$$\int \left(\cos\left(fx+e\right)a\right)^m \left(A+\cos\left(fx+e\right)B+C\left(\cos^2\left(fx+e\right)\right)\right) dx$$

```
[In] int((cos(f*x+e)*a)^m*(A+cos(f*x+e)*B+C*cos(f*x+e)^2),x)
```

```
[Out] int((cos(f*x+e)*a)^m*(A+cos(f*x+e)*B+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx = \int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)+A\right)\left(a\cos\left(fx+e\right)\right)^m dx$$

```
[In] integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

Sympy [F]

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (a\cos{(e+fx)})^m \left(A+B\cos{(e+fx)}+C\cos^2{(e+fx)}\right)\,dx$$

```
[In] integrate((a*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)
[Out] Integral((a*cos(e + f*x))**m*(A + B*cos(e + f*x) + C*cos(e + f*x)**2), x)
```

Maxima [F]

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos{(fx+e)}^2+B\cos{(fx+e)}+A\right)(a\cos{(fx+e)})^m\,dx$$

```
[In] integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="ma
xima")
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos{(fx+e)}^2+B\cos{(fx+e)}+A\right)(a\cos{(fx+e)})^m\,dx$$

```
[In] integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="gi
ac")
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (a\,\cos{(e+f\,x)})^m\,\left(C\cos{(e+f\,x)}^2+B\,\cos{(e+f\,x)}+A\right)\,dx$$

```
[In] int((a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2),x)
[Out] int((a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

3.239 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C$

Optimal result

Integrand size = 41, antiderivative size = 209

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{10bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*B*(b*cos(d*x+c))^(5/
2)*sin(d*x+c)/b^2/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^3/d+10/21*b*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b*cos
(d*x+c))^(1/2)/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3102, 2827, 2715, 2721, 2719, 2720}

$$\begin{aligned}
&\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\
&=\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45bd}+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}\\
&+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^3d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}\\
&+\frac{10B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

```
[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[C
os[c + d*x]]) + (10*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d
*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) +
(2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d) + (2*B*(b*Cos[
c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c
 + d*x])/(9*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^2}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}+\frac{2\int (b\cos(c+dx))^{5/2}\left(\frac{1}{2}b(9A+7C)+\frac{9}{2}bB\cos(c+dx)\right)\,dx}{9b^3}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}+\frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^3}\\
&\quad+\frac{(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx}{9b^2}\\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd}\\
&\quad+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}\\
&\quad+\frac{(5B)\int (b\cos(c+dx))^{3/2}\,dx}{7b}+\frac{1}{15}(9A+7C)\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd}\\
&\quad+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}\\
&\quad+\frac{1}{21}(5bB)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx+\frac{\left((9A+7C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d} + \frac{\left(5bB\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{10bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.60

$$\int\cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$= \frac{\sqrt{b\cos(c+dx)}\left(84(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(7(36A+\right.}{630d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*Ell
ipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*x] +
 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))/(6
30*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 16.00 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.83

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\right.\right.\right.}}$

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-1120*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)
^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126
*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/
((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{-75i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-}{}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqr
t(2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(35*C*cos(d*x + c)^3 + 45*B*cos(d*x
 + c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x + c))*sin(d*x +
 c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(
d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(
d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.240 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C$

Optimal result

Integrand size = 39, antiderivative size = 180

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}$$
$$+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}$$
$$+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*
x+c)/b^2/d+2/21*b*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1
/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*B*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

$= \{16, 3102, 2827, 2715, 2721, 2720, 2719\}$

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d} + \frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d} + \frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd} + \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*S
qrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2
1*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (2*C*(b*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}+\frac{2\int (b\cos(c+dx))^{3/2}\left(\frac{1}{2}b(7A+5C)+\frac{7}{2}bB\cos(c+dx)\right)\,dx}{7b^2}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}+\frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^2} + \frac{(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx}{7b}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{1}{5}(3B)\int\sqrt{b\cos(c+dx)}\,dx+\frac{1}{21}(b(7A+5C))\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}+\frac{\left(b(7A+5C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}} + \frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}$$

$$= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}$$

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.62

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{3/2}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(70A+6\right.}{105bd\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d
*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*b*d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 14.39 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.95

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(240*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
```

```
*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2
*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{5\sqrt{2}(7i\,A+5i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-7i\,A-5i\,C)\sqrt{b}}{}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*B*sqrt(b)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 63*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*C*cos(d*x + c)^2 + 21*B*cos(d*x +
 c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(
d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(
d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2), x)
```

3.241 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+d$

Optimal result

Integrand size = 33, antiderivative size = 145

$$\begin{aligned}&\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\&=\frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\\&\quad+\frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}\end{aligned}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/3*b*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2/5*(5*A+
3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3102, 2827, 2721, 2719, 2715, 2720}

$$\begin{aligned}&\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\&=\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\\&\quad+\frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\end{aligned}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Co
s[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sq
rt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*
(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2\int\sqrt{b\cos(c+dx)}\left(\frac{1}{2}b(5A+3C)+\frac{5}{2}bB\cos(c+dx)\right)\,dx}{5b}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{B\int(b\cos(c+dx))^{3/2}\,dx}{b} + \frac{1}{5}(5A+3C)\int\sqrt{b\cos(c+dx)}\,dx$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{1}{3}(bB)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx + \frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{\left(bB\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{2\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5B+3C\,\text{co}\right.}{15d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Elli
pticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c +
 d*x]))/(15*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 11.84 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.19

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10B+}{}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2B\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\right.\right.\right.}}{}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(24*cos(1/
2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1
/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/
2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\,dx$$
$$=\frac{-5i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*
(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(3*C*cos(d*x + c) + 5*B)*sqrt(b*cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.242 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 112

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b(3A+C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 2/3*b*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x
+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {16, 3102, 2827, 2721, 2720, 2719}

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2b(3A+C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x],x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
 + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b
*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = b\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}b(3A+C)+\frac{3}{2}bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + B\int \sqrt{b\cos(c+dx)}\,dx + \frac{1}{3}(b(3A+C))\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left(b(3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \frac{b\left(6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx\right.}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x],x]
```

```
[Out] (b*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)]))/(3*d*Sqrt[b*Cos
[c + d*x]])
```

Maple [A] (verified)

Time = 9.82 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.53

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\right.\right.\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}+\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{}\right.\right.\right.}}{\sqrt{-b}}$

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.33

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(3i\,A+i\,C)\sqrt{b}\text{weierstra}}{}$$

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/d

Sympy [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \sqrt{b\cos\left(c+dx\right)}\left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)*(b*cos(d*x+c))**(1/2)
,x)
[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(
c + d*x), x)
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c), x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x
, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x),x)
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x), x)
```

3.243 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 109

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d
/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^
(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {16, 3100, 2827, 2721, 2720, 2719}

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = -\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^2,x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c +
 d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Co
s[c + d*x]]) + (2*A*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2\int\frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{2\int\frac{\frac{b^2B}{2}-\frac{1}{2}b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b}\\
&= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+(bB)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx+(-A+C)\int\sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{\left(bB\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left((-A+C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\frac{2b\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\sin(c+dx\right.}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^2,x]
```

```
[Out] (2*b*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*
x]])
```

Maple [A] (verified)

Time = 9.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.39

method	result
default	$\frac{2b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}}$
parts	$-\frac{2Ab\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(cos(d*x+c)*b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2*b*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)
/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.65

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\frac{-i\sqrt{2}B\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B\sqrt{b}\cos(dx+c)\,\mathrm{w}}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x
+ c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + sqrt(2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*co
s(d*x + c))*A*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2*(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2, x)
```

3.244 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 140

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx$$
$$= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/3*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b*B*sin(d*x+c)/d/(b*cos(d*x+c
))^(1/2)+2/3*b*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2
*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used

$= \{16, 3100, 2827, 2716, 2721, 2719, 2720\}$

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}$$
$$+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^3,x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*b*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2
*b*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b^3\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2}{3}\int \frac{\frac{3b^2B}{2}+\frac{1}{2}b^2(A+3C)\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\frac{1}{3}(b(A+3C))\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-B\int \sqrt{b\cos(c+dx)}\,dx + \frac{\left(b(A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$

$$= \frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\frac{2b\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(A+3B\cos(c\right.}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^3,x]
```

```
[Out] (2*b*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*x]
))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(176) = 352.

Time = 10.98 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.61

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12B\right.}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{d}{2}\right.}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(cos(d*x+c)*b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
```

```
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*
c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.42

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\frac{\sqrt{2}(-iA-3iC)\sqrt{b}\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(iA+3i}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(
2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*(3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3*(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^3, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3, x)
```

3.245 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 181

$$
\begin{aligned}
&\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx \\
&= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} \\
&\quad+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

```
[Out] 2/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^2*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(3/2)+2/5*b*(3*A+5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b*B*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$\begin{aligned}&\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx\\&=\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}\\&\quad-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}\\&\quad+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\end{aligned}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^4,x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[C
os[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*S
qrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2))
+ (2*b^2*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b*(3*A + 5*C)*Si
n[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^4\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx\\
&= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{1}{5}(2b)\int \frac{\frac{5b^2B}{2}+\frac{1}{2}b^2(3A+5C)\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx\\
&= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}\\
&\quad+\left(b^3B\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx+\frac{1}{5}\left(b^2(3A+5C)\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{1}{3}(bB)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx+\frac{1}{5}(-3A-5C)\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}\\
&\quad+\frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}+\frac{\left(bB\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left((-3A-5C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = -\frac{\sqrt{b\cos(c+dx)}\sec^2(c+dx)\left(6(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-10B\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}\right.\right.}{15d}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^4,x]
```

```
[Out] -1/15*(Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(6*(3*A + 5*C)*Cos[c + d*x]^(3/2
)*EllipticE[(c + d*x)/2, 2] - 10*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2
, 2] - 10*B*Sin[c + d*x] - 9*A*Sin[2*(c + d*x)] - 15*C*Sin[2*(c + d*x)] - 6
*A*Tan[c + d*x]))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. 2(209) = 418.

Time = 14.96 (sec) , antiderivative size = 801, normalized size of antiderivative = 4.43

method	result	size
parts	Expression too large to display	801
default	Expression too large to display	805

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(cos(d*x+c)*b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/5*A*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(
```

```
1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d-2/3*B*(-2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/
2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-
1)*b)^(1/2)/d-2*C*b*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin
(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/
2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*
c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.22

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\frac{-5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\sqrt{b}\cos(dx+}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*A + 5*I*C
)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4*(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^4, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4,x)
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4, x)
```

3.246 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 210

$$\begin{aligned}
&\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx \\
&= -\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} \\
&\quad+\frac{2b^3B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{6bB\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

```
[Out] 2/7*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*b^3*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(5/2)+2/21*b^2*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*b*B*s
in(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/21*b*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)-6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\begin{aligned}&\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx\\&=\frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}\\&\quad+\frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}\\&\quad+\frac{6bB\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}\end{aligned}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^5,x]
```

```
[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) + (2*b*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*
Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2))
 + (2*b^3*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^2*(5*A + 7*C)
*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b*B*Sin[c + d*x])/(5*d*Sq
rt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^5 \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}\left(2b^2\right)\int \frac{\frac{7b^2B}{2}+\frac{1}{2}b^2(5A+7C)\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} \\
&\quad+\left(b^4B\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx+\frac{1}{7}\left(b^3(5A+7C)\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^3B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&\quad+\frac{1}{5}\left(3b^2B\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\frac{1}{21}(b(5A+7C))\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^3B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&\quad+\frac{6bB\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{1}{5}(3B)\int \sqrt{b\cos(c+dx)}\,dx+\frac{\left(b(5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}
\end{aligned}$$

$$= \frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6bB\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}$$

$$= -\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6bB\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.68

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$

$$= \frac{2\sqrt{b\cos(c+dx)}\sec^3(c+dx)\left(-63B\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A+7C)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\right.}{}$$

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^5,x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(-63*B*Cos[c + d*x]^(5/2)*EllipticE[
(c + d*x)/2, 2] + 5*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2
] + 21*B*Sin[c + d*x] + 63*B*Cos[c + d*x]^2*Sin[c + d*x] + (25*A*Sin[2*(c +
 d*x)])/2 + (35*C*Sin[2*(c + d*x)])/2 + 15*A*Tan[c + d*x]))/(105*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(234) = 468$.

Time = 16.14 (sec) , antiderivative size = 726, normalized size of antiderivative = 3.46

method	result	size
default	Expression too large to display	726
parts	Expression too large to display	1001

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(cos(d*x+c)*b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(A*(-1/56*
cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2)))+1/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x
+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1
/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)+C*(-1/6*cos(1/2*d*x+1/2*c)/
b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2
*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b
)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.10

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx =$$

$$-\frac{5\sqrt{2}(5iA+7iC)\sqrt{b}\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-5i\ldots}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 63*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*B*sqrt(b)*cos
(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*cos(d*x + c)^2
+ 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5*(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^5\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^5, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^5\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^5}\,dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5, x)
```

3.247 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\right.$

Optimal result

Integrand size = 39, antiderivative size = 210

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{2b(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{10b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+c))^(5/2)
*sin(d*x+c)/b/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^2/d+10/21*b^2*B*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*b*B*sin(d*x+c)*(b*cos
(d*x+c))^(1/2)/d+2/15*b*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x
+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {16, 3102, 2827, 2715, 2721, 2719, 2720}

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d} + \frac{2b(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{10b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^2d} + \frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd} + \frac{10bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (2*b*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt
[Cos[c + d*x]]) + (10*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(
21*d*Sqrt[b*Cos[c + d*x]]) + (10*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21
*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*B*(b*
Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin
[c + d*x])/(9*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}+\frac{2\int (b\cos(c+dx))^{5/2}\left(\frac{1}{2}b(9A+7C)+\frac{9}{2}bB\cos(c+dx)\right)\,dx}{9b^2}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}+\frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^2}\\
&\quad+\frac{(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx}{9b}\\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}\\
&\quad+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}\\
&\quad+\frac{1}{7}(5B)\int (b\cos(c+dx))^{3/2}\,dx+\frac{1}{15}(b(9A+7C))\int \sqrt{b\cos(c+dx)}\,dx
\end{aligned}$$

$$= \frac{10bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d} + \frac{1}{21}(5b^2B)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx + \frac{\left(b(9A+7C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}dx}{15\sqrt{\cos(c+dx)}}$$

$$= \frac{2b(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{10bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d} + \frac{\left(5b^2B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{21\sqrt{b\cos(c+dx)}}$$

$$= \frac{2b(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{10b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}$$

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.61

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{(b\cos(c+dx))^{5/2}\left(84(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{}\right.}{}$$

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*x]
 + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))/
(630*b*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 15.45 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.83

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos\right.\right.}}$

```
[In] int(cos(d*x+c)*(cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-1120
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*
c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c
)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.93

$$\int\cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{-75i\sqrt{2}Bb^{\frac{3}{2}}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}Bb^{\frac{3}{2}}\,\text{w}}{}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 21*I*sqrt(2)*(9*A + 7*C)*b^(3/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt
(2)*(9*A + 7*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
```

```
os(d*x + c) - I*sin(d*x + c))) + 2*(35*C*b*cos(d*x + c)^3 + 45*B*b*cos(d*x
+ c)^2 + 7*(9*A + 7*C)*b*cos(d*x + c) + 75*B*b)*sqrt(b*cos(d*x + c))*sin(d*
x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right) \,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*co
s(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right) \,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*co
s(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2),x)
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2), x)
```

3.248 $\int (b\cos(c+dx))^{3/2} (A + B\cos(c+dx) + C\cos^2(c+d$

Optimal result

Integrand size = 33, antiderivative size = 181

$$\begin{aligned}\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)\right.&\\ \left.+C\cos^2(c+dx)\right)\,dx &= \frac{6bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}\\ &+\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}\\ &+\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}\\ &+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}\end{aligned}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+
c)/b/d+2/21*b^2*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2
)+2/21*b*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*b*B*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3102, 2827, 2715, 2721, 2720, 2719}

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21
*d*Sqrt[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*
x])/(21*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*C*(b*Cos[
c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} \\
&\quad + \frac{2\int (b\cos(c+dx))^{3/2}\left(\frac{1}{2}b(7A+5C)+\frac{7}{2}bB\cos(c+dx)\right)\,dx}{7b} \\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b} \\
&\quad + \frac{1}{7}(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx \\
&= \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} \\
&\quad + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} \\
&\quad + \frac{1}{5}(3bB)\int \sqrt{b\cos(c+dx)}\,dx + \frac{1}{21}\left(b^2(7A+5C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} \\
&\quad + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{\left(b^2(7A+5C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{\left(3bB\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{6bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.60

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{3/2}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\ldots}\right.}{105d\cos^{\frac{3}{2}}(c\ldots}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d
*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 15.25 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.95

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\ldots\right.}{\ldots}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ldots\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\ldots}}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(240*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1
/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.01

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{-5i\sqrt{2}(7A+5C)b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*B*b^(3/2)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6
3*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) + 2*(15*C*b*cos(d*x + c)^2 + 21*B*b*cos(d*x +
c) + 5*(7*A + 5*C)*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx) +C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x
)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx) +C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx) +C\cos^2(c+dx)\right) \, dx = \int {\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right) \,d x$$

```
[In] int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.249 $\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 146

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2b(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/3*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+2/3*b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2/5*b*(
5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

= {16, 3102, 2827, 2721, 2719, 2715, 2720}

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```

```
[Out] (2*b*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[
Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*
d*Sqrt[b*Cos[c + d*x]]) + (2*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) +
 (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}\\
&\quad+\frac{2}{5}\int \sqrt{b\cos(c+dx)}\left(\frac{1}{2}b(5A+3C)+\frac{5}{2}bB\cos(c+dx)\right)\,dx\\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}\\
&\quad+B\int (b\cos(c+dx))^{3/2}\,dx+\frac{1}{5}(b(5A+3C))\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}\\
&\quad+\frac{1}{3}\left(b^2B\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&\quad+\frac{\left(b(5A+3C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}\\
&= \frac{2b(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}\\
&\quad+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{\left(b^2B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}
\end{aligned}$$

$$= \frac{2b(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2b\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5\right.}{15d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (2*b*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*El
lipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c
 + d*x]))/(15*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 13.84 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.18

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10\right.}{}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d} - \frac{2B\sqrt{\left(2\left(\cos^2\left(\right.\right.\right.}}{}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(24*cos(
1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
```

```
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+
1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.13

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{-5i\sqrt{2}Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}Bb^{\frac{3}{2}}\text{weierstrassPInver}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*I*sqrt(2)*(5*A + 3*C)*b^(3/2)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(
5*A + 3*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(3*C*b*cos(d*x + c) + 5*B*b)*sqrt(b*cos(d*x +
 c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x), x)
```

3.250 $\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 116

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{2bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 2/3*b^2*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b*C*
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/c
os(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {16, 3102, 2827, 2721, 2720, 2719}

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2bC\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```

```
[Out] (2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (2*b^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sq
rt[b*Cos[c + d*x]]) + (2*b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = b^2 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}(2b)\int \frac{\frac{1}{2}b(3A+C)+\frac{3}{2}bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + (bB)\int \sqrt{b\cos(c+dx)}\,dx + \frac{1}{3}\left(b^2(3A+C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left(b^2(3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{b^2\left(6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2$$

$$3d\sqrt{b\cos(c+dx)}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (b^2*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)]))/(3*d*Sqrt[b*C
os[c + d*x]])
```

Maple [A] (verified)

Time = 9.86 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.46

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\right.\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}+\frac{2B\sqrt{\left(2\left(\cos^2\right.\right.}}{\sqrt{}}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/s
in(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.28

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{-i\sqrt{2}(3A+C)b^{\frac{3}{2}}\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+C)b^{\frac{3}{2}}\mathrm{we}}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*
B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*b*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{(b\,\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^2}\,dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2, x)
```

3.251 $\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 114

$$\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx = -\frac{2b(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)-2*b*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {16, 3100, 2827, 2721, 2720, 2719}

$$\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx = \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{2b(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] (-2*b*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c
 + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[
b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^3\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+2\int \frac{\frac{b^2B}{2}-\frac{1}{2}b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\left(b^2B\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx-(b(A-C))\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{\left(b^2B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}\\
&\quad-\frac{\left(b(A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{2b(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{2b^2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$
\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{2b^2\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\right)+B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\sin
$$

$$
d\sqrt{b\cos(c+dx)}
$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (2*b^2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c +
d*x]])
```

Maple [A] (verified)

Time = 9.71 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.30

method	result
default	$\frac{2b^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$
parts	$-\frac{2A\,b^2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\mathrm{s}\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/
2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.57

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{-i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}Bb^{\frac{3}{2}}\cos(d\ldots}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*(A - C)*b^(3/2)*cos(d*x + c
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + I*sqrt(2)*(A - C)*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x
 + c))*A*b*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3,x)
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3, x)
```

3.252 $\int (b\cos(c+dx))^{3/2} (A + B\cos(c+dx) + C\cos^2(c+dx)) \sec^4(c+dx)\, dx$

Optimal result

Integrand size = 41, antiderivative size = 145

$$\begin{aligned}\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = &-\frac{2bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} \\ &+\frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} \\ &+\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

```
[Out] 2/3*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b^2*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(1/2)+2/3*b^2*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/
2)-2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used

$= \{16, 3100, 2827, 2716, 2721, 2719, 2720\}$

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] (-2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*b^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*S
qrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
+ (2*b^2*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^4\int\frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx\\
&=\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{1}{3}(2b)\int\frac{\frac{3b^2B}{2}+\frac{1}{2}b^2(A+3C)\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx\\
&=\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\left(b^3B\right)\int\frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\frac{1}{3}\left(b^2(A+3C)\right)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&=\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\\
&\quad-(bB)\int\sqrt{b\cos(c+dx)}\,dx+\frac{\left(b^2(A+3C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\
&=\frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}\\
&\quad+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\
&=-\frac{2bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{2b^2\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(A\cdots\right.}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (2*b^2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*
x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(181) = 362.

Time = 11.60 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.50

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\cdots\right.}{\cdots}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx\cdots}{2}}\cdots\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\cdots}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d
*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
```

```
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/
2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)
/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.38

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{-i\sqrt{2}(A+3C)b^{\frac{3}{2}}\cos(dx+c)^2 \,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*C)*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(3/2)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B
*b^(3/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^2*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
) + 2*(3*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4, x)
```

3.253 $\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) \sec^5(c + dx)\, dx$

Optimal result

Integrand size = 41, antiderivative size = 186

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = -\frac{2b(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^3*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(3/2)+2/5*b^2*(3*A+5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*B*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2/5*b*(3*A+5*C)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{2b(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```

```
[Out] (-2*b*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt
[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3
*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/
2)) + (2*b^3*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^2*(3*A + 5
*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^5 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(2b^2\right)\int \frac{\frac{5b^2B}{2} + \frac{1}{2}b^2(3A+5C)\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} \\
&\quad + \left(b^4B\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx + \frac{1}{5}\left(b^3(3A+5C)\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{1}{3}\left(b^2B\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx - \frac{1}{5}(b(3A+5C))\int \sqrt{b\cos(c+dx)}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} \\
&\quad + \frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} + \frac{\left(b^2B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} \\
&\quad - \frac{\left(b(3A+5C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$= -\frac{2b(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.66

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx =$$

$$-\frac{(b\cos(c+dx))^{3/2}\sec^3(c+dx)\left(6(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-10B\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}\right.\right.}{15d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] -1/15*((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(6*(3*A + 5*C)*Cos[c + d*x]^(3
/2)*EllipticE[(c + d*x)/2, 2] - 10*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)
/2, 2] - 10*B*Sin[c + d*x] - 9*A*Sin[2*(c + d*x)] - 15*C*Sin[2*(c + d*x)] -
 6*A*Tan[c + d*x]))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(214) = 428.

Time = 15.01 (sec) , antiderivative size = 806, normalized size of antiderivative = 4.33

method	result	size
default	Expression too large to display	806
parts	Expression too large to display	806

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2
*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*
x+1/2*c)^4+120*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-60*C*(sin(1/2*d*x+
```

```
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-
120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/
2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.20

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{-5i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}Bb^{\frac{3}{2}}\cos}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(3*A + 5*C)*
b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(3*A + 5*C)*b^(3/2)*cos(d*x + c
)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*(3*A + 5*C)*b*cos(d*x + c)^2 + 5*B*b*cos(d*x + c) + 3*A*b
)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^5, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^5}\,dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5,x)
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5, x)
```

3.254 $\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 215

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = -\frac{6bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6b^2B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/7*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*b^4*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(5/2)+2/21*b^3*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*b^2*B
*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/21*b^2*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^4B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^2B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```

```
[Out] (-6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*b^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(2
1*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7
/2)) + (2*b^4*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^3*(5*A +
7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b^2*B*Sin[c + d*x])/(
5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^6 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(2b^3\right)\int \frac{\frac{7b^2B}{2} + \frac{1}{2}b^2(5A+7C)\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} \\
&\quad + \left(b^5B\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \frac{1}{7}\left(b^4(5A+7C)\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&\quad + \frac{1}{5}\left(3b^3B\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx + \frac{1}{21}\left(b^2(5A+7C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&\quad + \frac{6b^2B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{1}{5}(3bB)\int \sqrt{b\cos(c+dx)}\,dx + \frac{\left(b^2(5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,d}{21\sqrt{b\cos(c+dx)}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} \\
&+ \frac{2b^4B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&+ \frac{6b^2B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3bB\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}} \\
&= -\frac{6bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} \\
&+ \frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} \\
&+ \frac{2b^4B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6b^2B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$
\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{(b\cos(c+dx))^{3/2}\sec^5(c+dx)\left(-504B\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40(5A+7C)\cos^{\frac{7}{2}}(c+dx)\right.}{}
$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^5*(-504*B*Cos[c + d*x]^(7/2)*EllipticE
[(c + d*x)/2, 2] + 40*(5*A + 7*C)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2,
 2] + 2*(110*A + 70*C + 273*B*Cos[c + d*x] + 10*(5*A + 7*C)*Cos[2*(c + d*x)
] + 63*B*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. 2(239) = 478.

Time = 16.03 (sec) , antiderivative size = 728, normalized size of antiderivative = 3.39

method	result	size
default	Expression too large to display	728
parts	Expression too large to display	1006

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(
1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))+1/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d
*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin
(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)+C*(-1/6*cos(1/2*d*x+1/2*c
)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.09

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{-5i\sqrt{2}(5A+7C)b^{\frac{3}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\cdots}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(5*A + 7*C)*b^(3/2)*cos(d*x + c)^4*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(5*A + 7*C)*b^(3/2)*cos
(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*
I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 63*I*sqrt(2)*B*b^(3/2)*cos(d*x
+ c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(63*B*b*cos(d*x + c)^3 + 5*(5*A + 7*C)*b*cos(d*x + c)^2 +
 21*B*b*cos(d*x + c) + 15*A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**6,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^6\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^6, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^6\,dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\int \frac{(b\,\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^6}\,dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^6,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^6, x)
```

3.255 $\int (b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+d$

Optimal result

Integrand size = 33, antiderivative size = 212

$$\begin{aligned}&\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)\right.\\&\left.+C\cos^2(c+dx)\right)\,dx=\frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}\\&+\frac{10b^3B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}\\&+\frac{10b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}\\&+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}\end{aligned}$$

```
[Out] 2/45*b*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+c))^(5/
2)*sin(d*x+c)/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d+10/21*b^3*B*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*b^2*B*sin(d*x+c)*(b*cos
(d*x+c))^(1/2)/d+2/15*b^2*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3102, 2827, 2715, 2721, 2719, 2720}

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{2b^2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2b(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}+\frac{10b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sq
rt[Cos[c + d*x]]) + (10*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])
/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])
/(21*d) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2
*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (2*C*(b*Cos[c + d*x])^(7/2)
*Sin[c + d*x])/(9*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} \\
&\quad + \frac{2\int(b\cos(c+dx))^{5/2}\left(\frac{1}{2}b(9A+7C)+\frac{9}{2}bB\cos(c+dx)\right)\,dx}{9b} \\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{B\int(b\cos(c+dx))^{7/2}\,dx}{b} \\
&\quad + \frac{1}{9}(9A+7C)\int(b\cos(c+dx))^{5/2}\,dx \\
&= \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} \\
&\quad + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} \\
&\quad + \frac{1}{7}(5bB)\int(b\cos(c+dx))^{3/2}\,dx + \frac{1}{15}\left(b^2(9A+7C)\right)\int\sqrt{b\cos(c+dx)}\,dx \\
&= \frac{10b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} \\
&\quad + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} \\
&\quad + \frac{1}{21}\left(5b^3B\right)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx + \frac{\left(b^2(9A+7C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{10b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{\left(5b^3B\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}$$

$$= \frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{10b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.59

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{5/2}\left(84(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\cdots\right.}{\cdots}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*x]
 + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))/
(630*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 18.69 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.81

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\right.\right.}}$

[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,method=_RETURNVE RBOSE)

[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-1120 *C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2* c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2 *d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1 26*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d* x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/ 2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^ (1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1 /2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))) /(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.96

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{-75i\sqrt{2}Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}Bb^{\frac{5}{2}}\text{wei}}{}$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm ="fricas")

[Out] 1/315*(-75*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I* sin(d*x + c)) + 75*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqrt(2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4 , 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt (2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c os(d*x + c) - I*sin(d*x + c))) + 2*(35*C*b^2*cos(d*x + c)^3 + 45*B*b^2*cos(d*x + c)^2 + 7*(9*A + 7*C)*b^2*cos(d*x + c) + 75*B*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x
)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int (b\,\cos\left(c+d\,x\right))^{5/2}\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.256 $\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 183

$$\begin{aligned}
&\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{6b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}\\
&+\frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}\\
&+\frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}\\
&+\frac{2bB(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}
\end{aligned}$$

```
[Out] 2/5*b*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*
x+c)/d+2/21*b^3*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2
)+2/21*b^2*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*b^2*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {16, 3102, 2827, 2715, 2721, 2720, 2719}

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{6b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```

```
[Out] (6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*b^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(
21*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c
+ d*x])/(21*d) + (2*b*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*C*(
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}\\
&\quad+\frac{2}{7}\int (b\cos(c+dx))^{3/2}\left(\frac{1}{2}b(7A+5C)+\frac{7}{2}bB\cos(c+dx)\right)\,dx\\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}\\
&\quad+B\int (b\cos(c+dx))^{5/2}\,dx+\frac{1}{7}(b(7A+5C))\int (b\cos(c+dx))^{3/2}\,dx\\
&= \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}\\
&\quad+\frac{2bB(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}\\
&\quad+\frac{1}{5}\left(3b^2B\right)\int \sqrt{b\cos(c+dx)}\,dx+\frac{1}{21}\left(b^3(7A+5C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx
\end{aligned}
$$

$$= \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2bB(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{\left(b^3(7A+5C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{21\sqrt{b\cos(c+dx)}} + \frac{\left(3b^2B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}$$

$$= \frac{6b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2bB(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.60

$$\int(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{b(b\cos(c+dx))^{3/2}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\right.}{105d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (b*(b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)
*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c +
 d*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 22.06 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.93

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(240*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1
/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.03

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{-5i\sqrt{2}(7A+5C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(7A+5C}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(7*A + 5*C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(7*A + 5*C)*b^(5/2)*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*B*b^(5/2)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6
3*I*sqrt(2)*B*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) + 2*(15*C*b^2*cos(d*x + c)^2 + 21*B*b^2*cos(d*
x + c) + 5*(7*A + 5*C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x), x)
```

3.257 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 151

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*b*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/3*b^3*B*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2/5
*b^2*(5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used

$= \{16, 3102, 2827, 2721, 2719, 2715, 2720\}$

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{2b^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d} + \frac{2bC\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```

```
[Out] (2*b^2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqr
t[Cos[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(
3*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*
d) + (2*b*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx \\
&= \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} \\
&\quad + \frac{1}{5}(2b)\int \sqrt{b\cos(c+dx)}\left(\frac{1}{2}b(5A+3C)+\frac{5}{2}bB\cos(c+dx)\right)\,dx \\
&= \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} \\
&\quad + (bB)\int (b\cos(c+dx))^{3/2}\,dx + \frac{1}{5}\left(b^2(5A+3C)\right)\int \sqrt{b\cos(c+dx)}\,dx \\
&= \frac{2b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} \\
&\quad + \frac{1}{3}\left(b^3B\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&\quad + \frac{\left(b^2(5A+3C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}} \\
&= \frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} \\
&\quad + \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left(b^3B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{2b^2\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5\cdots\right.}{15d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*
EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin
[c + d*x]))/(15*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 24.78 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.11

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10\cdots\right.}{\cdots}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d} - \frac{2B\sqrt{\left(2\left(\cos^2\left(\cdots\right.\right.\right.}}{\cdots}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(24*cos(
1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
```

```
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+
1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.12

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{-5i\sqrt{2}Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}Bb^{\frac{5}{2}}\text{weierstrassPInver}}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*I*sqrt(2)*(5*A + 3*C)*b^(5/2)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(
5*A + 3*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(3*C*b^2*cos(d*x + c) + 5*B*b^2)*sqrt(b*cos(d
*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2,x)
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2, x)
```

3.258 $\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 120

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{2b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 2/3*b^3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*
C*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {16, 3102, 2827, 2721, 2720, 2719}

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] (2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*
x]]) + (2*b^3*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*
Sqrt[b*Cos[c + d*x]]) + (2*b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = b^3 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}(2b^2)\int \frac{\frac{1}{2}b(3A+C) + \frac{3}{2}bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + (b^2B)\int \sqrt{b\cos(c+dx)}\,dx + \frac{1}{3}(b^3(3A+C))\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left(b^3(3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{2(b\cos(c+dx))^{5/2}\left(3BE\left(\frac{1}{2}(c+dx)\middle|2\right)+(3A+C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sqrt{\cos(c+dx)}\sin\right.}{3d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(5/2)*(3*B*EllipticE[(c + d*x)/2, 2] + (3*A + C)*Ellipt
icF[(c + d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d*Cos[c + d*x]
^(5/2))
```

Maple [A] (verified)

Time = 67.32 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.38

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^{3}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\text{co}\right.\right.}{3\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}\right.\right.\right.}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^{3}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}+\frac{2B\sqrt{\left(2\left(\cos^{2}\left(\right.\right.\right.}}{\sqrt{}}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/s
in(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\sec^{3}(c+dx)\,dx=\frac{-i\sqrt{2}(3A+C)b^{\frac{5}{2}}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}(3A+C)b^{\frac{5}{2}}\,\text{we}}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*b^(5/2)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*
B*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*b^2*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3,x)
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3, x)
```

3.259 $\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 116

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = -\frac{2b^2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b^3*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)-2*b^2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {16, 3100, 2827, 2721, 2720, 2719}

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] (-2*b^2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos
[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqr
t[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b^4 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + (2b)\int \frac{\frac{b^2B}{2} - \frac{1}{2}b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \left(b^3B\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx - \left(b^2(A-C)\right)\int \sqrt{b\cos(c+dx)}\,dx$$

$$= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(b^3B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}} - \frac{\left(b^2(A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{2b^2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int (b\cos(c+dx))^{5/2}\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{2b^3\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right) + B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (2*b^3*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c +
d*x]])
```

Maple [A] (verified)

Time = 192.87 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.26

method	result
default	$\frac{2b^{3}\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$
parts	$-\frac{2A\,b^{3}\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(s\right.}\right.}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/
2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.56

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\sec^{4}(c+dx)\,dx=\frac{-i\sqrt{2}Bb^{\frac{5}{2}}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}Bb^{\frac{5}{2}}\cos(d\ldots}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*(A - C)*b^(5/2)*cos(d*x + c
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + I*sqrt(2)*(A - C)*b^(5/2)*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x
 + c))*A*b^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \frac{(b\,\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^4}\,dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4,x)
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4, x)
```

3.260 $\int (b\cos(c+dx))^{5/2} (A + B\cos(c + dx) + C\cos^2(c + dx)) \sec^5(c + dx)\, dx$

Optimal result

Integrand size = 41, antiderivative size = 147

$$\begin{aligned}\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \\ -\frac{2b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} \\ +\frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} \\ +\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

```
[Out] 2/3*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b^3*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(1/2)+2/3*b^3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/
2)-2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used

$= \{16, 3100, 2827, 2716, 2721, 2719, 2720\}$

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```

```
[Out] (-2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d
*x]]) + (2*b^3*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d
*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)
) + (2*b^3*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^5 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}\left(2b^2\right)\int \frac{\frac{3b^2B}{2} + \frac{1}{2}b^2(A+3C)\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \left(b^4B\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx + \frac{1}{3}\left(b^3(A+3C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} \\
&\quad - \left(b^2B\right)\int \sqrt{b\cos(c+dx)}\,dx + \frac{\left(b^3(A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} \\
&\quad + \frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{2b^3\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(A\right.}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] (2*b^3*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*
x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(183) = 366.

Time = 4.54 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.46

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2
*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-
3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+
1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/
2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.39

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{-i\sqrt{2}(A+3C)b^{\frac{5}{2}}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B
*b^(5/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^2*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
) + 2*(3*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^5, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^5\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^5}\,dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5, x)
```

3.261 $\int (b\cos(c+dx))^{5/2} (A + B\cos(c+dx) + C\cos^2(c+dx)) \sec^6(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 188

$$\int (b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^6(c+dx)\,dx = -\frac{2b^2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^4*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(3/2)+2/5*b^3*(3*A+5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b^3*B*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2/5*b^2*(3*A+5*C)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{2b^2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```

```
[Out] (-2*b^2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sq
rt[Cos[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(
5/2)) + (2*b^4*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^3*(3*A +
 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^6 \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx\\
&= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{1}{5}\left(2b^3\right)\int \frac{\frac{5b^2B}{2}+\frac{1}{2}b^2(3A+5C)\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx\\
&= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}\\
&\quad+\left(b^5B\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx+\frac{1}{5}\left(b^4(3A+5C)\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{1}{3}\left(b^3B\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx-\frac{1}{5}\left(b^2(3A+5C)\right)\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}\\
&\quad+\frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}+\frac{\left(b^3B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\
&\quad-\frac{\left(b^2(3A+5C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$= -\frac{2b^2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.64

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = -\frac{2b^4\left(3(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-5B\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-5B\sin(c+dx\right.}{15d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] (-2*b^4*(3*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - 5*B*C
os[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] - 5*B*Sin[c + d*x] - (9*A*Sin[2
*(c + d*x)])/2 - (15*C*Sin[2*(c + d*x)])/2 - 3*A*Tan[c + d*x]))/(15*d*(b*Co
s[c + d*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(216) = 432$.

Time = 6.37 (sec) , antiderivative size = 808, normalized size of antiderivative = 4.30

Expression too large to display

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] -2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4+120*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-60*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/
```

```
2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2-120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(
1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.22

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{-5i\sqrt{2}Bb^{\frac{5}{2}}\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}Bb^{\frac{5}{2}}\cos}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(3*A + 5*C)*
b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(3*A + 5*C)*b^(5/2)*cos(d*x + c
)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*(3*A + 5*C)*b^2*cos(d*x + c)^2 + 5*B*b^2*cos(d*x + c) + 3
*A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**6,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^6, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{5/2}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^6}\,dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^6,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^6, x)
```

3.262 $\int (b\cos(c+dx))^{5/2} (A + B\cos(c+dx) + C\cos^2(c+dx)) \sec^7(c+dx)\, dx$

Optimal result

Integrand size = 41, antiderivative size = 217

$$\int (b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^7(c+dx)\, dx = -\frac{6b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^5B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6b^3B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/7*A*b^6*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*b^5*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(5/2)+2/21*b^4*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*b^3*B
*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/21*b^3*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/c
os(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^5B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^3B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^7,x]
```

```
[Out] (-6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
 d*x]]) + (2*b^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^
(7/2)) + (2*b^5*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^4*(5*A
+ 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b^3*B*Sin[c + d*x])
/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^7 \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}\left(2b^4\right)\int \frac{\frac{7b^2B}{2}+\frac{1}{2}b^2(5A+7C)\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} \\
&\quad +\left(b^6B\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx+\frac{1}{7}\left(b^5(5A+7C)\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^5B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&\quad +\frac{1}{5}\left(3b^4B\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\frac{1}{21}\left(b^3(5A+7C)\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^5B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{6b^3B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} \\
&\quad -\frac{1}{5}\left(3b^2B\right)\int \sqrt{b\cos(c+dx)}\,dx+\frac{\left(b^3(5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}
\end{aligned}$$

$$= \frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^5B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6b^3B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3b^2B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}$$

$$= -\frac{6b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^5B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6b^3B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{(b\cos(c+dx))^{5/2}\sec^6(c+dx)\left(-504B\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40(5A+7C)\cos^{\frac{7}{2}}(c+dx)\right.}{}$$

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^7,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^6*(-504*B*Cos[c + d*x]^(7/2)*EllipticE
[(c + d*x)/2, 2] + 40*(5*A + 7*C)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2,
 2] + 2*(110*A + 70*C + 273*B*Cos[c + d*x] + 10*(5*A + 7*C)*Cos[2*(c + d*x)
] + 63*B*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. 2(241) = 482.

Time = 3137.82 (sec) , antiderivative size = 728, normalized size of antiderivative = 3.35

method	result	size
default	Expression too large to display	728
parts	Expression too large to display	1008

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(
1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))+1/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d
*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin
(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)+C*(-1/6*cos(1/2*d*x+1/2*c
)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.12

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{-5i\sqrt{2}(5A+7C)b^{\frac{5}{2}}\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{}}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7
,x, algorithm="fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(5*A + 7*C)*b^(5/2)*cos(d*x + c)^4*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(5*A + 7*C)*b^(5/2)*cos
(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*
I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 63*I*sqrt(2)*B*b^(5/2)*cos(d*x
+ c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(63*B*b^2*cos(d*x + c)^3 + 5*(5*A + 7*C)*b^2*cos(d*x + c)
^2 + 21*B*b^2*cos(d*x + c) + 15*A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(
d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**7,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^7\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^7, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^7\,dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7
,x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^7}\,dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^7,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^7, x)
```

3.263 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 214

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*B*(b*cos(d*x+c))^(
5/2)*sin(d*x+c)/b^3/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^4/d+10/21*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b*cos
(d*x+c))^(1/2)/b/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d
*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2715, 2721, 2719, 2720}

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^2d}+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$
$$+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^4d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}$$
$$+\frac{10B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21bd}+\frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c +
 d*x]],x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt
[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d
*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d)
+ (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^2*d) + (2*B*(b*
Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d) + (2*C*(b*Cos[c + d*x])^(7/2)*S
in[c + d*x])/(9*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^3}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}+\frac{2\int (b\cos(c+dx))^{5/2}\left(\frac{1}{2}b(9A+7C)+\frac{9}{2}bB\cos(c+dx)\right)\,dx}{9b^4}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}+\frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^4}\\
&\quad+\frac{(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx}{9b^3}\\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d}\\
&\quad+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}\\
&\quad+\frac{(5B)\int (b\cos(c+dx))^{3/2}\,dx}{7b^2}+\frac{(9A+7C)\int\sqrt{b\cos(c+dx)}\,dx}{15b}\\
&= \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d}\\
&\quad+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}\\
&\quad+\frac{1}{21}(5B)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx+\frac{\left((9A+7C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{15b\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{\left(5B\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{21\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.59

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{168(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+600B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(7(36A+43}{1260d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*C
os[c + d*x]],x]
```

```
[Out] (168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] + 5
*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)])/(
1260*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 14.62 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.78

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\right.\right.}}$

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A
-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((
2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{-75i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(\cdots}{}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqr
t(2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(35*C*cos(d*x + c)^3 + 45*B*cos(d*x
 + c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x + c))*sin(d*x +
 c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos
(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos
(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/2), x)
```

3.264 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 185

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b^3/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1
/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d+6/5*B*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*
cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used

$= \{16, 3102, 2827, 2715, 2721, 2720, 2719\}$

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21bd} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d} + \frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d} + \frac{6BE\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c +
 d*x]],x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*
x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*S
qrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2
1*b*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d) + (2*C*(b*Cos[
c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^2}\\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}+\frac{2\int (b\cos(c+dx))^{3/2}\left(\frac{1}{2}b(7A+5C)+\frac{7}{2}bB\cos(c+dx)\right)\,dx}{7b^3}\\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}+\frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^3}\\
&\quad+\frac{(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx}{7b^2}\\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd}\\
&\quad+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}\\
&\quad+\frac{(3B)\int\sqrt{b\cos(c+dx)}\,dx}{5b}+\frac{1}{21}(7A+5C)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}\\
&\quad+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}+\frac{\left((7A+5C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} \\
&\quad + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.58

$$
\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx
$$

$$
= \frac{\sqrt{\cos(c+dx)}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(70A+65C\right.}{105d\sqrt{b\cos(c+dx)}}
$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*C
os[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x]
+ 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 14.37 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.89

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\right.}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)bd}}$

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*co
```

```
s(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c
)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{5\sqrt{2}(7i\,A+5i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-7i\,A-5i\,C)\sqrt{b}\text{w}}{}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*B*sqrt(b)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 63*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*C*cos(d*x + c)^2 + 21*B*cos(d*x +
 c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos
(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos
(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/2), x)
```

3.265 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 150

$$\begin{aligned}
&\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}
\end{aligned}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d+2/5*(5*
A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {16, 3102, 2827, 2721, 2719, 2715, 2720}

$$\begin{aligned}
&\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d} \\
&\quad + \frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d
*x]],x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[
Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sq
rt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d) + (2*
C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\,dx}{b}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{2\int \sqrt{b\cos(c+dx)}\left(\frac{1}{2}b(5A+3C)+\frac{5}{2}bB\cos(c+dx)\right)\,dx}{5b^2}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b^2} + \frac{(5A+3C)\int \sqrt{b\cos(c+dx)}\,dx}{5b}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{1}{3}B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx + \frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.65

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5B+3C\cos\right.}{15bd\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos
[c + d*x]],x]
```

[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d*x]))/(15*b*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 12.44 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.11

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10B+\right.}{}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}{}$

[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

$$\int\frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{-5i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-}{}$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)

```
 - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*
(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(3*C*cos(d*x + c) + 5*B)*sqrt(b*cos(d*x
+ c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d
*x + c)), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d
*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(1/2),x)
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(1/2), x)
```

3.266 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 117

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(d*
x+c)*(b*cos(d*x+c))^(1/2)/b/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d
*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3102, 2827, 2721, 2720, 2719}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]
]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b
*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}+\frac{2\int\frac{\frac{1}{2}b(3A+C)+\frac{3}{2}bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{3b}\\
&=\frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}+\frac{B\int\sqrt{b\cos(c+dx)}\,dx}{b}+\frac{1}{3}(3A+C)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx
\end{aligned}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*d*Sqrt[b*Cos[c +
 d*x]])
```

Maple [A] (verified)

Time = 9.40 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.41

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}\cdots\right.\right.\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\cdots\right.\right.\right.}}$
parts	$\frac{2A\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\sqrt{2}\right)}{d\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}+\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\cdots\right.\right.\right.}}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(3i\,A+i\,C)\sqrt{b}\text{weierstr}}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt
(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx&=\frac{2\,C\,\sin(c+d\,x)\,\sqrt{b\,\cos(c+d\,x)}}{3\,b\,d}\\&\quad+\frac{2\,A\,\sqrt{\cos(c+d\,x)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos(c+d\,x)}}\\&\quad+\frac{2\,B\,\sqrt{\cos(c+d\,x)}\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos(c+d\,x)}}\\&\quad+\frac{2\,C\,\sqrt{\cos(c+d\,x)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d\,\sqrt{b\,\cos(c+d\,x)}}\end{aligned}$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*C*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2)
*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*B*cos(c + d*x
)^(1/2)*ellipticE(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*C*cos(
c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))
```

3.267 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 110

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*
cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1
/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 3100, 2827, 2721, 2720, 2719}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d
*x]],x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c
 + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{2\int \frac{\frac{b^2B}{2}-\frac{1}{2}b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}\\
&= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx-\frac{(A-C)\int \sqrt{b\cos(c+dx)}\,dx}{b}\\
&= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}\\
&\quad-\frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}}\\
&= -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}\\
&\quad+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.37 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.54

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{b\cos(c+dx)}(B+C\cos(c+dx)+A\sec(c+dx))\left(\frac{\csc(c)\left(-3(A-C)\cos(c-dx-\arctan(\tan(c)))\sec(c)-(A-C)\cos(c+dx\cdots\right.}{\sqrt{\mathrm{se}\cdots}}\right.}{\cdots}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos
[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*((Csc[c]*(-3*(A
 - C)*Cos[c - d*x - ArcTan[Tan[c]]]*Sec[c] - (A - C)*Cos[c + d*x + ArcTan[T
an[c]]]*Sec[c] + 2*((2*A - C)*Cos[d*x] - C*Cos[2*c + d*x])*Sqrt[Sec[c]^2]))
/Sqrt[Sec[c]^2] - 4*B*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[C
sc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + (2*(A - C)*HypergeometricPFQ[{-1/2, -1/4
}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]])/(S
qrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(b*d*(2*A + C + 2*B*Cos[
c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [A] (verified)

Time = 12.71 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.35

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}$
parts	$-\frac{2A\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\right.\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.66

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{-i\sqrt{2}B\sqrt{b}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B\sqrt{b}\cos(dx+c)\,\text{w}}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x
+ c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + sqrt(2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*co
s(d*x + c))*A*sin(d*x + c))/(b*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/2)
,x)
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/sqrt(b*cos(c
 + d*x)), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sec(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d
*x + c)), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sec(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x
, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d
*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \int \frac{C\cos{(c + d\,x)}^2 + B\,\cos{(c + d\,x)} + A}{\cos{(c + d\,x)}\,\sqrt{b\,\cos{(c + d\,x)}}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(1/2)), x)
```

3.268 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 139

$$\begin{aligned}\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\\&\quad+\frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

```
[Out] 2/3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/d/(b*cos(d*x+c))^(
1/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*B*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\begin{aligned}\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}\\&\quad+\frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}\end{aligned}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c +
 d*x]],x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x
]]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B
*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b^2\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3b^2B}{2}+\frac{1}{2}b^2(A+3C)\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx}{3b}$$
$$= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+(bB)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\frac{1}{3}(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b} + \frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$
$$= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}}$$
$$= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.73 (sec) , antiderivative size = 757, normalized size of antiderivative = 5.45

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\cos^3(c+dx)\left(C+B\sec(c+dx)+A\sec^2(c+dx)\right)\left(\frac{4B\csc(c)\sec(c)}{d}+\frac{4A\sec(c)\sec^2(c+dx)\sin(dx)}{3d}+\frac{4\sec(c)\sec(c+dx)(A}{3d}\right.}{\sqrt{b\cos(c+dx)}(2A+C+2B\cos(c+dx)+C\cos(2c+2dx))}$$
$$-\frac{4A\cos^{\frac{5}{2}}(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\left(C+B\sec(c+dx)+A\sec^2(c+dx)\right)\sec(}{3d\sqrt{b\cos(c+dx)}(2A+C}$$
$$-\frac{4C\cos^{\frac{5}{2}}(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\left(C+B\sec(c+dx)+A\sec^2(c+dx)\right)\sec(}{d\sqrt{b\cos(c+dx)}(2A+C}$$
$$+\frac{2B\cos^{\frac{5}{2}}(c+dx)\csc(c)\left(C+B\sec(c+dx)+A\sec^2(c+dx)\right)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(t\right.}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c}}\right.}{d\sqrt{b\cos(c+dx)}(2A+C+2B\cos}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*C
os[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((4*B*Csc[c]*Sec[c]
)/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]*(
A*Sin[c] + 3*B*Sin[d*x]))/(3*d)))/(Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[
c + d*x] + C*Cos[2*c + 2*d*x])) - (4*A*Cos[c + d*x]^(5/2)*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x]
 + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos
[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^(5/
2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*Cos[c + d*x]]*
(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (2*
B*Cos[c + d*x]^(5/2)*Csc[c]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((Hyper
geometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c
]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
+ Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d
*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(175) = 350$.

Time = 13.68 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.66

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right.}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin(}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d
*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/
2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)
/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.45

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}(-i\,A - 3i\,C)\sqrt{b}\cos{(dx+c)}^2\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)} + i\,\sin{(dx+c)}) + \sqrt{2}(i\,A + 3i}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(
2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*(3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*co
s(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(A + B\cos{(c + dx)} + C\cos^2{(c + dx)})\sec^2{(c + dx)}}{\sqrt{b\cos{(c + dx)}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2/sqrt(b*co
s(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(C\cos{(dx + c)}^2 + B\cos{(dx + c)} + A)\sec{(dx + c)}^2}{\sqrt{b\cos{(dx + c)}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos
(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \int \frac{\left(C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A\right)\sec\left(dx + c\right)^2}{\sqrt{b\cos\left(dx + c\right)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos
(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \int \frac{C\cos\left(c + d\,x\right)^2 + B\,\cos\left(c + d\,x\right) + A}{\cos\left(c + d\,x\right)^2\,\sqrt{b\,\cos\left(c + d\,x\right)}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(1/2)), x)
```

3.269 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 180

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*
x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}} + \frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c +
 d*x]],x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt
[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*S
qrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2))
+ (2*b*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[c
+ d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^3\int\frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx\\
&= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2}{5}\int\frac{\frac{5b^2B}{2}+\frac{1}{2}b^2(3A+5C)\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx\\
&= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}\\
&\quad+\left(b^2B\right)\int\frac{1}{(b\cos(c+dx))^{5/2}}\,dx+\frac{1}{5}(b(3A+5C))\int\frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{1}{3}B\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx-\frac{(3A+5C)\int\sqrt{b\cos(c+dx)}\,dx}{5b}\\
&= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}\\
&\quad+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\
&\quad-\frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+9A\sin(c+\right.}{15d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*C
os[c + d*x]],x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[b*Cos[c
 + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(208) = 416.

Time = 17.98 (sec) , antiderivative size = 802, normalized size of antiderivative = 4.46

method	result	size
parts	Expression too large to display	802
default	Expression too large to display	808

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/5*A*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
```

```
cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*si
n(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d-2/3*B*(-2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/
2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-
1)*b)^(1/2)/d-2*C*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1
/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*
c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)
^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.24

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{-5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\sqrt{b}\cos(dx+c}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*A + 5*I*C
)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos
(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos
(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^3\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(1/2)),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(1/2)), x)
```

3.270 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 209

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/7*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*b^2*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(5/2)+2/21*b*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*B*sin(d
*x+c)/d/(b*cos(d*x+c))^(1/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(
b*cos(d*x+c))^(1/2)-6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1
/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c +
 d*x]],x]
```

```
[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d
*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*
Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2))
 + (2*b^2*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*(5*A + 7*C)*S
in[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*d*Sqrt[b
*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^4 \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}(2b)\int \frac{\frac{7b^2B}{2}+\frac{1}{2}b^2(5A+7C)\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} \\
&\quad +\left(b^3B\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx+\frac{1}{7}\left(b^2(5A+7C)\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&\quad +\frac{1}{5}(3bB)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\frac{1}{21}(5A+7C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&\quad +\frac{6B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{(3B)\int \sqrt{b\cos(c+dx)}\,dx}{5b}+\frac{\left((5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}
\end{aligned}$$

$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}}$$

$$= -\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\left(-63B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+63B\sin(c+\right.}{105d\sqrt{b\,c}}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*C
os[c + d*x]],x]
```

```
[Out] (2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c +
 d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c + d
*x]^2*Tan[c + d*x]))/(105*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(233) = 466$.

Time = 20.22 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.48

method	result	size
default	Expression too large to display	727
parts	Expression too large to display	874

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(cos(d*x+c)*b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/56*co
s(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+
1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2)))+2/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1
/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2
*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*C*(-1/6*cos(1/2*d*x+1/2*c)/
b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2
*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b
)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.12

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx =$$

$$-\frac{5\sqrt{2}(5i\,A + 7i\,C)\sqrt{b}\cos{(dx+c)}^4\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)} + i\,\sin{(dx+c)}) + 5\sqrt{2}(-5}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 63*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*B*sqrt(b)*cos
(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*cos(d*x + c)^2
+ 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x
 + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sec\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^4/sqrt(b*cos
(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sec\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^4/sqrt(b*cos
(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^4\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x)
)^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x)
)^(1/2)), x)
```

3.271 $\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 217

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}}+\frac{10B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/7*B*(b*cos(d*x+c))^(
5/2)*sin(d*x+c)/b^4/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^5/d+10/21*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b*c
os(d*x+c))^(1/2)/b^2/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d
/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3102, 2827, 2715, 2721, 2719, 2720}

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^3d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^5d} + \frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d} + \frac{10B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d} + \frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sq
rt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21
*b*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b
^2*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^3*d) + (2
*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d) + (2*C*(b*Cos[c + d*x])^(
7/2)*Sin[c + d*x])/(9*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^4}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{2\int (b\cos(c+dx))^{5/2}\left(\frac{1}{2}b(9A+7C)+\frac{9}{2}bB\cos(c+dx)\right)\,dx}{9b^5}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^5} + \frac{(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx}{9b^4}$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{(5B)\int (b\cos(c+dx))^{3/2}\,dx}{7b^3} + \frac{(9A+7C)\int \sqrt{b\cos(c+dx)}\,dx}{15b^2}$$

$$= \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{(5B)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b} + \frac{\left((9A+7C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{15b^2\sqrt{\cos(c+dx)}}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{\left(5B\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{21b\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}$$

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.60

$$\int\frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{168(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+}{}$$

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] + 5
*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)])/(
1260*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 16.94 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.77

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}{\cdots}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots\right.}{5b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\cdots\right.\right.\right.}}$

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-1120*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)
^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126
*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/
((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-75i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx\cdots\right.}{\cdots}$$

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqr
t(2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(35*C*cos(d*x + c)^3 + 45*B*cos(d*x
 + c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x + c))*sin(d*x +
 c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3
/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\cos(c}{(b\cos(dx+c))^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x
 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\cos(c}{(b\cos(dx+c))^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x
 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^4\,\left(C\cos(c+d\,x)^2+B\,\cos(c+c\right.}{(b\,\cos(c+d\,x))^{3/2}}$$

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2),x)
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2), x)
```

3.272 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 188

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}$$
$$+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}}$$
$$+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}$$
$$+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b^4/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^
(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d+6/5*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used

$= \{16, 3102, 2827, 2715, 2721, 2720, 2719\}$

$$\int \frac{\cos^3(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d} + \frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d} + \frac{6BE\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
d*x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b
*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]
)/(21*b^2*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d) + (2*C*(
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^3}\\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}+\frac{2\int (b\cos(c+dx))^{3/2}\left(\frac{1}{2}b(7A+5C)+\frac{7}{2}bB\cos(c+dx)\right)\,dx}{7b^4}\\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}+\frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^4}\\
&\quad+\frac{(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx}{7b^3}\\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}\\
&\quad+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}\\
&\quad+\frac{(3B)\int \sqrt{b\cos(c+dx)}\,dx}{5b^2}+\frac{(7A+5C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b}\\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}\\
&\quad+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}+\frac{\left((7A+5C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}$$

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.57

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+\right.}{}$$

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x]
+ 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 15.41 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.88

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\,c}}$

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(240*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
```

```
+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2
*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{5\sqrt{2}(7i\,A+5i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\sqrt{2}(-7i\,A-5i\,C)\sqrt{b}\text{weie}}{}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*B*sqrt(b)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 63*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*C*cos(d*x + c)^2 + 21*B*cos(d*x +
 c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(c\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(c\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}$$

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2),x)
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2), x)
```

3.273 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 153

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d+2/5
*(5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3102, 2827, 2721, 2719, 2715, 2720}

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqr
t[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*
d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
 + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\,dx}{b^2}$$
$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{2\int \sqrt{b\cos(c+dx)}\left(\frac{1}{2}b(5A+3C)+\frac{5}{2}bB\cos(c+dx)\right)\,dx}{5b^3}$$
$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b^3} + \frac{(5A+3C)\int \sqrt{b\cos(c+dx)}\,dx}{5b^2}$$
$$= \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b} + \frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}}$$
$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}}$$
$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\cos^{\frac{3}{2}}(c+dx)\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5\right.}{}$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Ellipt
icF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d
*x]))/(15*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 12.84 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.08

method	result
default	$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10B+\right.$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}{}$

[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,met
hod=_RETURNVERBOSE)

[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(24*cos(1/
2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1
/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/
2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{-5i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx\ldots}{}$$

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")

[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*
(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(3*C*cos(d*x + c) + 5*B)*sqrt(b*cos(d*x
+ c))*sin(d*x + c))/(b^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3
/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}$$

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2),x)
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2), x)
```

3.274 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 120

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d
/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 3102, 2827, 2721, 2720, 2719}

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}$$

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])
^(3/2),x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*
x]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sq
rt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2\int \frac{\frac{1}{2}b(3A+C)+\frac{3}{2}bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}$$

$$
=\frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}+\frac{B\int\sqrt{b\cos(c+dx)}\,dx}{b^2}+\frac{(3A+C)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b}
$$

$$
=\frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}+\frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}}
+\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}}
$$

$$
=\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}
+\frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}}
+\frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$
\int\frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cdots}}{3bd\sqrt{\cdots}}
$$

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(3/2),x]
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*b*d*Sqrt[b*Cos[c
 + d*x]])
```

Maple [A] (verified)

Time = 10.71 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.38

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\cdots\right.\right.\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}\cdots\right.\right.\right.}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}+\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{d}{\cdots}\right.\right.\right.}}{b\sqrt{-\cdots}}$

[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,metho
d=_RETURNVERBOSE)

[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(4*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{\cos(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x
, algorithm="fricas")

[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt
(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/(b^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2)
,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}$$

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(3/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(3/2), x)
```

3.275 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 116

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d
/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x
+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3100, 2827, 2721, 2720, 2719}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos
[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{2\int \frac{\frac{b^2B}{2}-\frac{1}{2}b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^3}\\ &= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b} - \frac{(A-C)\int \sqrt{b\cos(c+dx)}\,dx}{b^2}\end{aligned}$$

$$= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b\sqrt{b\cos(c+dx)}} - \frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+B\sqrt{\cos(c+dx}\cdots}{bd\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b*d*Sqrt[b*Cos[c + d*
x]])
```

Maple [A] (verified)

Time = 11.63 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.26

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.\right.}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots\right.}}$
parts	$-\frac{2A\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\cdots\right.\right.}\right.}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\cdots}}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/b*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
```

```
,2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)
/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.58

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}B\sqrt{b}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x
+ c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + sqrt(2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*co
s(d*x + c))*A*sin(d*x + c))/(b^2*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x
)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}+A}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{C\cos{(c+d\,x)}^2+B\,\cos{(c+d\,x)}+A}{{(b\,\cos{(c+d\,x)})}^{3/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2), x)
```

3.276 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 144

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/b/d/(b*cos(d*x+c))^(
1/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-2*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} - \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])
^(3/2),x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d
*x]]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*S
qrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2
*B*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2\int \frac{\frac{3b^2B}{2}+\frac{1}{2}b^2(A+3C)\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx}{3b^2} \\
&= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+B\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b} \\
&= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b^2} \\
&\quad+\frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}} \\
&= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} \\
&\quad+\frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}} \\
&= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} \\
&\quad+\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.17 (sec) , antiderivative size = 761, normalized size of antiderivative = 5.28

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^3(c+dx)(C+B\sec(c+dx)+A\sec^2(c+dx))\left(\frac{4B\csc(c)\sec(c)}{d}+\frac{4A\sec}{}\right.}{\sqrt{b\cos(c+dx)}(2A+C+2B\cos(c}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] ((Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((4*B*Csc[c]*Sec[c
])/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]*
(A*Sin[c] + 3*B*Sin[d*x]))/(3*d)))/(Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos
[c + d*x] + C*Cos[2*c + 2*d*x])) - (4*A*Cos[c + d*x]^(5/2)*Csc[c]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x
] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Co
s[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^(5
/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*Cos[c + d*x]]
*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (2
*B*Cos[c + d*x]^(5/2)*Csc[c]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])))/
b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(180) = 360$.

Time = 13.45 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.53

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin(}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2
*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-
3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+
1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/
2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-iA-3iC)\sqrt{b}\cos(dx+c)^2\ \text{weierstrassPIn}}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x
, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
```

```
c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(
2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*(3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*
cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos{(c+dx)}+C\cos^2{(c+dx)})\sec{(c+}}{(b\cos{(c+dx)})^{\frac{3}{2}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(3/2)
,x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d
*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A\right)\sec{(dx}}{(b\cos{(dx+c)})^{\frac{3}{2}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A\right)\sec{(dx}}{(b\cos{(dx+c)})^{\frac{3}{2}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(3/2)),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(3/2)), x)
```

3.277 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 183

$$\begin{aligned}
&\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \\
&-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} \\
&+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} \\
&+\frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}
\end{aligned}$$

```
[Out] 2/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/d/(b*cos(d*x+c))
^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*
x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sq
rt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b
*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)
) + (2*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[c
+ d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2\int \frac{\frac{5b^2B}{2}+\frac{1}{2}b^2(3A+5C)\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx}{5b} \\
&= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + (bB)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx + \frac{1}{5}(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b} - \frac{(3A+5C)\int \sqrt{b\cos(c+dx)}\,dx}{5b^2} \\
&= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} \\
&\quad + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}} \\
&\quad - \frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.65

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b*d*Sqrt[b*Cos
[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(211) = 422$.

Time = 18.52 (sec) , antiderivative size = 808, normalized size of antiderivative = 4.42

method	result	size
default	Expression too large to display	808
parts	Expression too large to display	808

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(3/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4+120*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-60*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
```

```
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2-120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(
1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.22

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-5i\,\sqrt{2}B\sqrt{b}\cos{(dx+c)}^3\,\text{weierstrassPInverse}(-4$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*A + 5*I*C
)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos{(c+dx)}+C\cos^2{(c+dx)})\sec^2{(c}}{(b\cos{(c+dx)})^{\frac{3}{2}}}$$

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)

[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A\right)\sec{(d}}{(b\cos{(dx+c)})^{\frac{3}{2}}}$$

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A\right)\sec{(d}}{(b\cos{(dx+c)})^{\frac{3}{2}}}$$

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + B\,\cos\left(c + d\,x\right) + A}{\cos\left(c + d\,x\right)^2\,\left(b\,\cos\left(c + d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(3/2)), x)
```

3.278 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 212

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2bB\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6B\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*b*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(5/2)+2/21*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*B*sin(d*x+c
)/b/d/(b*cos(d*x+c))^(1/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(
b*cos(d*x+c))^(1/2)-6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^
(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} - \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2bB\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6B\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
 d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*
b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7
/2)) + (2*b*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(5*A + 7*C)*S
in[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*b*d*Sqrt
[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^3 \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2}{7}\int \frac{\frac{7b^2B}{2}+\frac{1}{2}b^2(5A+7C)\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} \\
&\quad + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \frac{1}{7}(b(5A+7C))\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2bB\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&\quad + \frac{1}{5}(3B)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx + \frac{(5A+7C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b} \\
&= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2bB\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} \\
&\quad + \frac{6B\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} - \frac{(3B)\int \sqrt{b\cos(c+dx)}\,dx}{5b^2} \\
&\quad + \frac{\left((5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2bB\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6B\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} - \frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}}$$

$$= -\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2bB\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6B\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(-63B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A+\right.}{}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c +
 d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c + d
*x]^2*Tan[c + d*x]))/(105*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. 2(236) = 472.

Time = 19.62 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	730
parts	Expression too large to display	1008

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(3/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(2*A*(-1/56*
cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2)))+2/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x
+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1
/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*C*(-1/6*cos(1/2*d*x+1/2*c
)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.10

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx =$$

$$-\frac{5\sqrt{2}(5i\,A + 7i\,C)\sqrt{b}\cos{(dx + c)}^4 \,\text{weierstrassPInverse}(-4, 0, \cos{(dx + c)} + i\,\sin{(dx + c)}) + 5\sqrt{2}(-5i\,A}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 63*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*B*sqrt(b)*cos
(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*cos(d*x + c)^2
+ 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d
*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(3
/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x
 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x
 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^3\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(3/2)),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(3/2)), x)
```

3.279 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 217

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}}+\frac{10B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*B*(b*cos(d*x+c))^(
5/2)*sin(d*x+c)/b^5/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^6/d+10/21*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b
*cos(d*x+c))^(1/2)/b^3/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3
/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2715, 2721, 2719, 2720}

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^4d}$$
$$+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^6d}$$
$$+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}+\frac{10B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d}$$
$$+\frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sq
rt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21
*b^2*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21
*b^3*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^4*d) +
(2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d) + (2*C*(b*Cos[c + d*x])
^(7/2)*Sin[c + d*x])/(9*b^6*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^5}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}+\frac{2\int (b\cos(c+dx))^{5/2}\left(\frac{1}{2}b(9A+7C)+\frac{9}{2}bB\cos(c+dx)\right)\,dx}{9b^6}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}+\frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^6}\\
&\quad+\frac{(9A+7C)\int (b\cos(c+dx))^{5/2}\,dx}{9b^5}\\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d}\\
&\quad+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}\\
&\quad+\frac{(5B)\int (b\cos(c+dx))^{3/2}\,dx}{7b^4}+\frac{(9A+7C)\int \sqrt{b\cos(c+dx)}\,dx}{15b^3}\\
&= \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d}\\
&\quad+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}\\
&\quad+\frac{(5B)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b^2}+\frac{\left((9A+7C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{15b^3\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{\left(5B\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}$$

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.60

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{168(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+60$$

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] + 5
*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)])/(
1260*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 15.92 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.77

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{5b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos\right.\right.}}$

```
[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-1120
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*
c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c
)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-75i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx\right.}{}$$

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqr
t(2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(35*C*cos(d*x + c)^3 + 45*B*cos(d*x
 + c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x + c))*sin(d*x +
 c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5
/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x
 + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x
 + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^5\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}$$

```
[In] int((cos(c + d*x)^5*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2),x)
[Out] int((cos(c + d*x)^5*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2), x)
```

3.280 $\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 188

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b^5/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c)
)^(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+6/5*B*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used

$= \{16, 3102, 2827, 2715, 2721, 2720, 2719\}$

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d} + \frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d} + \frac{6BE\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
d*x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b
^2*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*
x])/(21*b^3*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d) + (2*C
*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^4}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}+\frac{2\int (b\cos(c+dx))^{3/2}\left(\frac{1}{2}b(7A+5C)+\frac{7}{2}bB\cos(c+dx)\right)\,dx}{7b^5}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}+\frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^5}+\frac{(7A+5C)\int (b\cos(c+dx))^{3/2}\,dx}{7b^4}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}+\frac{(3B)\int \sqrt{b\cos(c+dx)}\,dx}{5b^3}+\frac{(7A+5C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b^2}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}+\frac{\left((7A+5C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^2\sqrt{b\cos(c+dx)}}+\frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}}$$

$$= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.59

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\Big(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+$$

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x]
+ 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 14.90 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.88

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(1}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2}}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(240*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
```

```
*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1
/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{5\sqrt{2}(7i\,A+5i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\sqrt{2}(-7i\,A-5i\,C)\sqrt{b}\text{we}}{}$$

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*B*sqrt(b)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 63*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*C*cos(d*x + c)^2 + 21*B*cos(d*x +
 c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos(dx}{(b\cos(dx+c))^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x
 + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos(dx}{(b\cos(dx+c))^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x
 + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^4\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,}{(b\,\cos(c+d\,x))^{5/2}}$$

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2), x)
```

3.281 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 153

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+2
/5*(5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3102, 2827, 2721, 2719, 2715, 2720}

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqr
t[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^
2*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*
d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\,dx}{b^3}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{2\int \sqrt{b\cos(c+dx)}\left(\frac{1}{2}b(5A+3C)+\frac{5}{2}bB\cos(c+dx)\right)\,dx}{5b^4}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b^4} + \frac{(5A+3C)\int \sqrt{b\cos(c+dx)}\,dx}{5b^3}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2} + \frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63

$$\int \frac{\cos^3(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\Big(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+$$

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Ellipt
icF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d
*x]))/(15*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 13.12 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.08

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)C\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10B+6}{}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d} - \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}{}$

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(24*cos(
1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+
1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-5i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+\right.}{}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*
(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(3*C*cos(d*x + c) + 5*B)*sqrt(b*cos(d*x
+ c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5
/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(c\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(c\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}$$

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2),x)
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2), x)
```

3.282 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 120

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*C*si
n(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3
/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {16, 3102, 2827, 2721, 2720, 2719}

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}$$

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*
x]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*
Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2\int \frac{\frac{1}{2}b(3A+C)+\frac{3}{2}bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{3b^3}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{B\int\sqrt{b\cos(c+dx)}\,dx}{b^3} + \frac{(3A+C)\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{b^3\sqrt{\cos(c+dx)}}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int\frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\ }\cdots}{3b^2d\sqrt{\ }\cdots}$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*b^2*d*Sqrt[b*Cos
[c + d*x]])
```

Maple [A] (verified)

Time = 10.99 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.38

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}\cdots\right.\right.\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\cdots\right.\right.\right.}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}+\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\cdots\right.\right.\right.}}{b^2\sqrt{-b\cdots}}$

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/s
in(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cdots}{\cdots}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt
(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}$$

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2), x)
```

3.283 $\int \frac{\cos(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 116

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b
^2/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos
(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 3100, 2827, 2721, 2720, 2719}

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])
^(5/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos
[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqr
t[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx}{b}$$

$$= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{2\int \frac{\frac{b^2B}{2}-\frac{1}{2}b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^4}$$

$$= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b^2} - \frac{(A-C)\int \sqrt{b\cos(c+dx)}\,dx}{b^3}$$

$$= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2\sqrt{b\cos(c+dx)}} - \frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^3\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.58 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.41

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\cos(c+dx)}(B+C\cos(c+dx)+A\sec(c+d}{}$$

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*((Csc[c]*(-3*(A
 - C)*Cos[c - d*x - ArcTan[Tan[c]]]*Sec[c] - (A - C)*Cos[c + d*x + ArcTan[T
an[c]]]*Sec[c] + 2*((2*A - C)*Cos[d*x] - C*Cos[2*c + d*x])*Sqrt[Sec[c]^2]))
/Sqrt[Sec[c]^2] - 4*B*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[C
sc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + (2*(A - C)*HypergeometricPFQ[{-1/2, -1/4
}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]])/(S
qrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(b^3*d*(2*A + C + 2*B*Co
s[c + d*x] + C*Cos[2*(c + d*x)]))
```

Maple [A] (verified)

Time = 11.40 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.26

method	result
default	$\frac{2\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))b+b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(2A\cos(\frac{dx}{2}+\frac{c}{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,E(\cos(\frac{dx}{2}+\frac{c}{2}}{b^2\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})}$
parts	$-\frac{2A(-2\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))b+b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(\sin^2(\frac{dx}{2}+\frac{c}{2}))+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{-2(\sin^4(}{b^2\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}}$

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/
2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.58

$$\int \frac{\cos(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}B\sqrt{b}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0}{}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x
+ c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + sqrt(2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*co
s(d*x + c))*A*sin(d*x + c))/(b^3*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2)
,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(5/2),x)
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(5/2), x)
```

3.284 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 147

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/b^2/d/(b*cos(d*x+c
))^(1/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
-2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3100, 2827, 2716, 2721, 2719, 2720}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} - \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d
*x]]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d
*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
+ (2*B*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3b^2B}{2}+\frac{1}{2}b^2(A+3C)\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx}{3b^3}$$
$$= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{B\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{b} + \frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}$$
$$= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b^3} + \frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^3\sqrt{\cos(c+dx)}}$$
$$= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx}\right.}{3b^2d\sqrt{b\cos}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*x]))
/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(183) = 366$.

Time = 13.72 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.46

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin(}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2
*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-
3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+
1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/
2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.37

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}(-i\,A - 3i\,C)\sqrt{b}\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\text{c}}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
```

```
c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(
2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*(3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*
cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x
)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos{(c+d\,x)}^2+B\,\cos{(c+d\,x)}+A}{{(b\,\cos{(c+d\,x)})}^{5/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2), x)
```

3.285 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 185

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/b/d/(b*cos(d*x+c))
^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*co
s(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])
^(5/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sq
rt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b
^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)
) + (2*B*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[
c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx\\
&= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2\int \frac{\frac{5b^2B}{2}+\frac{1}{2}b^2(3A+5C)\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx}{5b^2}\\
&= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+B\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx+\frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b}\\
&= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}-\frac{(3A+5C)\int \sqrt{b\cos(c+dx)}\,dx}{5b^3}\\
&= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\\
&\quad+\frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}\\
&\quad-\frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\right.}{}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b^2*d*Sqrt[b*C
os[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. 2(213) = 426.

Time = 19.29 (sec) , antiderivative size = 808, normalized size of antiderivative = 4.37

method	result	size
default	Expression too large to display	808
parts	Expression too large to display	808

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4+120*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-60*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/
```

```
2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2-120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(
1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.21

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \frac{-5i\sqrt{2}B\sqrt{b}\cos(dx + c)^3 \,\text{weierstrassPInverse}(-4$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*A + 5*I*C
)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(5/2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(5/2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(5/2)), x)
```

3.286 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 212

$$\begin{aligned}
&\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \\
&-\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} \\
&+\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} \\
&+\frac{2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}+\frac{6B\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

```
[Out] 2/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*B*sin(d*x+c)/d/(b*cos(d*x+c))
^(5/2)+2/21*(5*A+7*C)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+6/5*B*sin(d*x+c)/
b^2/d/(b*cos(d*x+c))^(1/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d
/(b*cos(d*x+c))^(1/2)-6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c
)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{21b^2d\sqrt{b\cos(c+dx)}}$$
$$+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}+\frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}$$
$$-\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{6B\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
 d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*
b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7
/2)) + (2*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(5*A + 7*C)*Sin
[c + d*x])/(21*b*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*b^2*d*Sq
rt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2\int \frac{\frac{7b^2B}{2}+\frac{1}{2}b^2(5A+7C)\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx}{7b} \\
&= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+(bB)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx+\frac{1}{7}(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} \\
&\quad+\frac{(3B)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b}+\frac{(5A+7C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b^2} \\
&= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} \\
&\quad+\frac{6B\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}-\frac{(3B)\int \sqrt{b\cos(c+dx)}\,dx}{5b^3} \\
&\quad+\frac{\left((5A+7C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^2\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{6B\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} - \frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}}$$

$$= -\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{6B\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-63B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A\right.}{}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c +
 d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c + d
*x]^2*Tan[c + d*x]))/(105*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. 2(236) = 472.

Time = 19.69 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	730
parts	Expression too large to display	1008

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(5/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(2*A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(
1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))+2/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d
*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin
(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*C*(-1/6*cos(1/2*d*x+1/2
*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x
+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-
1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.10

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx =$$
$$-\frac{5\sqrt{2}(5iA+7iC)\sqrt{b}\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-5iA-$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 63*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*B*sqrt(b)*cos
(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*cos(d*x + c)^2
+ 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d
*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(5
/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec (d}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec (d}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(5/2)),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(5/2)), x)
```

3.287 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 188

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^3d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/b^2/d/(b*cos(d*x
+c))^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^3/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)/b^3/d/(b*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(
b*cos(d*x+c))^(1/2)/b^4/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{3100, 2827, 2716, 2721, 2720, 2719\}$

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\,dx = -\frac{2(3A + 5C)E\left(\frac{1}{2}(c + dx)\middle|\,2\right)\sqrt{b\cos(c + dx)}}{5b^4d\sqrt{\cos(c + dx)}} + \frac{2(3A + 5C)\sin(c + dx)}{5b^3d\sqrt{b\cos(c + dx)}} + \frac{2A\sin(c + dx)}{5bd(b\cos(c + dx))^{5/2}} + \frac{2B\sqrt{\cos(c + dx)}\,\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3b^3d\sqrt{b\cos(c + dx)}} + \frac{2B\sin(c + dx)}{3b^2d(b\cos(c + dx))^{3/2}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sq
rt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b
^3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/
2)) + (2*B*Sin[c + d*x])/(3*b^2*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*
Sin[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2\int \frac{\frac{5b^2B}{2}+\frac{1}{2}b^2(3A+5C)\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx}{5b^3} \\
&= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx}{b} + \frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b^2} \\
&= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^3} - \frac{(3A+5C)\int \sqrt{b\cos(c+dx)}\,dx}{5b^4} \\
&= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} \\
&\quad + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^3\sqrt{b\cos(c+dx)}} \\
&\quad - \frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^4\sqrt{\cos(c+dx)}} \\
&= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^3d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.63

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2\Big(-3(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sqrt{\cos(c+$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b^3*d*Sqrt[b*C
os[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. 2(216) = 432.

Time = 18.90 (sec) , antiderivative size = 808, normalized size of antiderivative = 4.30

method	result	size
default	Expression too large to display	808
parts	Expression too large to display	808

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4+120*cos(1/2*d*x+1/2*c)*C*sin(1/2*d*x+1/2*c)^6-60*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2-120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(si
```

```
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(
1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.19

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{-5i\sqrt{2}B\sqrt{b}\cos\left(dx+c\right)^3 \text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm
="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*A + 5*I*C
)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^4*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\, dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c))^{\frac{7}{2}}}\, dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x
)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\, dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c))^{\frac{7}{2}}}\, dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\, dx = \int \frac{C\cos(c + d\,x)^2 + B\,\cos(c + d\,x) + A}{(b\,\cos(c + d\,x))^{7/2}}\, dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2), x)
```

3.288 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C$

Optimal result

Integrand size = 43, antiderivative size = 223

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$
$$+\frac{3B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}$$
$$+\frac{B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$
$$+\frac{C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}-\frac{(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] 1/4*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/5*C*cos(d*x+c)^(
7/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x
+c)^(1/2)+1/5*(5*A+4*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-
1/15*(5*A+4*C)*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+3/8*B*s
in(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {17, 3102, 2827, 2713, 2715, 8}

$$\begin{aligned}
&\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx \\
&= -\frac{(5A+4C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{(5A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d} \\
&\quad+\frac{3B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d} \\
&\quad+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{5d}
\end{aligned}$$

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (3*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + ((5*A + 4*C)*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*B*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (B*Cos[c + d*x]^(5/2)*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*
x]]*Sin[c + d*x])/(5*d) - ((5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)
/(15*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}\\
&\quad+\frac{\sqrt{b\cos(c+dx)}\int\cos^3(c+dx)(5A+4C+5B\cos(c+dx))\,dx}{5\sqrt{\cos(c+dx)}}\\
&= \frac{C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}+\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\cos^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&\quad+\frac{\left((5A+4C)\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)\,dx}{5\sqrt{\cos(c+dx)}}\\
&= \frac{B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}\\
&\quad+\frac{C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}\\
&\quad+\frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}\\
&\quad-\frac{\left((5A+4C)\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{5d\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= \frac{(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{3B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}$$
$$+ \frac{B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$
$$+ \frac{C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}$$
$$- \frac{(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{\cos(c+dx)}}$$
$$= \frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$
$$+ \frac{3B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}$$
$$+ \frac{B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$
$$+ \frac{C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}$$
$$- \frac{(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.49

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}(180Bc+180Bdx+60(6A+5C)\sin(c+dx)+120B\sin(2(c+dx))+40A\sin(3(c+dx)}{480d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x] +
120*B*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)] + 15
*B*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.60

method	result
default	$\frac{\sqrt{\cos(dx+c)b}\,(24C\sin(dx+c)(\cos^4(dx+c))+30B\sin(dx+c)(\cos^3(dx+c))+40A\sin(dx+c)(\cos^2(dx+c))+32C(\cos^2(dx+c))\sin(dx+c)}{120d\sqrt{\cos(dx+c)}}$
parts	$\frac{A(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}+\frac{B\sqrt{\cos(dx+c)b}\,(2\sin(dx+c)(\cos^3(dx+c))+3\cos(dx+c)\sin(dx+c)+3dx+3c)}{8d\sqrt{\cos(dx+c)}}+\frac{C(3}{}$
risch	$\frac{3\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{i(dx+c)}xB}{4(e^{2i(dx+c)}+1)}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{6i(dx+c)}C}{80(e^{2i(dx+c)}+1)d}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{5i(dx+c)}B}{32(e^{2i(dx+c)}+1)d}$

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/120/d*(cos(d*x+c)*b)^(1/2)*(24*C*sin(d*x+c)*cos(d*x+c)^4+30*B*sin(d*x+c)*
cos(d*x+c)^3+40*A*sin(d*x+c)*cos(d*x+c)^2+32*C*cos(d*x+c)^2*sin(d*x+c)+45*B
*sin(d*x+c)*cos(d*x+c)+80*A*sin(d*x+c)+45*B*(d*x+c)+64*sin(d*x+c)*C)/cos(d*
x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.31

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$=\left[\frac{45\,B\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2\,(}{}\right.$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(45*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*cos(d*x + c
)^4 + 30*B*cos(d*x + c)^3 + 8*(5*A + 4*C)*cos(d*x + c)^2 + 45*B*cos(d*x + c
) + 80*A + 64*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*c
os(d*x + c)), 1/120*(45*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (24*C*cos(d*x + c)^4 + 30*B*co
s(d*x + c)^3 + 8*(5*A + 4*C)*cos(d*x + c)^2 + 45*B*cos(d*x + c) + 80*A + 64
*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)B\sqrt{b}+2\,C\sqrt{b}(3\,\mathrm{s}}{}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/480*(15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 2*C*sqrt(b)*(3*sin(5*d*x + 5*c) + 25
*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 150*sin(1/5*arctan2
(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))) + 40*A*sqrt(b)*(sin(3*d*x + 3*c) + 9
*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(187) = 374$.

Time = 5.36 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.84

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{45\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+225\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+240\,A\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-150\,B\sqrt{b}\tan}{}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] 1/120*(45*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^10 + 225*B*sqrt(b)*d*x*tan(1/2
*d*x + 1/2*c)^8 + 240*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^9 - 150*B*sqrt(b)*tan(
1/2*d*x + 1/2*c)^9 + 240*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^9 + 450*B*sqrt(b)*d
*x*tan(1/2*d*x + 1/2*c)^6 + 640*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 - 60*B*sqr
t(b)*tan(1/2*d*x + 1/2*c)^7 + 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 + 450*B*
sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 800*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 +
 928*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 225*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2
*c)^2 + 640*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 60*B*sqrt(b)*tan(1/2*d*x + 1
/2*c)^3 + 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 45*B*sqrt(b)*d*x + 240*A*s
qrt(b)*tan(1/2*d*x + 1/2*c) + 150*B*sqrt(b)*tan(1/2*d*x + 1/2*c) + 240*C*sq
rt(b)*tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^10 + 5*d*tan(1/2*d*x +
1/2*c)^8 + 10*d*tan(1/2*d*x + 1/2*c)^6 + 10*d*tan(1/2*d*x + 1/2*c)^4 + 5*d*
tan(1/2*d*x + 1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.63

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(120\,B\,\sin\left(c+d\,x\right)+400\,A\,\sin\left(2\,c+2\,d\,x\right)+40\,A\,\sin\left(4\,c+4\,d\,x\right)+135\right.}{}$$

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(120*B*sin(c + d*x) + 400*A*sin(
2*c + 2*d*x) + 40*A*sin(4*c + 4*d*x) + 135*B*sin(3*c + 3*d*x) + 15*B*sin(5*
c + 5*d*x) + 350*C*sin(2*c + 2*d*x) + 56*C*sin(4*c + 4*d*x) + 6*C*sin(6*c +
 6*d*x) + 360*B*d*x*cos(c + d*x)))/(480*d*(cos(2*c + 2*d*x) + 1))
```

3.289 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C$

Optimal result

Integrand size = 43, antiderivative size = 184

$$\begin{aligned}&\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\&=\frac{(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\\&\quad+\frac{(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}\\&\quad+\frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}-\frac{B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}\end{aligned}$$

```
[Out] 1/4*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*(4*A+3*C)*x*(b
*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)-1/3*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/
8*(4*A+3*C)*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used

$= \{17, 3102, 2827, 2715, 8, 2713\}$

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{x(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}$$
$$-\frac{B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$
$$+\frac{C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4*A + 3*C)*Sqrt[Cos[c +
 d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Cos[c + d*x]^(5/2)*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x
]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}\\
&\quad+\frac{\sqrt{b\cos(c+dx)}\int\cos^2(c+dx)(4A+3C+4B\cos(c+dx))\,dx}{4\sqrt{\cos(c+dx)}}\\
&= \frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}+\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&\quad+\frac{\left((4A+3C)\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}\\
&= \frac{(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}\\
&\quad+\frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}+\frac{\left((4A+3C)\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{\cos(c+dx)}}\\
&\quad-\frac{\left(B\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}\\
&\quad+\frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}-\frac{B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.50

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\sin(2(c+dx))+8B\sin(3}{96d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c +
 d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c +
 d*x)]))/(96*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 8.72 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.62

method	result
default	$\frac{\sqrt{\cos(dx+c)b}\,(6C(\cos^3(dx+c))\sin(dx+c)+8B\sin(dx+c)(\cos^2(dx+c))+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A(dx}{24d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{B(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}+\frac{C\sqrt{\cos(dx+c)b}\,(2\sin(dx+c)(\cos^3(d}{8d\sqrt{}}$
risch	$\frac{\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{i(dx+c)}x(8A+6C)}{8\,e^{2i(dx+c)}+8}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{5i(dx+c)}C}{32(e^{2i(dx+c)}+1)d}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{4i(dx+}}{12(e^{2i(dx+c)}+1)d}$

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(cos(d*x+c)*b)^(1/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*sin(d*x+c)*cos
(d*x+c)^2+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)
+16*B*sin(d*x+c)+9*C*(d*x+c))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.50

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\left[\frac{3\,(4\,A+3\,C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\right.}{}\right.$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*cos(
d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b
*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(3*(
4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*
(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.63

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A\sqrt{b}+3\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c\ldots\right.\right.}{96\ldots}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + 3*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*
sqrt(b) + 8*B*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(156) = 312.

Time = 4.11 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.33

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{12\,A\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+9\,C\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+48\,A\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,C\sqrt{b}dx\tan\left(\frac{1}{2}\right.}{}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] 1/24*(12*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^8 + 9*C*sqrt(b)*d*x*tan(1/2*d*x
 + 1/2*c)^8 + 48*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 36*C*sqrt(b)*d*x*ta
n(1/2*d*x + 1/2*c)^6 - 24*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 + 48*B*sqrt(b)*t
an(1/2*d*x + 1/2*c)^7 - 30*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 + 72*A*sqrt(b)*
d*x*tan(1/2*d*x + 1/2*c)^4 + 54*C*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 - 24*A
*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 80*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 18*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 48*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2
+ 36*C*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 24*A*sqrt(b)*tan(1/2*d*x + 1/2*
c)^3 + 80*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 - 18*C*sqrt(b)*tan(1/2*d*x + 1/2
*c)^3 + 12*A*sqrt(b)*d*x + 9*C*sqrt(b)*d*x + 24*A*sqrt(b)*tan(1/2*d*x + 1/2
*c) + 48*B*sqrt(b)*tan(1/2*d*x + 1/2*c) + 30*C*sqrt(b)*tan(1/2*d*x + 1/2*c)
)/(d*tan(1/2*d*x + 1/2*c)^8 + 4*d*tan(1/2*d*x + 1/2*c)^6 + 6*d*tan(1/2*d*x
+ 1/2*c)^4 + 4*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+d\,x\right)+24\,C\,\sin\left(c+d\,x\right)+24\,A\,\sin\left(3\,c+3\,d\,x\right)+80\,B\,\sin\left(\right.\right.}{96}$$

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
+ d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*d*
x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x) +
 72*C*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

3.290 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+$

Optimal result

Integrand size = 43, antiderivative size = 143

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\ =\frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{(3A+2C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}\\ +\frac{B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}\\ +\frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

```
[Out] 1/3*C*cos(d*x+c)^(3/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/2*B*x*(b*cos(d*x
+c))^(1/2)/cos(d*x+c)^(1/2)+1/3*(3*A+2*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
/cos(d*x+c)^(1/2)+1/2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {17, 3102, 2813}

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\ =\frac{(3A+2C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}\\ +\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}\\ +\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{3d}\end{aligned}$$

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (C*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[
c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}\\
&\quad+\frac{\sqrt{b\cos(c+dx)}\int\cos(c+dx)(3A+2C+3B\cos(c+dx))\,dx}{3\sqrt{\cos(c+dx)}}\\
&= \frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{(3A+2C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}\\
&\quad+\frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}(6Bc+6Bdx+3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+C\sin(3(c+dx)))}{12d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*S
in[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.91 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

method	result
default	$\frac{\sqrt{\cos(dx+c)b}\,(2C(\cos^2(dx+c))\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+6A\sin(dx+c)+3B(dx+c)+4\sin(dx+c)C)}{6d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{B\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{C(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}xB}{\mathrm{e}^{2i(dx+c)}+1}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{4i(dx+c)}C}{12(\mathrm{e}^{2i(dx+c)}+1)d}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{3i(dx+c)}B}{4(\mathrm{e}^{2i(dx+c)}+1)d}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(cos(d*x+c)*b)^(1/2)*(2*C*cos(d*x+c)^2*sin(d*x+c)+3*B*sin(d*x+c)*cos(
d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.65

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\left[\frac{3\,B\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2}{12\,d\cos(dx+}\right.$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*cos(d*x + c)^2
 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*
sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x + c)
^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [A] (verification not implemented)

Time = 29.67 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.69

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\begin{cases} x\sqrt{b\cos{(c)}}(A+B\cos{(c)}+C\cos^2{(c)})\sqrt{\cos{(c)}} \\ 0 \\ \frac{A\sqrt{b\cos{(c+dx)}}\sin{(c+dx)}}{d\sqrt{\cos{(c+dx)}}}+\frac{Bx\sqrt{b\cos{(c+dx)}}\sin^2{(c+dx)}}{2\sqrt{\cos{(c+dx)}}}+\frac{Bx\sqrt{b\cos{(c+dx)}}\cos^{\frac{3}{2}}{(c+dx)}}{2}+\frac{B\sqrt{b\cos{(c+dx)}}\sin{(c+dx)}\sqrt{\cos{(c+dx)}}}{2d} \end{cases}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)*(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(b*cos(c))*(A + B*cos(c) + C*cos(c)**2)*sqrt(cos(c)), Eq(d
, 0)), (0, Eq(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (A*sqrt(b*cos(c + d*
x))*sin(c + d*x)/(d*sqrt(cos(c + d*x))) + B*x*sqrt(b*cos(c + d*x))*sin(c +
d*x)**2/(2*sqrt(cos(c + d*x))) + B*x*sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/
2)/2 + B*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d) + 2*C*s
qrt(b*cos(c + d*x))*sin(c + d*x)**3/(3*d*sqrt(cos(c + d*x))) + C*sqrt(b*cos
(c + d*x))*sin(c + d*x)*cos(c + d*x)**(3/2)/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})B\sqrt{b}+C\sqrt{b}\big(\sin{(3\,dx+3\,c)}+9\,\sin{\left(\tfrac{1}{3}\,\arctan{(\sin{(3\,dx+3\,c)}\,,\cos{(3\,dx+}}\right.}}{12\,d}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*sqrt(b) + C*sqrt(b)*(sin(3*d*x +
 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 12*A*sqrt
(b)*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(119) = 238.

Time = 2.94 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.70

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+9\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+12\,A\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,B\sqrt{b}\tan\left(\frac{1}{2}\,dx+\right.}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] 1/6*(3*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 9*B*sqrt(b)*d*x*tan(1/2*d*x +
 1/2*c)^4 + 12*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 - 6*B*sqrt(b)*tan(1/2*d*x +
 1/2*c)^5 + 12*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 9*B*sqrt(b)*d*x*tan(1/2*d
*x + 1/2*c)^2 + 24*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 8*C*sqrt(b)*tan(1/2*d
*x + 1/2*c)^3 + 3*B*sqrt(b)*d*x + 12*A*sqrt(b)*tan(1/2*d*x + 1/2*c) + 6*B*s
qrt(b)*tan(1/2*d*x + 1/2*c) + 12*C*sqrt(b)*tan(1/2*d*x + 1/2*c))/(d*tan(1/2
*d*x + 1/2*c)^6 + 3*d*tan(1/2*d*x + 1/2*c)^4 + 3*d*tan(1/2*d*x + 1/2*c)^2 +
 d)
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,B\,\sin\left(c+d\,x\right)+12\,A\,\sin\left(2\,c+2\,d\,x\right)+3\,B\,\sin\left(3\,c+3\,d\,x\right)+10\,C\,\mathrm{si}\right.}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*c
 + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*d*
x) + 12*B*d*x*cos(c + d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.291 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 123

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

```
[Out] A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*C*x*(b*cos(d*x+c))^(1/2)/cos(
d*x+c)^(1/2)+B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*C*sin
(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 2717, 2715, 8}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos
[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (C*x*Sqrt[b*Cos[c + d*x]])/
(2*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&\quad+\frac{\left(C\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}+\frac{\left(C\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(c+dx)}(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(
c + d*x)]))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\sqrt{\cos(dx+c)b}\,(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))}{2d\sqrt{\cos(dx+c)}}$	63
risch	$\frac{\sqrt{\cos(dx+c)b}\,x(4A+2C)}{4\sqrt{\cos(dx+c)}}+\frac{B\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)b}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	92
parts	$\frac{A\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{B\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{C\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}$	101

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(cos(d*x+c)*b)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x
+c)+C*(d*x+c))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \left[\frac{(2A+C)\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2 - 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c) - b\right.}{4d\cos(dx+c)} \right.$$

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*cos(d*x + c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [A] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \begin{cases} \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}\sin^2(c+dx)}{2\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}{2} + \frac{C\sqrt{b\cos(c+dx)}\mathrm{s}}{} \\ \frac{x\sqrt{b\cos(c)}(A+B\cos(c)+C\cos^2(c))}{\sqrt{\cos(c)}} \end{cases}$$

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)

[Out] Piecewise((A*x*sqrt(b*cos(c + d*x))/sqrt(cos(c + d*x)) + B*sqrt(b*cos(c + d*x))*sin(c + d*x)/(d*sqrt(cos(c + d*x))) + C*x*sqrt(b*cos(c + d*x))*sin(c + d*x)**2/(2*sqrt(cos(c + d*x))) + C*x*sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)/2 + C*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d), Ne(d, 0)), (x*sqrt(b*cos(c))*(A + B*cos(c) + C*cos(c)**2)/sqrt(cos(c)), True))

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))C\sqrt{b}+8\,A\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+4\,B\sqrt{b}\sin\left(dx+c\right)}{4\,d}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C*sqrt(b) + 8*A*sqrt(b)*arctan(sin(d*
x + c)/(cos(d*x + c) + 1)) + 4*B*sqrt(b)*sin(d*x + c))/d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/sqrt
(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(4\,B\,\sin\left(c+d\,x\right)+C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(1/2),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(4*B*sin(c + d*x) + C*sin(2*c + 2*d*x) + 4*A*d*x +
2*C*d*x))/(4*d*cos(c + d*x)^(1/2))
```

3.292 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*arctanh(sin(d*x+c))*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(1/2)+C*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3102, 2814, 3855}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(3/2),x]
```

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sq
rt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[b*Cos[c + d*x]]*Sin[c
+ d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}\left(Bc+Bdx-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}\right.\right.\right.}{d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*Sqr
t[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 10.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result
default	$-\frac{(2A\ \mathrm{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c)-\sin(dx+c)C)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$
parts	$\frac{C\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}-\frac{2A\ \mathrm{arctanh}(\cot(dx+c)-\csc(dx+c))\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{B\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$
risch	$\frac{Bx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}-\frac{i\sqrt{\cos(dx+c)b}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{i\sqrt{\cos(dx+c)b}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}-\frac{\sqrt{\cos(dx+c)b}\,A\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}}{\sqrt{\mathrm{co}}}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2),x
,method=_RETURNVERBOSE)
[Out] -1/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*(cos(d*x+c
)*b)^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.27

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \left[-\frac{2A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)-B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2-2\sqrt{\right.}}{2d\cos(d}\right.$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c) - B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x +
 c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b
) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos
(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c
)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*
cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos\left(c+dx\right)}\left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(3/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/cos(
c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{A\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\sin\left(d\right.\right.\right.}{2d}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*B*sqrt(b)*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*sqrt(b)*sin(d*x + c))/d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(
d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(3/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(3/2), x)
```

3.293 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+C*x*(b*cos(d*x+c))^(1/
2)/cos(d*x+c)^(1/2)+B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 3100, 2814, 3855}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(5/2),x]
```

```
[Out] (C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (B*ArcTanh[Sin[c + d*x]]*Sq
rt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c
+ d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\cos(c+dx)}\int (B+C\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}(Cdx\cos(c+dx)+B\text{arctanh}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c +
 d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 9.87 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\sqrt{\cos(dx+c)b}\,(-2B\cos(dx+c)\,\text{arctanh}(\cot(dx+c)-\csc(dx+c))+C\cos(dx+c)(dx+c)+A\sin(dx+c))}{d\cos(dx+c)^{\frac{3}{2}}}$	70
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}-\frac{2B\sqrt{\cos(dx+c)b}\,\text{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}}+\frac{C\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	99
risch	$\frac{Cx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{2i\sqrt{\cos(dx+c)b}\,A}{\sqrt{\cos(dx+c)}\,d\,(\mathrm{e}^{2i(dx+c)}+1)}+\frac{\sqrt{\cos(dx+c)b}\,B\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{\sqrt{\cos(dx+c)b}\,B\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}$	134

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(cos(d*x+c)*b)^(1/2)*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+C*
cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 312, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[-\frac{2\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^2-C\sqrt{-b}\cos\left(dx+c\right)^2\log\left(2\,b\cos\left(dx+c\right)^2-2\right.}{2\,d\cos\left(dx\right.}\right.$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d
*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
 - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(
b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*log(-(b*c
os(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x +
 c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos
(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{B\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\right.\right.}{2\,d}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(B*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*C*sqrt(b)*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*A*sqrt(b)*sin(2*d*x + 2*c)/(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(
d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(5/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(5/2), x)
```

3.294 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 111

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+B*sin(d*x+c)*(b*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*(A+2*C)*arctanh(sin(d*x+c))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {17, 3100, 2827, 3852, 8, 3855}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(7/2),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*
x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (B*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b\cos(c+dx)}\int(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\sqrt{b\cos(c+dx)}\int(2B+(A+2C)\cos(c+dx))\sec^2(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left((A+2C)\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}$$

$$= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} - \frac{\left(B\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}((A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+dx))}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(7/2),x]

[Out] (Sqrt[b*Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A
+ 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 10.23 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

method	result
default	$\frac{\left(A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)-4C\left(\cos^2(dx+c)\right)\operatorname{arctanh}(\cot(dx+c}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\left(-\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)\right)\sqrt{\cos(dx+c)b}}{2d\cos(dx+c)^{\frac{5}{2}}}+\frac{B\sin}{}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}\left(A\,\mathrm{e}^{3i(dx+c)}-2B\,\mathrm{e}^{2i(dx+c)}-A\,\mathrm{e}^{i(dx+c)}-2B\right)}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{\sqrt{\cos(dx+c)b}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)b}\,(A+2C)\,\mathrm{l}}{2\sqrt{\cos(dx+c}}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot(d
*x+c)+csc(d*x+c)-1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+2*B*sin
(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2\,B\cos\left(dx\right.\right.}{4\,d\cos\left(dx+c\right)^3}\right.$$
$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-(2\,B\cos\left(dx+c\right)+A)\sqrt{b\cos\left(dx+c\right)}\sqrt{}}{2\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
7/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt
(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)
^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(95) = 190$.

Time = 0.74 (sec) , antiderivative size = 780, normalized size of antiderivative = 7.03

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
7/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*C*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
 - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) - (4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
 + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2
*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*
x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + 8*
B*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(7/2),x)

[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(7/2), x)

3.295 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 152

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/2*B*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/3*(2*A+3*C)*sin(d*x+c)*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {17, 3100, 2827, 3853, 3855, 3852, 8}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(9/2),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (B*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + ((2*A + 3*C)*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\sqrt{b\cos(c+dx)}\int(3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)\,dx}{3\sqrt{\cos(c+dx)}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left((2A+3C)\sqrt{b\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{3\sqrt{\cos(c+dx)}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}} - \frac{\left((2A+3C)\sqrt{b\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{B\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}(3B\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c+dx)+(2A+3C)\cos(2(c+\cdots}{6d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*
C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Co
s[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 10.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

method	result
default	$\frac{(3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+4A\sin(dx+c)(\cos^2(dx+c))+6C\cdots}{6d\cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{A(2(\cos^2(dx+c))+1)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{B(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\cdots}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}\,(3B\,e^{5i(dx+c)}-6C\,e^{4i(dx+c)}-12A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-3B\,e^{i(dx+c)}-4A-6C)}{3\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^3}+\frac{\sqrt{\cos(dx+c)b}\,B\ln(e^{i(dx+c)}\cdots}{2\sqrt{\cos(dx+c)}\,d}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(9/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*B*cos(d*x+c)^3*ln(-c
ot(d*x+c)+csc(d*x+c)-1)+4*A*sin(d*x+c)*cos(d*x+c)^2+6*C*cos(d*x+c)^2*sin(d*
x+c)+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x
+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \left[\frac{3\,B\sqrt{b}\cos{(dx+c)}^4\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,(2\,(2\,A+3\,C)\cos}{12\,d\cos{(dx+c)}^4}\right.$$
$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos{(dx+c)}^4-\left(2\,(2\,A+3\,C)\cos{(dx+c)}^2+3\,B\cos{(dx+c}\right.}{6\,d\cos{(dx+c)}^4}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
9/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(3*
B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*
x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c)
 + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(9/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. $2(128) = 256$.

Time = 0.59 (sec) , antiderivative size = 1009, normalized size of antiderivative = 6.64

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
9/2),x, algorithm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
 + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)
*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*
d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*si
n(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c
)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c
) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4
*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)
+ 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +
 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(
2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + 24*C*sqrt(b)*sin(2*d*x + 2*c)/(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(
d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos(c+dx)}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^{9/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2), x)
```

3.296 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 193

$$
\begin{aligned}
&\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx \\
&= \frac{(3A+4C)\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}
\end{aligned}
$$

```
[Out] 1/4*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*(3*A+4*C)*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+B*sin(d*x+c)*(b*cos(d*x+c))^
(1/2)/d/cos(d*x+c)^(3/2)+1/3*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+
c)^(7/2)+1/8*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used

= {17, 3100, 2827, 3852, 3853, 3855}

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(11/2),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c +
d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (
(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (
B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (B*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{b\cos(c+dx)}\int\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\sqrt{b\cos(c+dx)}\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left((3A+4C)\sqrt{b\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}\\
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left((3A+4C)\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{8\sqrt{\cos(c+dx)}}\\
&\quad - \frac{\left(B\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}\\
&\quad + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}(3(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\,(6A+3(3A+4C)\cos^2(c+$$
$$24d\cos^{\frac{9}{2}}(c+dx)$$

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]

[Out] (Sqrt[b*Cos[c + d*x]]*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(9/2))

Maple [A] (verified)

Time = 10.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.10

method	result
default	$\frac{(-9A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+9A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-12C(\cos^4(dx+c))\ln(-\cot(d}{}$
parts	$\frac{A(-3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+3(\cos^2(dx+c))\sin(dx+c)+2\sin}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}\,(9A\,e^{7i(dx+c)}+12C\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12C\,e^{3i(dx+c)}-64B\,e}{12\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^4}$

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)

[Out] 1/24/d*(-9*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+9*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)+16*B*sin(d*x+c)*cos(d*x+c)^3+9*A*sin(d*x+c)*cos(d*x+c)^2+12*C*cos(d*x+c)^2*sin(d*x+c)+8*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(9/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \left[\frac{3\,(3\,A+4\,C)\sqrt{b}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\,(16\,B\cos}{48\,d\cos}\right.$$
$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{5}-\left(16\,B\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)\,\mathrm{co}}{24\,d\cos\left(dx+c\right)^{5}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
11/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 +
 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x
 + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (16*B
*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(11/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. 2(165) = 330.

Time = 0.62 (sec) , antiderivative size = 2611, normalized size of antiderivative = 13.53

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
11/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
 + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
```

```
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*
x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*
d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2
*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*
c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos
(2*d*x + 2*c) + 1) - 64*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*c
os(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c)
 - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*B*sqrt(b)/(2*(3*cos(4*d*x + 4*c) +
3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c
)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x
+ 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*
sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 12*(4*(sin(4*d*x + 4*c) + 2*
sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*
(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*
x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*
d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*c
os(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
C*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$=\int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(
d*x + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$=\int \frac{\sqrt{b\cos(c+dx)}\,(C\cos(c+dx)^2+B\cos(c+dx)+A)}{\cos(c+dx)^{11/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2), x)
```

3.297 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\right.$

Optimal result

Integrand size = 43, antiderivative size = 229

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{3bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}+\frac{bC\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}-\frac{b(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] 1/4*b*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/5*b*C*cos(d*x+
c)^(7/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b*B*x*(b*cos(d*x+c))^(1/2)/c
os(d*x+c)^(1/2)+1/5*b*(5*A+4*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)-1/15*b*(5*A+4*C)*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/
2)+3/8*b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {17, 3102, 2827, 2713, 2715, 8}

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{b(5A+4C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{b(5A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} +\frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d} +\frac{3bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d} +\frac{bC\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{5d}$$

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (3*b*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*(5*A + 4*C)*Sqrt
[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*b*B*Sqrt[Cos[c
 + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^(5/2)
*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b*C*Cos[c + d*x]^(7/2)*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(5*d) - (b*(5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin
[c + d*x]^3)/(15*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{bC\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)(5A+4C+5B\cos(c+dx))\,dx}{5\sqrt{\cos(c+dx)}}$$

$$= \frac{bC\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int\cos^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b(5A+4C)\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)\,dx}{5\sqrt{\cos(c+dx)}}$$

$$= \frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{bC\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{\left(3bB\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{4\sqrt{\cos(c+dx)}} - \frac{\left(b(5A+4C)\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{b(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{3bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{bC\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d} - \frac{b(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{\left(3bB\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{\cos(c+dx)}}$$

$$= \frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{3bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{bC\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d} - \frac{b(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.48

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{3/2}(180Bc+180Bdx+60(6A+5C)\sin(c+dx)+120B\sin(2(c+dx))}{480d\cos}$$

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x]
+ 120*B*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)] +
15*B*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 8.94 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.59

method	result
default	$\frac{b\sqrt{\cos(dx+c)b}\,(24C\sin(dx+c)(\cos^4(dx+c))+30B\sin(dx+c)(\cos^3(dx+c))+40A\sin(dx+c)(\cos^2(dx+c))+32C(\cos^2(dx+c))\sin(dx+c}{120d\sqrt{\cos(dx+c)}}$
parts	$\frac{Ab(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}+\frac{Bb\sqrt{\cos(dx+c)b}\,(2\sin(dx+c)(\cos^3(dx+c))+3\cos(dx+c)\sin(dx+c)+3dx+3c)}{8d\sqrt{\cos(dx+c)}}+\frac{Cb}{}$
risch	$\frac{3b\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{i(dx+c)}xB}{4(e^{2i(dx+c)}+1)}-\frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{6i(dx+c)}C}{80(e^{2i(dx+c)}+1)d}-\frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{5i(dx+c)}B}{32(e^{2i(dx+c)}+1)d}$

```
[In] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/120*b/d*(cos(d*x+c)*b)^(1/2)*(24*C*sin(d*x+c)*cos(d*x+c)^4+30*B*sin(d*x+c
)*cos(d*x+c)^3+40*A*sin(d*x+c)*cos(d*x+c)^2+32*C*cos(d*x+c)^2*sin(d*x+c)+45
*B*sin(d*x+c)*cos(d*x+c)+80*A*sin(d*x+c)+45*B*(d*x+c)+64*sin(d*x+c)*C)/cos(
d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.35

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \left[\frac{45\,B\sqrt{-b}b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\right.}{}\right.$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)^2),x, algorithm="fricas")
```

```
[Out] [1/240*(45*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(
d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*b*cos(d*x
 + c)^4 + 30*B*b*cos(d*x + c)^3 + 8*(5*A + 4*C)*b*cos(d*x + c)^2 + 45*B*b*c
os(d*x + c) + 16*(5*A + 4*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)), 1/120*(45*B*b^(3/2)*arctan(sqrt(b*cos(d*x + c)
)*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (24*C*b*cos(d*x
 + c)^4 + 30*B*b*cos(d*x + c)^3 + 8*(5*A + 4*C)*b*cos(d*x + c)^2 + 45*B*b*c
os(d*x + c) + 16*(5*A + 4*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)**2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.74

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{40\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+15\,(}{}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(40*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))*A*sqrt(b) + 15*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 2*(3*b*sin(
5*d*x + 5*c) + 25*b*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) +
150*b*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*C*sqrt(b))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(193) = 386.

Time = 5.33 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.80

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{\left(45\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+225\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+240\,A\sqrt{b}\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)^2),x, algorithm="giac")
```

```
[Out] 1/120*(45*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^10 + 225*B*sqrt(b)*d*x*tan(1/2
*d*x + 1/2*c)^8 + 240*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^9 - 150*B*sqrt(b)*tan(
1/2*d*x + 1/2*c)^9 + 240*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^9 + 450*B*sqrt(b)*d
*x*tan(1/2*d*x + 1/2*c)^6 + 640*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 - 60*B*sqr
t(b)*tan(1/2*d*x + 1/2*c)^7 + 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 + 450*B*
sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 800*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 +
 928*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 225*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2
*c)^2 + 640*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 60*B*sqrt(b)*tan(1/2*d*x + 1
/2*c)^3 + 320*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 45*B*sqrt(b)*d*x + 240*A*s
qrt(b)*tan(1/2*d*x + 1/2*c) + 150*B*sqrt(b)*tan(1/2*d*x + 1/2*c) + 240*C*sq
rt(b)*tan(1/2*d*x + 1/2*c))*b/(d*tan(1/2*d*x + 1/2*c)^10 + 5*d*tan(1/2*d*x
+ 1/2*c)^8 + 10*d*tan(1/2*d*x + 1/2*c)^6 + 10*d*tan(1/2*d*x + 1/2*c)^4 + 5*
d*tan(1/2*d*x + 1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.62

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(120\,B\,\sin\left(c+d\,x\right)+400\,A\,\sin\left(2\,c+2\,d\,x\right)+40\,A\right.}{}$$

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(120*B*sin(c + d*x) + 400*A*si
n(2*c + 2*d*x) + 40*A*sin(4*c + 4*d*x) + 135*B*sin(3*c + 3*d*x) + 15*B*sin(
5*c + 5*d*x) + 350*C*sin(2*c + 2*d*x) + 56*C*sin(4*c + 4*d*x) + 6*C*sin(6*c
 + 6*d*x) + 360*B*d*x*cos(c + d*x)))/(480*d*(cos(2*c + 2*d*x) + 1))
```

3.298 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)\right.$ -

Optimal result

Integrand size = 43, antiderivative size = 189

$$\begin{aligned}
&\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)\right. \\
&\left.+C\cos^2(c+dx)\right)\,dx=\frac{b(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\
&+\frac{b(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} \\
&+\frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}-\frac{bB\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

```
[Out] 1/4*b*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*b*(4*A+3*C)*
x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)-1/3*b*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^
(1/2)+1/8*b*(4*A+3*C)*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used

$= \{17, 3102, 2827, 2715, 8, 2713\}$

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{bx(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d} - \frac{bB\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bC\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (b*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b
*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b*(4*A + 3*C)*Sqrt[C
os[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*C*Cos[c + d*x]^(
5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b*B*Sqrt[b*Cos[c + d*x]]*S
in[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)(4A+3C+4B\cos(c+dx))\,dx}{4\sqrt{\cos(c+dx)}}$$

$$= \frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b(4A+3C)\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$

$$= \frac{b(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left(b(4A+3C)\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{\cos(c+dx)}} - \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{b(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{b(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{bB\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.49

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2} (A+B\cos(c+dx) +C\cos^2(c+dx)) \, dx = \frac{(b\cos(c+dx))^{3/2}(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\mathrm{s}}{96d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c
 + d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c
 + d*x)]))/(96*d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 8.63 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.61

method	result
default	$\frac{b\sqrt{\cos(dx+c)b}\,(6C(\cos^3(dx+c))\sin(dx+c)+8B\sin(dx+c)(\cos^2(dx+c))+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A}{24d\sqrt{\cos(dx+c)}}$
parts	$\frac{Ab\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}} + \frac{Bb(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}} + \frac{Cb\sqrt{\cos(dx+c)b}\,(2\sin(dx+c)(c}{8}$
risch	$\frac{b\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}x(8A+6C)}{8\,\mathrm{e}^{2i(dx+c)}+8} - \frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{5i(dx+c)}C}{32(\mathrm{e}^{2i(dx+c)}+1)d} - \frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{4}}{12(\mathrm{e}^{2i(dx+c)}+1)d}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24*b/d*(cos(d*x+c)*b)^(1/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*sin(d*x+c)*c
os(d*x+c)^2+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+
c)+16*B*sin(d*x+c)+9*C*(d*x+c))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.51

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\left[\frac{3\,(4\,A+3\,C)\sqrt{-b}b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(\right.}\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(4*A + 3*C)*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqr
t(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*b*
cos(d*x + c)^3 + 8*B*b*cos(d*x + c)^2 + 3*(4*A + 3*C)*b*cos(d*x + c) + 16*B
*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)),
 1/24*(3*(4*A + 3*C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*b*cos(d*x + c)^3 + 8*B*b*cos(d
*x + c)^2 + 3*(4*A + 3*C)*b*cos(d*x + c) + 16*B*b)*sqrt(b*cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.67

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{24\,(2\,(dx+c)b+b\sin\left(2\,dx+2\,c\right))A\sqrt{b}+8\,\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\right.\right.\right.\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + 8*(b*sin(3*d*x +
3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqrt(b)
+ 3*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(161) = 322$.

Time = 4.13 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.27

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\left(12\,A\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+9\,C\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+48\,A\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] 1/24*(12*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^8 + 9*C*sqrt(b)*d*x*tan(1/2*d*x
 + 1/2*c)^8 + 48*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 36*C*sqrt(b)*d*x*ta
n(1/2*d*x + 1/2*c)^6 - 24*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 + 48*B*sqrt(b)*t
an(1/2*d*x + 1/2*c)^7 - 30*C*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 + 72*A*sqrt(b)*
d*x*tan(1/2*d*x + 1/2*c)^4 + 54*C*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 - 24*A
*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 80*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 18*
C*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 48*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2
+ 36*C*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 24*A*sqrt(b)*tan(1/2*d*x + 1/2*
c)^3 + 80*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 - 18*C*sqrt(b)*tan(1/2*d*x + 1/2
*c)^3 + 12*A*sqrt(b)*d*x + 9*C*sqrt(b)*d*x + 24*A*sqrt(b)*tan(1/2*d*x + 1/2
*c) + 48*B*sqrt(b)*tan(1/2*d*x + 1/2*c) + 30*C*sqrt(b)*tan(1/2*d*x + 1/2*c)
)*b/(d*tan(1/2*d*x + 1/2*c)^8 + 4*d*tan(1/2*d*x + 1/2*c)^6 + 6*d*tan(1/2*d*
x + 1/2*c)^4 + 4*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{b\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+d\,x\right)+24\,C\,\sin\left(c+d\,x\right)+24\,A\,\sin\left(3\right.\right.}{}$$

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(
c + d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*
d*x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x)
 + 72*C*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

3.299 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 147

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b(3A+2C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}+\frac{bC\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 1/3*b*C*cos(d*x+c)^(3/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/2*b*B*x*(b*cos
(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/3*b*(3*A+2*C)*sin(d*x+c)*(b*cos(d*x+c))^(
1/2)/d/cos(d*x+c)^(1/2)+1/2*b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^
(1/2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used

$= \{17, 3102, 2813\}$

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{b(3A+2C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d} + \frac{bC\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```

```
[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*(3*A + 2*C)*Sqrt[b
*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d
*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (b*C*Cos[c + d*x]^(3/2)*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{bC\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)(3A+2C+3B\cos(c+dx))\,dx}{3\sqrt{\cos(c+dx)}}$$

$$= \frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b(3A+2C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{bC\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\int\frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{b\sqrt{b\cos(c+dx)}(6Bc+6Bdx+3(4A+3C}{12}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B
*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.82 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.57

method	result	s
default	$\frac{b\sqrt{\cos(dx+c)b}\,(2C(\cos^2(dx+c))\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+6A\sin(dx+c)+3B(dx+c)+4\sin(dx+c)C)}{6d\sqrt{\cos(dx+c)}}$	8
parts	$\frac{Ab\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{Bb\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{Cb(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}$	1
risch	$\frac{bBx\sqrt{\cos(dx+c)b}}{2\sqrt{\cos(dx+c)}}+\frac{b\sqrt{\cos(dx+c)b}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{\cos(dx+c)b}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{\cos(dx+c)b}\,B\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	1

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b/d*(cos(d*x+c)*b)^(1/2)*(2*C*cos(d*x+c)^2*sin(d*x+c)+3*B*sin(d*x+c)*co
s(d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.69

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{3\,B\sqrt{-b}b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b*cos(d*x +
c)^2 + 3*B*b*cos(d*x + c) + 2*(3*A + 2*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*b^(3/2)*arctan(sqrt(b*co
s(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*
b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*(3*A + 2*C)*b)*sqrt(b*cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{12\,Ab^{\frac{3}{2}}\sin\left(dx+c\right)+3\left(2\left(dx+c\right)b+b\sin\left(\right.\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(12*A*b^(3/2)*sin(d*x + c) + 3*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*B*
sqrt(b) + (b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))*C*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(}{\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/sq
rt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.48

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\,C\right.}{}$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(1/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + 3*B*sin(2
*c + 2*d*x) + C*sin(3*c + 3*d*x) + 6*B*d*x))/(12*d*cos(c + d*x)^(1/2))
```

3.300 $\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 127

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

```
[Out] A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b*C*x*(b*cos(d*x+c))^(1/2)/
cos(d*x+c)^(1/2)+b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2
*b*C*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 2717, 2715, 8}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bC\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```

```
[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*C*x*Sqrt[b*Cos[c + d*x
]])/(2*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqr
t[Cos[c + d*x]]) + (b*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x
])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&\quad+\frac{\left(bC\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{bC\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}+\frac{\left(bC\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}(2(2A+C)(c+dx)+4B\mathrm{s}}{4d\cos^{\frac{3}{2}}(c+dx}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2
*(c + d*x)]))/(4*d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 10.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{b\sqrt{\cos(dx+c)b}\,(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))}{2d\sqrt{\cos(dx+c)}}$	64
risch	$\frac{b\sqrt{\cos(dx+c)b}\,x(4A+2C)}{4\sqrt{\cos(dx+c)}}+\frac{bB\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{b\sqrt{\cos(dx+c)b}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	95
parts	$\frac{Cb\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{Ab\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{bB\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	104

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/d*(cos(d*x+c)*b)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d
*x+c)+C*(d*x+c))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.71

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{(2A+C)\sqrt{-b}b\cos(dx+c)\log\left(2b\cos(\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*A + C)*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*c
os(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*b*cos(d*x
 + c) + 2*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos
(d*x + c)), 1/2*((2*A + C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)
/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*b*cos(d*x + c) + 2*B*b)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.53

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{8Ab^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+4Bb^{\frac{3}{2}}\sin(dx}{4}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*B*b^(3/2)*sin(
d*x + c) + (2*(d*x + c)*b + b*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b$$

$$\cos\left(dx+c\right)^{\frac{3}{2}}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.43

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(4\,B\,\sin\left(c+d\,x\right)+C\,\sin\right.}{4\,d\,\sqrt{\cos\left(c+d\right.}}$$

[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(3/2),x)

[Out] (b*(b*cos(c + d*x))^(1/2)*(4*B*sin(c + d*x) + C*sin(2*c + 2*d*x) + 4*A*d*x + 2*C*d*x))/(4*d*cos(c + d*x)^(1/2))

3.301 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 96

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{A\text{barctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*arctanh(sin(d*x+c))*(b*cos(
d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b*C*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 3102, 2814, 3855}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{A\text{barctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bC\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```

```
[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*ArcTanh[Sin[c + d*x]
]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[b*Cos[c + d*x]]*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} \\
&\quad +\frac{bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}\left(Bc+Bdx-A\log\left(\cos\right.\right.}{}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*C
os[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 10.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

method	result
default	$-\frac{b\left(2A\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c)-\sin(dx+c)C\right)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$
parts	$-\frac{2A\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))b\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{Bb\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{bC\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$
risch	$\frac{bBx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}-\frac{ib\sqrt{\cos(dx+c)b}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{ib\sqrt{\cos(dx+c)b}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{\cos(dx+c)b}\,A\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{\cos(d}}{}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -b/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*(cos(d*x+c
)*b)^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.21

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[-\frac{2A\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c}{b\sqrt{\cos(dx+c)}}\right.}{}\right.\ldots$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="fricas")
```

[Out] [-1/2*(2*A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c) - B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/2*(2*B*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*b^(3/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\,Bb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+2\,Cb^{\frac{3}{2}}\sin\left(dx+\right.}{}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] 1/2*(4*B*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*b^(3/2)*sin(d*x + c) + (b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(}{\cos\left(dx+c\right)^{\frac{5}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\right.}{\cos\left(c+d\,x\right)^{5/2}}$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(5/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(5/2), x)
```

3.302 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 96

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b*C*x*(b*cos(d*x+c))
^(1/2)/cos(d*x+c)^(1/2)+b*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 3100, 2814, 3855}

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{bB\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(7/2),x]
```

```
[Out] (b*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*B*ArcTanh[Sin[c + d*x]
]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*
Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(b\sqrt{b\cos(c+dx)}\right)\int(B+C\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bB\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}(Cdx\cos(c+dx)+B\mathrm{arcta}}{d\cos^{\frac{5}{2}}(}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c
 + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 10.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{b\sqrt{\cos(dx+c)b}\,(-2B\cos(dx+c)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))+C\cos(dx+c)(dx+c)+A\sin(dx+c))}{d\cos(dx+c)^{\frac{3}{2}}}$	71
parts	$\frac{Ab\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}-\frac{2B\sqrt{\cos(dx+c)b}\,b\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}}+\frac{Cb\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	102
risch	$\frac{bCx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{2ib\sqrt{\cos(dx+c)b}\,A}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{b\sqrt{\cos(dx+c)b}\,B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{\cos(dx+c)b}\,B\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)}\,d}$	138

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
,method=_RETURNVERBOSE)
```

```
[Out] b/d*(cos(d*x+c)*b)^(1/2)*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+C*
cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.29

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left[-\frac{2\,B\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*
sqrt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*b*cos(d*x + c)^2*log(2*b*c
os(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x
+ c) - b) - 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^2), 1/2*(2*C*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*b^(3/2)*cos(d*x + c)^2*log
(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin
(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*b*
sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.53

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{4\,Cb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(b\log\left(\cos\left(dx\right.\right.\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*C*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b*log(cos(d*x +
 c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d
*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b) + 4*A*b^(3/2)*sin(2*d*x + 2*c)/(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b$$
$$\cos\left(dx+c\right)^{\frac{7}{2}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\right.}{\cos\left(c+d\,x\right)^{7/2}}$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(7/2), x)
```

3.303 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 114

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{b(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+}}{2d\sqrt{\cos(c+dx)}}$$
$$+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b*B*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*b*(A+2*C)*arctanh(sin(d*x+c))*(b
*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {17, 3100, 2827, 3852, 8, 3855}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{b(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+}}{2d\sqrt{\cos(c+dx)}}$$
$$+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```

```
[Out] (b*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c +
d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) +
 (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (2B+(A+2C)\cos(c+dx))\sec^2(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b(A+2C)\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}$$

$$= \frac{b(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} - \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{b(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.61

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}\left((A+2C)\text{arctanh}(\sin(c\right.}{2}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(9/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (
A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))
```

Maple [A] (verified)

Time = 10.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15

method	result
default	$\frac{b\left(A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)-4C\left(\cos^2(dx+c)\right)\text{arctanh}(\cot(d\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{Ab\left(-\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)\right)\sqrt{\cos(dx+c)b}}{2d\cos(dx+c)^{\frac{5}{2}}}+\frac{bB}{}$
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\left(A\,\mathrm{e}^{3i(dx+c)}-2B\,\mathrm{e}^{2i(dx+c)}-A\,\mathrm{e}^{i(dx+c)}-2B\right)}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{b\sqrt{\cos(dx+c)b}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{\cos(dx+c)b}\,(A+}{2\sqrt{\cos(}}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot
(d*x+c)+csc(d*x+c)-1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+2*B*s
in(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.11

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left[\frac{(A+2\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}}{}\right.}{}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{3}-\left(2\,Bb\cos\left(dx+c\right)+Ab\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{}}{2\,d\cos\left(dx+c\right)^{3}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b*arcta
n(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*
x + c)^3 - (2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(9/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(98) = 196.

Time = 0.52 (sec) , antiderivative size = 813, normalized size of antiderivative = 7.13

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log
(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) + 8*B*b^(
3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1) - (4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(
2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos
(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x +
2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*
c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(
2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)
*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*
c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*
x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(}{\cos\left(dx+c\right)^{\frac{9}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\right.}{\cos(c+d\,x)^{9/2}}$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2),x)
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2), x)
```

3.304 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 156

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{bB\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/3*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/2*b*B*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/3*b*(2*A+3*C)*sin(d*x+c)*(b*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*b*B*arctanh(sin(d*x+c))*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {17, 3100, 2827, 3853, 3855, 3852, 8}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{b(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{bB\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```

```
[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b*B*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b*(2*A + 3*C)*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)\,dx}{3\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b(2A+3C)\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{3\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}} - \frac{\left(b(2A+3C)\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{bB\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{b(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+dx)}(3B\operatorname{arctanh}(\sin(c+dx))\cos$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(11/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A +
3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*
Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 9.88 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

method	result
default	$\frac{b\left(3B\left(\cos^3(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-3B\left(\cos^3(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+4A\sin(dx+c)\left(\cos^2(dx+c)\right)+6C}{6d\cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{Ab\left(2\left(\cos^2(dx+c)\right)+1\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{Bb\left(-\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\left(3B\,e^{5i(dx+c)}-6C\,e^{4i(dx+c)}-12A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-3B\,e^{i(dx+c)}-4A-6C\right)}{3\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^3}+\frac{b\sqrt{\cos(dx+c)b}\,B\ln\left(e^{i(dx+}\right.}{2\sqrt{\cos(dx+c)}\,d}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),
x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b/d*(3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*B*cos(d*x+c)^3*ln(
-cot(d*x+c)+csc(d*x+c)-1)+4*A*sin(d*x+c)*cos(d*x+c)^2+6*C*cos(d*x+c)^2*sin(
d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d
*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.74

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \left[\frac{3\,Bb^{\frac{3}{2}}\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos}}{}\right.}{}\right.$$

$$-\frac{3\,B\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^4-\left(2\left(2\,A+3\,C\right)b\cos\left(dx+c\right)^2+3\,Bb\cos\left(dx+c\right)\right.}{6\,d\cos\left(dx+c\right)^4}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*b^(3/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b)*s
qrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1
/6*(3*B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqr
t(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*b*cos(d*x + c)^2 + 3*B*b*
cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(11/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(132) = 264$.

Time = 0.69 (sec) , antiderivative size = 1044, normalized size of antiderivative = 6.69

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1) - 16*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9
*b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x
 + 6*c) - 3*(3*b*cos(2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*co
s(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c
)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 +
9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*(4*(b*sin
(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos
```

```
(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos
(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c
) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x +
4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)
*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) -
 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c)
 + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x
+ 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin
(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b}{\cos\left(dx+c\right)^{\frac{11}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\right.}{\cos\left(c+d\,x\right)^{11/2}}$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2), x)
```

3.305 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 198

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c\cdots}}{8d\sqrt{\cos(c+dx)}}$$
$$+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$
$$+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

```
[Out] 1/4*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*b*(3*A+4*C)*
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b*B*sin(d*x+c)*(b*cos(d*
x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*b*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/
cos(d*x+c)^(7/2)+1/8*b*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d
/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {17, 3100, 2827, 3852, 3853, 3855}

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c\cdots}}{8d\sqrt{\cos(c+dx)}}$$
$$+\frac{b(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}$$
$$+\frac{bB\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(13/2),x]
```

```
[Out] (b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c
+ d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2))
 + (b*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2
)) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b*B*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b(3A+4C)\sqrt{b\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b(3A+4C)\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{8\sqrt{\cos(c+dx)}} - \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int(1+x^2)\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{b(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{bB\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.56

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+dx)}(3(3A+4C)\text{arctanh}(\sin(c$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(13/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4
 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 +
 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(9/2))
```

Maple [A] (verified)

Time = 10.04 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08

method	result
default	$-\frac{b\left(9A\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)-9A\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+12C\left(\cos^4(dx+c)\right)\ln(-\cot(d\right.}{}$
parts	$-\frac{Ab\left(3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)-3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-3\left(\cos^2(dx+c)\right)\sin(dx+c)-2\sin\right.}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\left(9A\,e^{7i(dx+c)}+12C\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12C\,e^{3i(dx+c)}-64B\,e^{\right.}{12\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}$

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),
x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*b/d*(9*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-9*A*cos(d*x+c)^4*l
n(-cot(d*x+c)+csc(d*x+c)+1)+12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-
12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-16*B*sin(d*x+c)*cos(d*x+c)^3
-9*A*sin(d*x+c)*cos(d*x+c)^2-12*C*cos(d*x+c)^2*sin(d*x+c)-8*B*sin(d*x+c)*co
s(d*x+c)-6*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.56

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \left[\frac{3\,(3\,A+4\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx-}{}\right.}{}\right.$$

$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left(16\,Bb\cos\left(dx+c\right)^3+3\,(3\,A+4\,C)b\cos\right.}{24\,d\cos\left(dx+c\right)^5}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*b*cos(d*x + c)^3 + 3*(3*A + 4*C)*b*cos(d*x + c)
^2 + 8*B*b*cos(d*x + c) + 6*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
```

```
n(d*x + c))/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*b*arctan(sqrt
(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)
^5 - (16*B*b*cos(d*x + c)^3 + 3*(3*A + 4*C)*b*cos(d*x + c)^2 + 8*B*b*cos(d*
x + c) + 6*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(13/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2732 vs. $2(170) = 340$.

Time = 0.63 (sec) , antiderivative size = 2732, normalized size of antiderivative = 13.80

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4
*c) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c)
+ 4*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) +
4*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)
^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)
^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*
b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b
*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d
*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x
```

+ 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c)
+ 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^
2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^
2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b
*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*
cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(
2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*
x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x +
 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) +
 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) -
12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*
cos(2*d*x + 2*c) + b)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*
b*cos(2*d*x + 2*c) + b)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) +
4*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c)
+ 4*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) +
 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*
x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)
^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2
*d*x + 2*c) + 1) + 64*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3
*(3*b*cos(2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*B*sqrt(b)/(2*(3*cos(4*d*x + 4
*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
 + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 12*(4*(b*sin(4*d*x + 4
*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
c))) - 4(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2
*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*
sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))

```
)^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4
*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
 + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b*cos(
4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(b)/(2*(2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 +
 sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2
*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(}{\cos\left(dx+c\right)^{\frac{13}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\right.}{\cos\left(c+d\,x\right)^{13/2}}$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(13/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(13/2), x)
```

3.306 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)\right.$ -

Optimal result

Integrand size = 43, antiderivative size = 241

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{3b^2B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{b^2B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}+\frac{b^2C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}-\frac{b^2(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] 1/4*b^2*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/5*b^2*C*cos(
d*x+c)^(7/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b^2*B*x*(b*cos(d*x+c))^(
1/2)/cos(d*x+c)^(1/2)+1/5*b^2*(5*A+4*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/c
os(d*x+c)^(1/2)-1/15*b^2*(5*A+4*C)*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)+3/8*b^2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {17, 3102, 2827, 2713, 2715, 8}

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{b^2(5A+4C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{b^2(5A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d} + \frac{3b^2B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d} + \frac{b^2C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{5d}$$

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (3*b^2*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*(5*A + 4*C)*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*b^2*B*Sqrt
[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*B*Cos[c + d*
x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b^2*C*Cos[c + d*x]^(7/
2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) - (b^2*(5*A + 4*C)*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x]^3)/(15*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{b^2C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}\\
&\quad+\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)(5A+4C+5B\cos(c+dx))\,dx}{5\sqrt{\cos(c+dx)}}\\
&= \frac{b^2C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}\\
&\quad+\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int\cos^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&\quad+\frac{\left(b^2(5A+4C)\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)\,dx}{5\sqrt{\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{b^2 B \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{b^2 C \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{\left(3b^2 B\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{\cos(c+dx)}} - \frac{\left(b^2(5A+4C)\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int (1-x^2)\,dx, x, -\sin(c+dx)\right)}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{3b^2 B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b^2 B \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{b^2 C \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d} - \frac{b^2(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{\left(3b^2 B\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{\cos(c+dx)}}$$

$$= \frac{3b^2 Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{3b^2 B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b^2 B \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{b^2 C \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d} - \frac{b^2(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.45

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{5/2}(180Bc+180Bdx+60(6A+5C)\sin(c+dx)+120B\sin(2(c+dx))}{480d\cos}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x]
+ 120*B*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)] +
15*B*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 8.92 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.57

method	result
default	$\frac{b^2\sqrt{\cos(dx+c)b}\,(24C\sin(dx+c)(\cos^4(dx+c))+30B\sin(dx+c)(\cos^3(dx+c))+40A\sin(dx+c)(\cos^2(dx+c))+32C(\cos^2(dx+c))\sin(dx+}{120d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\,b^2(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}+\frac{B\,b^2\sqrt{\cos(dx+c)b}\,(2\sin(dx+c)(\cos^3(dx+c))+3\cos(dx+c)\sin(dx+c)+3dx+3c)}{8d\sqrt{\cos(dx+c)}}+$
risch	$\frac{3b^2\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{i(dx+c)}xB}{4(e^{2i(dx+c)}+1)}-\frac{ib^2\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{6i(dx+c)}C}{80(e^{2i(dx+c)}+1)d}-\frac{ib^2\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))e^{5i(dx+}}{32(e^{2i(dx+c)}+1)d}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/120*b^2/d*(cos(d*x+c)*b)^(1/2)*(24*C*sin(d*x+c)*cos(d*x+c)^4+30*B*sin(d*x
+c)*cos(d*x+c)^3+40*A*sin(d*x+c)*cos(d*x+c)^2+32*C*cos(d*x+c)^2*sin(d*x+c)+
45*B*sin(d*x+c)*cos(d*x+c)+80*A*sin(d*x+c)+45*B*(d*x+c)+64*sin(d*x+c)*C)/co
s(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.37

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \left[\frac{45\,B\sqrt{-b}b^2\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(45*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*co
s(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*b^2*cos
(d*x + c)^4 + 30*B*b^2*cos(d*x + c)^3 + 8*(5*A + 4*C)*b^2*cos(d*x + c)^2 +
45*B*b^2*cos(d*x + c) + 16*(5*A + 4*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/120*(45*B*b^(5/2)*arctan(sqrt(b*
cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (24
*C*b^2*cos(d*x + c)^4 + 30*B*b^2*cos(d*x + c)^3 + 8*(5*A + 4*C)*b^2*cos(d*x
 + c)^2 + 45*B*b^2*cos(d*x + c) + 16*(5*A + 4*C)*b^2)*sqrt(b*cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.77

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{40\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+15}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/480*(40*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))))*A*sqrt(b) + 15*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c)
+ 8*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 2
*(3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d
*x + 5*c))) + 150*b^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))
*C*sqrt(b))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(205) = 410.

Time = 5.82 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.71

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{45\,Bb^{\frac{5}{2}}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+225\,Bb^{\frac{5}{2}}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+240\,Ab^{\frac{5}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] 1/120*(45*B*b^(5/2)*d*x*tan(1/2*d*x + 1/2*c)^10 + 225*B*b^(5/2)*d*x*tan(1/2
*d*x + 1/2*c)^8 + 240*A*b^(5/2)*tan(1/2*d*x + 1/2*c)^9 - 150*B*b^(5/2)*tan(
1/2*d*x + 1/2*c)^9 + 240*C*b^(5/2)*tan(1/2*d*x + 1/2*c)^9 + 450*B*b^(5/2)*d
*x*tan(1/2*d*x + 1/2*c)^6 + 640*A*b^(5/2)*tan(1/2*d*x + 1/2*c)^7 - 60*B*b^(
5/2)*tan(1/2*d*x + 1/2*c)^7 + 320*C*b^(5/2)*tan(1/2*d*x + 1/2*c)^7 + 450*B*
b^(5/2)*d*x*tan(1/2*d*x + 1/2*c)^4 + 800*A*b^(5/2)*tan(1/2*d*x + 1/2*c)^5 +
 928*C*b^(5/2)*tan(1/2*d*x + 1/2*c)^5 + 225*B*b^(5/2)*d*x*tan(1/2*d*x + 1/2
*c)^2 + 640*A*b^(5/2)*tan(1/2*d*x + 1/2*c)^3 + 60*B*b^(5/2)*tan(1/2*d*x + 1
/2*c)^3 + 320*C*b^(5/2)*tan(1/2*d*x + 1/2*c)^3 + 45*B*b^(5/2)*d*x + 240*A*b
^(5/2)*tan(1/2*d*x + 1/2*c) + 150*B*b^(5/2)*tan(1/2*d*x + 1/2*c) + 240*C*b^
(5/2)*tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^10 + 5*d*tan(1/2*d*x +
1/2*c)^8 + 10*d*tan(1/2*d*x + 1/2*c)^6 + 10*d*tan(1/2*d*x + 1/2*c)^4 + 5*d*
tan(1/2*d*x + 1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.60

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{b^2\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(120\,B\,\sin\left(c+d\,x\right)+400\,A\,\sin\left(2\,c+2\,d\,x\right)+40\,A\right.}{}$$

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(120*B*sin(c + d*x) + 400*A*
sin(2*c + 2*d*x) + 40*A*sin(4*c + 4*d*x) + 135*B*sin(3*c + 3*d*x) + 15*B*si
n(5*c + 5*d*x) + 350*C*sin(2*c + 2*d*x) + 56*C*sin(4*c + 4*d*x) + 6*C*sin(6
*c + 6*d*x) + 360*B*d*x*cos(c + d*x)))/(480*d*(cos(2*c + 2*d*x) + 1))
```

3.307 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 199

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{b^2B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] 1/4*b^2*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*b^2*(4*A+3
*C)*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*sin(d*x+c)*(b*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)+1/8*b^2*(4*A+3*C)*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^
(1/2)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3102, 2827, 2715, 8, 2713}

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2x(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d} - \frac{b^2B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```

```
[Out] (b^2*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*B*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b^2*(4*A + 3*C)*
Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*C*Cos[c
+ d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b^2*B*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} \\
&\quad+\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)(4A+3C+4B\cos(c+dx))\,dx}{4\sqrt{\cos(c+dx)}} \\
&= \frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} \\
&\quad+\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&\quad+\frac{\left(b^2(4A+3C)\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{4\sqrt{\cos(c+dx)}} \\
&= \frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} \\
&\quad+\frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} \\
&\quad+\frac{\left(b^2(4A+3C)\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{\cos(c+dx)}} \\
&\quad-\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}} \\
&= \frac{b^2(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} \\
&\quad+\frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}-\frac{b^2B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.46

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{(b\cos(c+dx))^{5/2}(48Ac+36cC+48Adx \cdots}{}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c
 + d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c
 + d*x)]))/(96*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 8.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.59

method	result
default	$\frac{b^2\sqrt{\cos(dx+c)b}\,\left(6C\left(\cos^3(dx+c)\right)\sin(dx+c)+8B\sin(dx+c)\left(\cos^2(dx+c)\right)+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A \cdots\right.}{24d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\,b^2\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{B\,b^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}+\frac{C\,b^2\sqrt{\cos(dx+c)b}\,(2\sin(dx+ \cdots}{}$
risch	$\frac{b^2\sqrt{\cos(dx+c)b}\,x(8A+6C)}{16\sqrt{\cos(dx+c)}}+\frac{3b^2B\sin(dx+c)\sqrt{\cos(dx+c)b}}{4d\sqrt{\cos(dx+c)}}+\frac{b^2\sqrt{\cos(dx+c)b}\,C\sin(4dx+4c)}{32\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{\cos(dx+c)b}\,B\sin(3dx+3c \cdots}{12\sqrt{\cos(dx+c)}\,d}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24*b^2/d*(cos(d*x+c)*b)^(1/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*sin(d*x+c)
*cos(d*x+c)^2+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*
x+c)+16*B*sin(d*x+c)+9*C*(d*x+c))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.52

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{3\,(4\,A+3\,C)\sqrt{-b}b^2\cos\,(dx+c)\log\left(2\,b \cdots\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(4*A + 3*C)*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*s
qrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*
b^2*cos(d*x + c)^3 + 8*B*b^2*cos(d*x + c)^2 + 3*(4*A + 3*C)*b^2*cos(d*x + c
) + 16*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)), 1/24*(3*(4*A + 3*C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
+ c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*b^2*cos(d*x + c)^3 +
 8*B*b^2*cos(d*x + c)^2 + 3*(4*A + 3*C)*b^2*cos(d*x + c) + 16*B*b^2)*sqrt(b
*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{24\,(2\,(dx+c)b^2+b^2\sin{(2\,dx+2\,c)})A\sqrt{b}+}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + 8*(b^2*sin(3*
d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*
sqrt(b) + 3*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A)(}{\sqrt{\cos{(dx+c)}}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/sq
rt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.47

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2\,\sqrt{b\,\cos{(c+d\,x)}}\,(72\,B\,\sin{(c+d\,x)}+24}{}$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(1/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(72*B*sin(c + d*x) + 24*A*sin(2*c + 2*d*x) + 8*
B*sin(3*c + 3*d*x) + 24*C*sin(2*c + 2*d*x) + 3*C*sin(4*c + 4*d*x) + 48*A*d*
x + 36*C*d*x))/(96*d*cos(c + d*x)^(1/2))
```

3.308 $\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 155

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2(3A+2C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{b^2C\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 1/3*b^2*C*cos(d*x+c)^(3/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/2*b^2*B*x*(b
*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/3*b^2*(3*A+2*C)*sin(d*x+c)*(b*cos(d*x
+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b^2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(
d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used

$= \{17, 3102, 2813\}$

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2(3A+2C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d} + \frac{b^2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*(3*A + 2*C)*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[Cos
[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (b^2*C*Cos[c + d*x]^(
3/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$

$$
\begin{aligned}
&= \frac{b^2 C \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} \\
&\quad + \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)(3A+2C+3B\cos(c+dx))\,dx}{3\sqrt{\cos(c+dx)}} \\
&= \frac{b^2 B x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2(3A+2C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{b^2 B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} \\
&\quad + \frac{b^2 C\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

$$
\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(6Bc+6Bdx+3(4A+3C}{12a}
$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B
*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 9.72 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

method	result
default	$\frac{b^2\sqrt{\cos(dx+c)b}\,\left(2C\left(\cos^2(dx+c)\right)\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+6A\sin(dx+c)+3B(dx+c)+4\sin(dx+c)C\right)}{6d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\,b^2\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{B\,b^2\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{C\,b^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}$
risch	$\frac{b^2 Bx\sqrt{\cos(dx+c)b}}{2\sqrt{\cos(dx+c)}}+\frac{b^2\sqrt{\cos(dx+c)b}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{\cos(dx+c)b}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{\cos(dx+c)b}\,B\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^2/d*(cos(d*x+c)*b)^(1/2)*(2*C*cos(d*x+c)^2*sin(d*x+c)+3*B*sin(d*x+c)*
cos(d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.70

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{3\,B\sqrt{-b}b^2\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+\right.\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(
d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b^2*cos(d*
x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*(3*A + 2*C)*b^2)*sqrt(b*cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*b^(5/2)*arctan(s
qrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)
 + (2*C*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*(3*A + 2*C)*b^2)*sqrt
(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{12\,Ab^{\frac{5}{2}}\sin\left(dx+c\right)+3\left(2\left(dx+c\right)b^2+b^2\,\mathrm{si}\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(12*A*b^(5/2)*sin(d*x + c) + 3*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c)
)*B*sqrt(b) + (b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))))*C*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b}{\cos\left(dx+c\right)^{\frac{3}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\,C\right.}{}$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(3/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + 3*B*sin
(2*c + 2*d*x) + C*sin(3*c + 3*d*x) + 6*B*d*x))/(12*d*cos(c + d*x)^(1/2))
```

3.309 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 135

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

```
[Out] A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b^2*C*x*(b*cos(d*x+c))^(1
/2)/cos(d*x+c)^(1/2)+b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/
2)+1/2*b^2*C*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 2717, 2715, 8}

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2C\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```

```
[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*C*x*Sqrt[b*Cos[c +
 d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin
[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&\quad+\frac{\left(b^2C\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{b^2C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}+\frac{\left(b^2C\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{\cos(c+dx)}}
\end{aligned}$$

$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(2(2A+C)(c+dx)+4B}{4d\cos^{\frac{5}{2}}(c+d}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2
*(c + d*x)]))/(4*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 10.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{b^2\sqrt{\cos(dx+c)b}\,(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))}{2d\sqrt{\cos(dx+c)}}$	66
risch	$\frac{b^2\sqrt{\cos(dx+c)b}\,x(4A+2C)}{4\sqrt{\cos(dx+c)}} + \frac{b^2B\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}} + \frac{b^2\sqrt{\cos(dx+c)b}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	101
parts	$\frac{A\,b^2\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}} + \frac{b^2B\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}} + \frac{C\,b^2\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}$	110

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/d*(cos(d*x+c)*b)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin
(d*x+c)+C*(d*x+c))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.68

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{(2A+C)\sqrt{-b}b^2\cos(dx+c)\log\left(2b\cos(\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*A + C)*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b
*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*b^2*cos
(d*x + c) + 2*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)), 1/2*((2*A + C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*b^2*cos(d*x + c) + 2
*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{8\,Ab^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+4\,Bb^{\frac{5}{2}}\sin(dx+}{4}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*B*b^(5/2)*sin(
d*x + c) + (2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(}{\cos\left(dx+c\right)^{\frac{5}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.42

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(4\,B\,\sin\left(c+d\,x\right)+C\,\mathrm{s}\right.}{4\,d\,\sqrt{\cos\left(c+\right.}}$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(5/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(4*B*sin(c + d*x) + C*sin(2*c + 2*d*x) + 4*A*d*
x + 2*C*d*x))/(4*d*cos(c + d*x)^(1/2))
```

3.310 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b^2*arctanh(sin(d*x+c))*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b^2*C*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/
d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 3102, 2814, 3855}

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{Ab^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(7/2),x]
```

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*ArcTanh[Sin[c +
d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[b*Cos[c +
d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int(A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab^2\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(Bc+Bdx-A\log\left(\cos\left(\right.\right.\right.}{}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*C
os[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 9.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result
default	$-\frac{b^2(2A\ \mathrm{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c)-\sin(dx+c)C)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$
parts	$-\frac{2A\ \mathrm{arctanh}(\cot(dx+c)-\csc(dx+c))b^2\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{B\,b^2\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{b^2C\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$
risch	$\frac{b^2Bx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}-\frac{ib^2\sqrt{\cos(dx+c)b}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{ib^2\sqrt{\cos(dx+c)b}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}-\frac{b^2\sqrt{\cos(dx+c)b}\,A\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}+ b^2\sqrt{\mathrm{c}}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -b^2/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*(cos(d*x
+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.10

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left[-\frac{2\,A\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(
b*sqrt(cos(d*x + c))))*cos(d*x + c) - B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*c
os(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x
+ c) - b) - 2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(
d*cos(d*x + c)), 1/2*(2*B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*b^(5/2)*cos(d*x + c)*log(-(b
*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x
 + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*b^2*sq
rt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{4\,Bb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+2\,Cb^{\frac{5}{2}}\sin\left(dx\right.}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*B*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*b^(5/2)*sin(
d*x + c) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/
d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b}{\cos\left(dx+c\right)^{\frac{7}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\right.}{\cos\left(c+d\,x\right)^{7/2}}$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(7/2), x)
```

3.311 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b^2*C*x*(b*cos(d*x
+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/
d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 3100, 2814, 3855}

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2B\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```

```
[Out] (b^2*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*B*ArcTanh[Sin[c +
d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c +
d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (B+C\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}\\
&\quad+\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\\
&\quad+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(Cdx\cos(c+dx)+B\operatorname{arc}}{d\cos}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(9/2),x]
[Out] ((b*Cos[c + d*x])^(5/2)*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c
 + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(7/2))
```

Maple [A] (verified)

Time = 9.74 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

method	result
default	$\frac{b^2\sqrt{\cos(dx+c)b}\,(-2B\cos(dx+c)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))+C\cos(dx+c)(dx+c)+A\sin(dx+c))}{d\cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A\,b^2\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}-\frac{2B\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))b^2\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{C\,b^2\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$
risch	$\frac{b^2Cx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{2ib^2\sqrt{\cos(dx+c)b}\,A}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}+\frac{b^2\sqrt{\cos(dx+c)b}\,B\ln\left(e^{i(dx+c)}+i\right)}{\sqrt{\cos(dx+c)}\,d}-\frac{b^2\sqrt{\cos(dx+c)b}\,B\ln\left(e^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)}\,d}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
,method=_RETURNVERBOSE)
[Out] b^2/d*(cos(d*x+c)*b)^(1/2)*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+
C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.18

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left[-\frac{2\,B\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+}{b\sqrt{\cos(dx+c)}}\right.}{}\right.$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(
b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*b^2*cos(d*x + c)^2*log(2
*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(
d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2), 1/2*(2*C*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
 + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*b^(5/2)*cos(d*x + c)
^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c
))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c)
)*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(9/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.48

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{4\,Cb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\frac{4\,Ab}{\cos(2\,dx+2\,c)^2+\sin(}}{}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*C*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*A*b^(5/2)*sin(
2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2
*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(}{\cos\left(dx+c\right)^{\frac{9}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\right.}{\cos\left(c+d\,x\right)^{9/2}}$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2), x)
```

3.312 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 120

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b^2(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+}}{2d\sqrt{\cos(c+dx)}}$$
$$+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b^2*B*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*b^2*(A+2*C)*arctanh(sin(d*x+
c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {17, 3100, 2827, 3852, 8, 3855}

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b^2(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+}}{2d\sqrt{\cos(c+dx)}}$$
$$+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```

```
[Out] (b^2*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c
+ d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2
)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (2B+(A+2C)\cos(c+dx))\sec^2(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}$$

$$
\begin{aligned}
&= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&\quad + \frac{\left(b^2(A+2C)\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}} \\
&= \frac{b^2(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} \\
&\quad - \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}} \\
&= \frac{b^2(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$
\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\left((A+2C)\text{arctanh}(\sin(c+\right.}{2d}
$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(11/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (
A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(9/2))
```

Maple [A] (verified)

Time = 10.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11

method	result
default	$\frac{b^2\left(A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)-4C\left(\cos^2(dx+c)\right)\text{arctanh}(\cot(dx\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\,b^2\left(-\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)\right)\sqrt{\cos(dx+c)b}}{2d\cos(dx+c)^{\frac{5}{2}}}+\frac{b^2}{}$
risch	$-\frac{ib^2\sqrt{\cos(dx+c)b}\left(A\,e^{3i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-2B\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}-\frac{b^2\sqrt{\cos(dx+c)b}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{\cos(dx+c)b}\,(A}{2\sqrt{\cos(}}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),
x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-c
ot(d*x+c)+csc(d*x+c)-1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+2*B
*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.08

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \left[\frac{(A+2\,C)b^{\frac{5}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)}{}\right.}{}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\left(2\,Bb^2\cos\left(dx+c\right)+Ab^2\right)\sqrt{b\cos\left(dx+c\right.}}{2\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b^2
*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*
cos(d*x + c)^3 - (2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(11/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. 2(104) = 208.

Time = 0.55 (sec) , antiderivative size = 873, normalized size of antiderivative = 7.28

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*B*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) + 2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*
sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c
) + 1))*C*sqrt(b) - (4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c)
+ 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x +
4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2
 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*
x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos
(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^
2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4
*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*
x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqr
t(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*
cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b}{\cos\left(dx+c\right)^{\frac{11}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(b\,\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\right.}{\cos(c+d\,x)^{11/2}}$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2), x)
```

3.313 $\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 164

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b^2B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{b^2(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/3*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/2*b^2*B*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/3*b^2*(2*A+3*C)*sin(d*x+c)
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*b^2*B*arctanh(sin(d*x+c))*(b*c
os(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {17, 3100, 2827, 3853, 3855, 3852, 8}

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b^2(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b^2B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(13/2),x]
```

```
[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]])
 + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b^
2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b^2*(2*A
 + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}\\
&\quad+\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int(3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)\,dx}{3\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&\quad+\frac{\left(b^2(2A+3C)\sqrt{b\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{3\sqrt{\cos(c+dx)}}\\
&= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}\\
&\quad+\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}\\
&\quad-\frac{\left(b^2(2A+3C)\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int1\,dx,x,-\tan(c+dx)\right)}{3d\sqrt{\cos(c+dx)}}\\
&= \frac{b^2B\,\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}\\
&\quad+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\,(3B\text{arctanh}(\sin(c+dx))$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(13/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A +
3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*
Cos[c + d*x]^(9/2))
```

Maple [A] (verified)

Time = 9.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

method	result
default	$\frac{b^2(3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+4A\sin(dx+c)(\cos^2(dx+c))}{6d\cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{A\,b^2(2(\cos^2(dx+c))+1)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{B\,b^2(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib^2\sqrt{\cos(dx+c)b}\,(3B\,e^{5i(dx+c)}-6C\,e^{4i(dx+c)}-12A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-3B\,e^{i(dx+c)}-4A-6C)}{3\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^3}+\frac{b^2\sqrt{\cos(dx+c)b}\,B\ln(e^{i}}{2\sqrt{\cos(dx+c)}}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),
x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^2/d*(3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*B*cos(d*x+c)^3*l
n(-cot(d*x+c)+csc(d*x+c)-1)+4*A*sin(d*x+c)*cos(d*x+c)^2+6*C*cos(d*x+c)^2*si
n(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos
(d*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.74

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \left[\frac{3\,Bb^{\frac{5}{2}}\cos(dx+c)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b}}{}\right.}{}\right.$$

$$-\frac{3\,B\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^4-\left(2\,(2\,A+3\,C)b^2\cos(dx+c)^2+3\,Bb^2\cos(dx+\right.}{6\,d\cos(dx+c)^4}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*b^(5/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*A*
b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^
4), -1/6*(3*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*b^2*cos(d*x + c)^
2 + 3*B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(13/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(140) = 280$.

Time = 0.62 (sec) , antiderivative size = 1112, normalized size of antiderivative = 6.78

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1) - 16*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) +
 9*b^2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*s
in(6*d*x + 6*c) - 3*(3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*sqrt
(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos
(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x
 + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) -
 3*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2
```

```
*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
 + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*
d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*
x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 +
 b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*s
in(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c
) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x +
 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^
2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(}{\cos\left(dx+c\right)^{\frac{13}{2}}}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\right.}{\cos\left(c+d\,x\right)^{13/2}}$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(13/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(13/2), x)
```

3.314 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 208

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{b^2(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b^2(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

```
[Out] 1/4*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*b^2*(3*A+4
*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b^2*B*sin(d*x+c)*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*b^2*B*sin(d*x+c)^3*(b*cos(d*x+c))^
(1/2)/d/cos(d*x+c)^(7/2)+1/8*b^2*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {17, 3100, 2827, 3852, 3853, 3855}

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{b^2(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{b^2(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b^2B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(15/2),x]
```

```
[Out] (b^2*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[
c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9
/2)) + (b^2*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x
]^(5/2)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
 + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2(3A+4C)\sqrt{b\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b^2(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b^2(3A+4C)\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{8\sqrt{\cos(c+dx)}} - \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int(1+x^2)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b^2(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.53

$$\int\frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(3(3A+4C)\text{arctanh}(\sin(c\right.}{}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(15/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4
 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 +
 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(13/2))
```

Maple [A] (verified)

Time = 10.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.03

method	result
default	$-\frac{b^2\left(9A\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)-9A\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+12C\left(\cos^4(dx+c)\right)\ln(-\text{co}\right.}{}$
parts	$\frac{A\,b^2\left(-3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+3\left(\cos^2(dx+c)\right)\sin(dx+c)+\right.}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib^2\sqrt{\cos(dx+c)}b\left(9A\,e^{7i(dx+c)}+12C\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12C\,e^{3i(dx+c)}-64\right.}{12\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),
x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*b^2/d*(9*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-9*A*cos(d*x+c)^4
*ln(-cot(d*x+c)+csc(d*x+c)+1)+12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1
)-12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-16*B*sin(d*x+c)*cos(d*x+c)
^3-9*A*sin(d*x+c)*cos(d*x+c)^2-12*C*cos(d*x+c)^2*sin(d*x+c)-8*B*sin(d*x+c)*
cos(d*x+c)-6*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.57

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \left[\frac{3\,(3\,A+4\,C)b^{\frac{5}{2}}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(d}{}\right.}{}\right.$$

$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left(16\,Bb^2\cos\left(dx+c\right)^3+3\,(3\,A+4\,C)\right.}{24\,d\cos\left(dx+c\right)^5}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*b^2*cos(d*x + c)^3 + 3*(3*A + 4*C)*b^2*cos(d*x
+ c)^2 + 8*B*b^2*cos(d*x + c) + 6*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
```

```
+ c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*b^2*a
rctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*co
s(d*x + c)^5 - (16*B*b^2*cos(d*x + c)^3 + 3*(3*A + 4*C)*b^2*cos(d*x + c)^2
+ 8*B*b^2*cos(d*x + c) + 6*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(15/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. $2(180) = 360$.

Time = 0.65 (sec) , antiderivative size = 2972, normalized size of antiderivative = 14.29

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx=\text{Too large to display}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d
*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*s
in(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6
*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*
c) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*
d*x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*
sin(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2
+ 48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*
b^2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x +
4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x
+ 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d
```

```
*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4
*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*
x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) + 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(
4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^
2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2
 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2
*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x +
 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x
 + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*
d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(b^2*cos(8*d*x
+ 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x
+ 2*c) + b^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(b^
2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^
2*cos(2*d*x + 2*c) + b^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 44*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x +
 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 12*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2
*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x
 + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8
*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*
d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x
+ 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*
c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(
4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2
 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*
d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1) + 64*(3*b^2*cos(6*d*x + 6*c)*sin(2*d
*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*
c) + b^2)*sin(6*d*x + 6*c) - 3*(3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4
*c))*B*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 1
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
2*c) + 1) + 12*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b
^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^
```

```
2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*
b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4
*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*
x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos
(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2
*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2
*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(b)/
(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2
*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\right.}{\cos\left(c+d\,x\right)^{15/2}}$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(15/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(15/2), x)
```

3.315 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 184

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\\ &\quad+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}}\\ &\quad+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}-\frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}\end{aligned}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/4*C*cos(
d*x+c)^(7/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(d*x+c)^(
1/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1
/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used

$= \{17, 3102, 2827, 2715, 8, 2713\}$

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d\sqrt{b\cos(c+dx)}}$$
$$-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$
$$+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos
[c + d*x]],x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c + d*x]
^(3/2)*Sin[c + d*x])/(8*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/2)*Sin
[c + d*x])/(4*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^
3)/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)(4A+3C+4B\cos(c+dx))\,dx}{4\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{b\cos(c+dx)}}$$

$$= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+dx)}\right)\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

$$= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}-\frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.50

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\sin(2(c+dx))+8B\sin(3(}{96d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d
*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d
*x)]))/(96*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.62

method	result
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(6C\left(\cos^3(dx+c)\right)\sin(dx+c)+8B\sin(dx+c)\left(\cos^2(dx+c)\right)+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A(dx}{24d\sqrt{\cos(dx+c)b}}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\left(\sqrt{\cos}(dx+c)\right)}{2d\sqrt{\cos(dx+c)b}}+\frac{B\left(2+\cos^2(dx+c)\right)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{3d\sqrt{\cos(dx+c)b}}+\frac{C\left(2\sin(dx+c)\left(\cos^3(dx+c)\right)+3\cos}{8d\sqrt{}}$
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)x(8A+6C)}{16\sqrt{\cos(dx+c)b}}+\frac{3B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{4d\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(4dx+4c)}{32\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(3dx+3c)}{12\sqrt{\cos(dx+c)b}\,d}+$

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*cos(d*x+c)^(1/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*sin(d*x+c)*cos(d*x
+c)^2+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)+16*
B*sin(d*x+c)+9*C*(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.53

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\left[-\frac{3\,(4\,A+3\,C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right.\right.}{}\right.$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt
(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(6*C*cos
(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(
b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)), 1/24*(
3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos
(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2 +
 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.63

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{\sqrt{b}} + \frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))))C}{\sqrt{b}} + \frac{8\,B(\sin(3\,dx+3\,c)+9\,\sin(\frac{1}{3}\,a}{}}{96\,d}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + 3*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/
sqrt(b) + 8*B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))/sqrt(b))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b
*cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.76

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+d\,x\right)+24\,C\,\sin\left(c+d\,x\right)+24\,A\,\sin\left(3\,c+3\,d\,x\right)+80\,B\,\sin\left(\right.\right.}{96}$$

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
+ d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*d*
x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x) +
 72*C*d*x*cos(c + d*x)))/(96*b*d*(cos(2*c + 2*d*x) + 1))
```

3.316 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 143

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/3*C*cos(d*x+c)^(
5/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*x+c)^(1/2)/(b*cos(d*x+
c))^(1/2)+1/3*(3*A+2*C)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {17, 3102, 2813}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{(3A+2C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos
[c + d*x]],x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(3*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(3A+2C+3B\cos(c+dx))\,dx}{3\sqrt{b\cos(c+dx)}}$$

$$= \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}(6Bc+6Bdx+3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+C\sin(3(c+dx)))}{12d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Sin
[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(2C\left(\cos^2(dx+c)\right)\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+6A\sin(dx+c)+3B(dx+c)+4\sin(dx+c)C\right)}{6d\sqrt{\cos(dx+c)b}}$	83
parts	$\frac{A\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}+\frac{B(\cos(dx+c)\sin(dx+c)+dx+c)\left(\sqrt{\cos}(dx+c)\right)}{2d\sqrt{\cos(dx+c)b}}+\frac{C\left(2+\cos^2(dx+c)\right)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{3d\sqrt{\cos(dx+c)b}}$	113
risch	$\frac{Bx\left(\sqrt{\cos}(dx+c)\right)}{2\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(2dx+2c)}{4\sqrt{\cos(dx+c)b}\,d}$	126

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*cos(d*x+c)^(1/2)*(2*C*cos(d*x+c)^2*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+
c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.69

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\left[-\frac{3\,B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-2}{12\,bd\cos\left(dx+\right.}\right.$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c)^
2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(b*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c
))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x +
 c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{\sqrt{b}}+\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{\sqrt{b}}+\frac{12\,A\sin(dx+c)}{\sqrt{b}}}{12\,d}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/sqrt(b) + C*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/sqrt(b) + 12*A*sin(
d*x + c)/sqrt(b))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b
*cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,B\,\sin\left(c+d\,x\right)+12\,A\,\sin\left(2\,c+2\,d\,x\right)+3\,B\,\sin\left(3\,c+3\,d\,x\right)+10\,C\,\sin\right.}{12\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*c
 + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*d*
x) + 12*B*d*x*cos(c + d*x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))
```

3.317 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 123

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)^(1/
2)/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*sin
(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2717, 2715, 8}

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos
[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (C*x*Sqrt[Cos[c + d*x]])/(2
*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[
c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{b\cos(c+dx)}}
\end{aligned}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c
+ d*x)]))/(4*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 10.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{(\sqrt{\cos}(dx+c))(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))}{2d\sqrt{\cos(dx+c)b}}$	63
risch	$\frac{(\sqrt{\cos}(dx+c))x(4A+2C)}{4\sqrt{\cos(dx+c)b}} + \frac{B\sin(dx+c)(\sqrt{\cos}(dx+c))}{d\sqrt{\cos(dx+c)b}} + \frac{(\sqrt{\cos}(dx+c))C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)b}\,d}$	92
parts	$\frac{A(\sqrt{\cos}(dx+c))(dx+c)}{d\sqrt{\cos(dx+c)b}} + \frac{B\sin(dx+c)(\sqrt{\cos}(dx+c))}{d\sqrt{\cos(dx+c)b}} + \frac{C(\cos(dx+c)\sin(dx+c)+dx+c)(\sqrt{\cos}(dx+c))}{2d\sqrt{\cos(dx+c)b}}$	101

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*cos(d*x+c)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+
C*(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\left[-\frac{(2\,A+C)\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right.}{4\,bd\cos(dx+c)}\right.$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*co
s(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x +
c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*
x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(s
qrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [A] (verification not implemented)

Time = 15.79 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\begin{cases}\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{Cx\sin^2(c+dx)\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{Cx\cos^{\frac{5}{2}}(c+dx)}{2\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}} & \text{for } d \neq \\ \frac{x(A+B\cos(c)+C\cos^2(c))\sqrt{\cos(c)}}{\sqrt{b\cos(c)}} & \text{otherw}\end{cases}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Piecewise(((A*x*sqrt(cos(c + d*x))/sqrt(b*cos(c + d*x)) + B*sin(c + d*x)*sqr
t(cos(c + d*x))/(d*sqrt(b*cos(c + d*x))) + C*x*sin(c + d*x)**2*sqrt(cos(c +
 d*x))/(2*sqrt(b*cos(c + d*x))) + C*x*cos(c + d*x)**(5/2)/(2*sqrt(b*cos(c +
 d*x))) + C*sin(c + d*x)*cos(c + d*x)**(3/2)/(2*d*sqrt(b*cos(c + d*x))), Ne
(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)*sqrt(cos(c))/sqrt(b*cos(c)), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{\sqrt{b}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{4\,B\sin(dx+c)}{\sqrt{b}}}{4\,d}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/sqrt(b) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/sqrt(b) + 4*B*sin(d*x + c)/sqrt(b))/d
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b
*cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)+4\,B\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(3\,c+3\,d\,x\right)+8\,A\,d\,x\,\cos\right.}{4\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + 4*B*sin(2*c +
2*d*x) + C*sin(3*c + 3*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))
/(4*b*d*(cos(2*c + 2*d*x) + 1))
```

3.318 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] B*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c)^
(1/2)/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {18, 3102, 2814, 3855}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{A\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[
c + d*x]]),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sq
rt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}\left(Bc+Bdx-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c-\right.\right.\right.}{d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[
b*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*Sqrt[
b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{(2A\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c)-\sin(dx+c)C)(\sqrt{\cos}(dx+c))}{d\sqrt{\cos(dx+c)b}}$	61
parts	$\frac{C\sin(dx+c)(\sqrt{\cos}(dx+c))}{d\sqrt{\cos(dx+c)b}}-\frac{2A(\sqrt{\cos}(dx+c))\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)b}}+\frac{B(\sqrt{\cos}(dx+c))(dx+c)}{d\sqrt{\cos(dx+c)b}}$	99
risch	$\frac{Bx(\sqrt{\cos}(dx+c))}{\sqrt{\cos(dx+c)b}}-\frac{(\sqrt{\cos}(dx+c))A\ln(e^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)b}\,d}+\frac{(\sqrt{\cos}(dx+c))A\ln(e^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)b}\,d}+\frac{C\sin(2dx+2c)}{2d\sqrt{\cos(dx+c)}\sqrt{\cos(dx+c)b}}$	129

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*cos(d*x+c)
^(1/2)/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.32

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$$
$$=\left[-\frac{2A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)+B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b}\right.}{2bd\cos(dx}\right.$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c) + B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x +
 c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b
) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x
+ c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*c
os(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x +
 c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*
b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c
))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx=\int \frac{A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)}{\sqrt{b\cos\left(c+dx\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*sqr
t(cos(c + d*x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{4\,B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{2\,C\sin(d}{\sqrt{b}}}{2\,d}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + 2*C*sin(d*x + c)/sqrt(b))/d
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A}{\sqrt{b\cos\left(dx + c\right)}\sqrt{\cos\left(dx + c\right)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*sqr
t(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + B\,\cos\left(c + d\,x\right) + A}{\sqrt{\cos\left(c + d\,x\right)}\,\sqrt{b\,\cos\left(c + d\,x\right)}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(1/2)), x)
```

3.319 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{B\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/2)/(
b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {18, 3100, 2814, 3855}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[
c + d*x]]),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (B*ArcTanh[Sin[c + d*x]]*Sq
rt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int (B+C\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{Cdx\cos(c+dx)+B\operatorname{arctanh}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[
b*Cos[c + d*x]]),x]
```

```
[Out] (C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{-2B\cos(dx+c)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))+C\cos(dx+c)(dx+c)+A\sin(dx+c)}{d\sqrt{\cos(dx+c)b}\sqrt{\cos(dx+c)}}$	70
parts	$\frac{A\sin(dx+c)}{d\sqrt{\cos(dx+c)}\sqrt{\cos(dx+c)b}}-\frac{2B(\sqrt{\cos}(dx+c))\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)b}}+\frac{C(\sqrt{\cos}(dx+c))(dx+c)}{d\sqrt{\cos(dx+c)b}}$	99
risch	$\frac{Cx(\sqrt{\cos}(dx+c))}{\sqrt{\cos(dx+c)b}}+\frac{ie^{-i(dx+c)}A}{\sqrt{\cos(dx+c)b}\sqrt{\cos(dx+c)}\,d}+\frac{(\sqrt{\cos}(dx+c))B\ln(e^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)b}\,d}-\frac{(\sqrt{\cos}(dx+c))B\ln(e^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)b}\,d}$	130

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+C*cos(d*x+c)*(d*x+c)+A*
sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.41

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \left[-\frac{2B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^2+C\sqrt{-b}\cos(dx+c)^2\log\left(2b\cos(dx+c)^2+2\right.}{2bd\cos(}\right.$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^2 + C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d
*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
 - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(
d*x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqr
t(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*log(-(b
*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x
 + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(c
os(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)}{\sqrt{b\cos\left(c+dx\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*cos
(c + d*x)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.60

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{B\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{4\,C\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{}{b\cos(2\,dx-}}{2\,d}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*C*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + 4*A*sqrt(b)*sin(2*d*x + 2*c)/(b*co
s(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c) + b))/d
```

Giac [F]

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos
(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos(c+d\,x)^2 + B\,\cos(c+d\,x) + A}{\cos(c+d\,x)^{3/2}\,\sqrt{b\,\cos(c+d\,x)}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(1/2)), x)
```

3.320 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 111

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/d/cos
(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*arctanh(sin(d*x+c))*cos(d*x+
c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {18, 3100, 2827, 3852, 8, 3855}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+2C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[
c + d*x]]),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*
x]]) + (A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\sqrt{\cos(c+dx)}\int(2B+(A+2C)\cos(c+dx))\sec^2(c+dx)\,dx}{2\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left((A+2C)\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2\sqrt{b\cos(c+dx)}}\\
&= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad-\frac{\left(B\sqrt{\cos(c+dx)}\right)\text{Subst}(\int 1\,dx,x,-\tan(c+dx))}{d\sqrt{b\cos(c+dx)}}\\
&= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.62

$$
\int\frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx
=\frac{(A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[
b*Cos[c + d*x]]),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Si
n[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 10.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

method	result
default	$\frac{A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)-4C(\cos^2(dx+c))\operatorname{arctanh}(\cot(dx+}{2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(A\,e^{2i(dx+c)}-A-4B\cos(dx+c))}{2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)d}-\frac{(\sqrt{\cos}(dx+c))(A+2C)\ln(e^{i(dx+c)}-i)}{2\sqrt{\cos(dx+c)b}\,d}+\frac{(\sqrt{\cos}(dx+c))(A+2C)\ln(e^{i(dx+c)}+i}{2\sqrt{\cos(dx+c)b}\,d}$
parts	$\frac{A(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c))}{2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}+\frac{B\sin(dx}{d\sqrt{\cos(dx+c)}\,\sqrt{}}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot(d
*x+c)+csc(d*x+c)-1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+2*B*sin
(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.15

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\left[\frac{(A+2\,C)\sqrt{b}\cos{(dx+c)}^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,(2\,B\cos{(dx}}{4\,bd\cos{(dx+c)}^3}\right.$$
$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos{(dx+c)}^3-(2\,B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}}\sqrt{}}{2\,bd\cos{(dx+c)}^3}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sq
rt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x +
c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c))/(b*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(95) = 190$.

Time = 0.53 (sec) , antiderivative size = 785, normalized size of antiderivative = 7.07

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(c
os(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 8*B*sqrt(b)
*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*
d*x + 2*c) + b) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x
+ 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^
2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(
2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c
)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*
x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((2*(2*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4
*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(
2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\, dx = \int \frac{C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A}{\sqrt{b\cos\left(dx + c\right)}\cos\left(dx + c\right)^{\frac{5}{2}}}\, dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos
(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\, dx = \int \frac{C\cos\left(c + d\,x\right)^2 + B\,\cos\left(c + d\,x\right) + A}{\cos\left(c + d\,x\right)^{5/2}\,\sqrt{b\,\cos\left(c + d\,x\right)}}\, dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(1/2)), x)
```

3.321 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 152

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{B\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/2*B*sin(d*x+c)/d
/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(d*x+c)/d/cos(d*x+c
)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(
b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used

$= \{18, 3100, 2827, 3853, 3855, 3852, 8\}$

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[
c + d*x]]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (
A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c +
d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c +
d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{\sqrt{\cos(c+dx)}\int (3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)\,dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{b\cos(c+dx)}} \\
&\quad - \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\operatorname{Subst}(\int 1\,dx, x, -\tan(c+dx))}{3d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{B \operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.57

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{3B \operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx) + (4A+3C+3B\cos(c+dx)+(2A+3C)\cos(2(c+dx)))\tan(c+}{6d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[
b*Cos[c + d*x]]),x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x] +
 (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x])/(6*d*Cos[c + d*x]^(3/2)*Sqrt[b
*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.82 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

method	result
default	$\frac{3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+4A\sin(dx+c)(\cos^2(dx+c))+6C}{6d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A(2(\cos^2(dx+c))+1)\sin(dx+c)}{3d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}+\frac{B(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+}{2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(3B\,e^{4i(dx+c)}-6C\,e^{3i(dx+c)}-3B+(-16A-18C)\cos(dx+c)+i(-8A-6C)\sin(dx+c))}{6\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^2 d}+\frac{(\sqrt{\cos}(dx+c))B\ln(e^{i(dx+c)}+i)}{2\sqrt{\cos(dx+c)b}\,d}-($

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*B*cos(d*x+c)^3*ln(-c
ot(d*x+c)+csc(d*x+c)-1)+4*A*sin(d*x+c)*cos(d*x+c)^2+6*C*cos(d*x+c)^2*sin(d*
x+c)+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x
+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.78

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{3\,B\sqrt{b}\cos(dx+c)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,(2\,(2\,A+3\,C)\cos(\right.}{12\,bd\cos(dx+c)^4}$$
$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^4-\left(2\,(2\,A+3\,C)\cos(dx+c)^2+3\,B\cos(dx+c)\right.}{6\,bd\cos(dx+c)^4}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^4), -1/6*(
3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(
d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x +
c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*
x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. 2(128) = 256.

Time = 0.49 (sec) , antiderivative size = 1014, normalized size of antiderivative = 6.67

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2
*c)^2 + 2*b*cos(2*d*x + 2*c) + b) + 16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x
+ 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*s
in(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d*x +
 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(
3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d
*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + si
n(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)) - 3*(4*(sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x
 + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x +
 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*
d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*B/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x
 + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))
/d
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos
(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
d*x))^(1/2)), x)
```

3.322 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 193

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(3A+4C)\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{B\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*A*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*sin(
d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*B*sin(d*x+c)^3/d/cos(d*x
+c)^(5/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c
))^(1/2)+1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {18, 3100, 2827, 3852, 3853, 3855}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(3A+4C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{8d\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[
c + d*x]]),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c +
d*x]]) + (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (
(3*A + 4*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (
B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*
x]^3)/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
```

```
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\sqrt{\cos(c+dx)}\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)\,dx}{4\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{4\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{8\sqrt{b\cos(c+dx)}}\\
&\quad-\frac{\left(B\sqrt{\cos(c+dx)}\right)\operatorname{Subst}\left(\int\left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{B\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.57

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{3(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\left(6A+3(3A+4C)\cos^2(c+dx)+24B\cos^3(c\right.}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[
b*Cos[c + d*x]]),x]
```

```
[Out] (3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3
*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c
+ d*x]^2))/(24*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 10.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.10

method	result
default	$\frac{-9A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+9A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-12C(\cos^4(dx+c))\ln(-\cot(dx+}{}$
parts	$-\frac{A\left(3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)-3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-3(\cos^2(dx+c))\sin(dx+c)-2\sin\right.}{8d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{i\left(9A\,e^{6i(dx+c)}+12C\,e^{6i(dx+c)}+33A\,e^{4i(dx+c)}+12C\,e^{4i(dx+c)}-48B\,e^{3i(dx+c)}-33A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-9A-12C-80B\cos(d\right.}{24\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)^3 d}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(cos(d*x+c)*b)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(-9*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+9*A*cos(d*x+c)^4*ln(
-cot(d*x+c)+csc(d*x+c)+1)-12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+12
*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)+16*B*sin(d*x+c)*cos(d*x+c)^3+9
*A*sin(d*x+c)*cos(d*x+c)^2+12*C*cos(d*x+c)^2*sin(d*x+c)+8*B*sin(d*x+c)*cos(
d*x+c)+6*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.58

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{3\,(3\,A+4\,C)\sqrt{b}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,(16\,B\,\mathrm{co}}{48\,bd\,\mathrm{c}}\right.$$
$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left(16\,B\cos\left(dx+c\right)^3+3\,(3\,A+4\,C)\,\mathrm{c}}{24\,bd\cos\left(dx+c\right)^5}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 +
 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(b*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d
*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (16
*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A)*
sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(165) = 330$.

Time = 0.57 (sec) , antiderivative size = 2611, normalized size of antiderivative = 13.53

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
 + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
```

```
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c
) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*co
s(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2
*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
+ 2*c) + 1)*sqrt(b)) - 64*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3
*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*
c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*B/((2*(3*cos(4*d*x + 4*c) + 3*cos
(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 +
 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c
)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2
*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)) + 12*(4*(sin(4*d*x + 4*c)
+ 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + co
s(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*s
in(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*C/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A}{\sqrt{b\cos\left(dx + c\right)}\cos\left(dx + c\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos
(d*x + c)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + B\,\cos\left(c + d\,x\right) + A}{\cos\left(c + d\,x\right)^{9/2}\,\sqrt{b\,\cos\left(c + d\,x\right)}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c +
d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c +
d*x))^(1/2)), x)
```

3.323 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 199

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}-\frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/4*C*co
s(d*x+c)^(7/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(d*x+
c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*
x+c))^(1/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {17, 3102, 2827, 2715, 8, 2713}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8bd\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c +
d*x]^(3/2)*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/
2)*Sin[c + d*x])/(4*b*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)} \int \cos^2(c+dx) \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \, dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)} \int \cos^2(c+dx)(4A+3C+4B\cos(c+dx))\,dx}{4b\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right) \int \cos^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right) \int \cos^2(c+dx)\,dx}{4b\sqrt{b\cos(c+dx)}}$$

$$= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right) \int 1\,dx}{8b\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+dx)}\right) \text{Subst}\left(\int (1-x^2)\,dx, x, -\sin(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}$$

$$= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} - \frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.46

$$\int \frac{\cos^{\frac{7}{2}}(c+dx) \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}} \, dx = \frac{\cos^{\frac{3}{2}}(c+dx)(48Ac + 36cC + 48Adx + 36Cdx$$

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d
*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d
*x)]))/(96*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 10.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.59

method	result
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(6C\left(\cos^3(dx+c)\right)\sin(dx+c)+8B\sin(dx+c)\left(\cos^2(dx+c)\right)+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A(dx\right.}{24bd\sqrt{\cos(dx+c)b}}$
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)(\cos(dx+c)\sin(dx+c)+dx+c)}{2db\sqrt{\cos(dx+c)b}}+\frac{B\left(2+\cos^2(dx+c)\right)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{3db\sqrt{\cos(dx+c)b}}+\frac{C\left(\sqrt{\cos}(dx+c)\right)\left(2\sin(dx+c)\left(\cos^3\right.\right.}{8db\sqrt{}}$
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)x(8A+6C)}{16b\sqrt{\cos(dx+c)b}}+\frac{3B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{4bd\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(4dx+4c)}{32b\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(3dx+3c)}{12b\sqrt{\cos(dx+c)b}\,d}+\frac{(\sqrt{}}{}$

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/b/d*cos(d*x+c)^(1/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*sin(d*x+c)*cos(d
*x+c)^2+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)+1
6*B*sin(d*x+c)+9*C*(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.42

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\left[-\frac{3\,(4\,A+3\,C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx\right.\right.}{}\right.$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt
(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(6*C*cos
(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(
b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)), 1/24
*(3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*c
os(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2
 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{3}{2}}} + \frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+}{}}{}$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(3/2) + 3*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/
b^(3/2) + 8*B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))/b^(3/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos
(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+d\,x\right.\right.}{}$$

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
+ d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*d*
x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x) +
 72*C*d*x*cos(c + d*x)))/(96*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.324 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 155

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/3*C*cos(d*x+c)
^(5/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*x+c)^(1/2)/b/(b*co
s(d*x+c))^(1/2)+1/3*(3*A+2*C)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {17, 3102, 2813}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{(3A+2C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sin[c
 + d*x])/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(3A+2C+3B\cos(c+dx))\,dx}{3b\sqrt{b\cos(c+dx)}}$$

$$= \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(6Bc+6Bdx+3(4A+3C)\sin(c$$

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]

[Out] (Cos[c + d*x]^(3/2)*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*(b*Cos[c + d*x])^(3/2))

Maple [A] (verified)

Time = 9.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

method	result	si
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(2C\left(\cos^2(dx+c)\right)\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+6A\sin(dx+c)+3B(dx+c)+4\sin(dx+c)C\right)}{6bd\sqrt{\cos(dx+c)b}}$	86
parts	$\frac{A\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{bd\sqrt{\cos(dx+c)b}}+\frac{B\left(\sqrt{\cos}(dx+c)\right)(\cos(dx+c)\sin(dx+c)+dx+c)}{2db\sqrt{\cos(dx+c)b}}+\frac{C\left(2+\cos^2(dx+c)\right)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{3db\sqrt{\cos(dx+c)b}}$	12
risch	$\frac{Bx\left(\sqrt{\cos}(dx+c)\right)}{2b\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)(4A+3C)\sin(dx+c)}{4b\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(3dx+3c)}{12b\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(2dx+2c)}{4b\sqrt{\cos(dx+c)b}\,d}$	13

[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/6/b/d*cos(d*x+c)^(1/2)*(2*C*cos(d*x+c)^2*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/(cos(d*x+c)*b)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.56

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{3\,B\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+\right.}{}\right.$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c)^

```
2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(b^2*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x +
 c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x
 + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{3}{2}}} + \frac{C\,(\sin(3\,dx+3\,c)+9\,\sin(\frac{1}{3}\,\text{arctan}}{b^{\frac{3}{2}}}}{12\,d}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(3/2) + C*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(3/2) + 12*A*sin(
d*x + c)/b^(3/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos
(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(3\,B\,\sin(c+d$$

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*c
 + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*d*
x) + 12*B*d*x*cos(c + d*x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.325 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 135

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)^(
1/2)/b/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)
+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 2717, 2715, 8}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]]
)/(2*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqr
```

```
t[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[
c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\cos(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int\cos^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2b\sqrt{b\cos(c+dx)}}\\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(2(2A+C)(c+dx)+4B\sin(c+dx)}{4d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c
+ d*x)]))/(4*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 10.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{(\sqrt{\cos}(dx+c))(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))}{2bd\sqrt{\cos(dx+c)b}}$	66
risch	$\frac{(\sqrt{\cos}(dx+c))x(4A+2C)}{4b\sqrt{\cos(dx+c)b}}+\frac{B\sin(dx+c)(\sqrt{\cos}(dx+c))}{bd\sqrt{\cos(dx+c)b}}+\frac{(\sqrt{\cos}(dx+c))C\sin(2dx+2c)}{4b\sqrt{\cos(dx+c)b}\,d}$	101
parts	$\frac{A(\sqrt{\cos}(dx+c))(dx+c)}{d\sqrt{\cos(dx+c)b}\,b}+\frac{B\sin(dx+c)(\sqrt{\cos}(dx+c))}{bd\sqrt{\cos(dx+c)b}}+\frac{C(\sqrt{\cos}(dx+c))(\cos(dx+c)\sin(dx+c)+dx+c)}{2db\sqrt{\cos(dx+c)b}}$	110

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/d*cos(d*x+c)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c
)+C*(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.61

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{(2A+C)\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+\right.}{}\right.$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*co
s(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x +
```

```
c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(
d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{3}{2}}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}+\frac{4\,B\ldots}{\ldots}}{4\,d}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(3/2) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/b^(3/2) + 4*B*sin(d*x + c)/b^(3/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(\ldots\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(C\,\sin(c+d\,x)\ -$$

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + 4*B*sin(2*c +
2*d*x) + C*sin(3*c + 3*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))
/(4*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.326 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] B*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c
)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d
*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 3102, 2814, 3855}

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{A\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]
]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(Bdx+A\operatorname{arctanh}(\sin(c+dx))+}{d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(B*d*x + A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*
(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 10.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result
default	$-\frac{(2A\ \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c)-\sin(dx+c)C)\left(\sqrt{\cos}(dx+c)\right)}{bd\sqrt{\cos(dx+c)b}}$
parts	$-\frac{2A\ \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))\left(\sqrt{\cos}(dx+c)\right)}{db\sqrt{\cos(dx+c)b}}+\frac{B\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\sqrt{\cos(dx+c)b}\,b}+\frac{C\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{bd\sqrt{\cos(dx+c)b}}$
risch	$\frac{Bx\left(\sqrt{\cos}(dx+c)\right)}{b\sqrt{\cos(dx+c)b}}-\frac{i\left(\sqrt{\cos}(dx+c)\right)C\,\mathrm{e}^{i(dx+c)}}{2b\sqrt{\cos(dx+c)b}\,d}+\frac{i\left(\sqrt{\cos}(dx+c)\right)C\,\mathrm{e}^{-i(dx+c)}}{2b\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{b\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+\right.}{b\sqrt{}}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*cos(d*x+
c)^(1/2)/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.03

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{2A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos}{}\right.$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c) + B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x +
```

```
 c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b
) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*
x + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)
*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x
 + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) -
2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c\right.\right.}{b^{\frac{3}{2}}}}{}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/b^(3/2) + 2*C*sin(d*x + c)/b^(3/2))/d
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\cos}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(c+dx)}\,\left(C\cos(c+dx)^2+B\,\cos(c+$$

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(3/2),x)
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(3/2), x)
```

3.327 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{B\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/2)
/b/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d
*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {18, 3100, 2814, 3855}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c +
d*x])^(3/2)),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (B*ArcTanh[Sin[c + d*x]
]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b*d*Sq
rt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int(B+C\cos(c+dx))\sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{\cos(c+dx)}(Cdx\cos(c+dx)+B\text{arctanh}(\sin(c+dx))\cos(c-}{d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d
*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 9.89 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{-2B\cos(dx+c)\,\text{arctanh}(\cot(dx+c)-\csc(dx+c))+C\cos(dx+c)(dx+c)+A\sin(dx+c)}{bd\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	73
parts	$\frac{A\sin(dx+c)}{bd\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}-\frac{2B\,\text{arctanh}(\cot(dx+c)-\csc(dx+c))\left(\sqrt{\cos}(dx+c)\right)}{db\sqrt{\cos(dx+c)b}}+\frac{C\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\sqrt{\cos(dx+c)b}\,b}$	108
risch	$\frac{Cx\left(\sqrt{\cos}(dx+c)\right)}{b\sqrt{\cos(dx+c)b}}+\frac{i\mathrm{e}^{-i(dx+c)}A}{b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{b\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{b\sqrt{\cos(dx+c)b}\,d}$	142

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+C*cos(d*x+c)*(d*x+c)+
A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.11

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\left[-\frac{2\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^2+C\sqrt{}}{}\right.$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^2 + C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d
```

```
*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
 - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*co
s(d*x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(s
qrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*log(-
(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d
*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt
(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/((b*cos(c + d*x))**(3/2)*
sqrt(cos(c + d*x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{4A\sqrt{b}\sin(2dx+2c)}{b^2\cos(2dx+2c)^2+b^2\sin(2dx+2c)^2+2b^2\cos(2dx+2c)+b^2} + \frac{B\left(\log\left(\cos(dx+c)^2+\cdots\right.\right.}{\cdots}}{\cdots}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + B*(log(cos(d*x + c)^2 + sin(d*x +
 c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d
*x + c) + 1))/b^(3/2) + 4*C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2)
)/d
```

Giac [F]

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos(dx+c)^2 + B\cos(dx+c) + A}{(b\cos(dx+c))^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*s
qrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos(c+dx)^2 + B\cos(c+dx) + A}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(3/2)), x)
```

3.328 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 120

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b/d
/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*arctanh(sin(d*x+c))*cos(
d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {18, 3100, 2827, 3852, 8, 3855}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{(A+2C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c +
d*x])^(3/2)),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c +
d*x]]) + (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) +
 (B*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int (2B+(A+2C)\cos(c+dx))\sec^2(c+dx)\,dx}{2b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left((A+2C)\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2b\sqrt{b\cos(c+dx)}}$$

$$= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+dx)}\right)\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}$$

$$= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+}{2d\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Si
n[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 9.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11

method	result
default	$\frac{A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)-4C\left(\cos^2(dx+c)\right)\text{arctanh}(\cot(dx+}{2bd\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i\left(A\,e^{2i(dx+c)}-A-4B\cos(dx+c)\right)}{2b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)d}-\frac{\left(\sqrt{\cos}(dx+c)\right)(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2b\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)(A+2C)\ln\left(e^{i(dx+c)}+\right.}{2b\sqrt{\cos(dx+c)b}\,d}$
parts	$\frac{A\left(-\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)\right)}{2db\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}+\frac{B\sin(dx}{bd\sqrt{\cos(dx+c)}\,\sqrt{}}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot
(d*x+c)+csc(d*x+c)-1)-4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))+2*B*s
in(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.99

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\ldots}{\cos(dx+c)\ldots}\right)\ldots}{\ldots}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\left(2\,B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\ldots}}{2\,b^2d\cos\left(dx+c\right)^3}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(
sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x
+ c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b^2*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(104) = 208$.

Time = 0.51 (sec) , antiderivative size = 802, normalized size of antiderivative = 6.68

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*
c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x
 + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b*co
s(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x +
 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b)) + 2*C*(log
(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2
 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2))/d
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c))^{\frac{3}{2}}\cos(dx + c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*c
os(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{3/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(3/2)), x)
```

3.329 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 164

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{B\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*A*sin(d*x+c)/b/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/2*B*sin(d*x+c)
/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(d*x+c)/b/d/cos
(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/
2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {18, 3100, 2827, 3853, 3855, 3852, 8}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c +
d*x])^(3/2)),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) +
 (A*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[
c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Si
n[c + d*x])/(3*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int(3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)\,dx}{3b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{3b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2b\sqrt{b\cos(c+dx)}} - \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{3bd\sqrt{b\cos(c+dx)}}$$

$$= \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{3B\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c+}{6d\sqrt{\cos(c+dx)}(b\cos(c+}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x] +
 (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(b*Cos
[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 8.70 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

method	result
default	$\frac{3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+4A\sin(dx+c)(\cos^2(dx+c))+6C(}{6bd\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A(2(\cos^2(dx+c))+1)\sin(dx+c)}{3db\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}-\frac{B((\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-}{2db\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(3B\,e^{4i(dx+c)}-6C\,e^{3i(dx+c)}-3B+(-16A-18C)\cos(dx+c)+i(-8A-6C)\sin(dx+c))}{6b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^2 d}+\frac{(\sqrt{\cos}(dx+c))B\ln(e^{i(dx+c)}+i)}{2b\sqrt{\cos(dx+c)b}\,d}-\frac{(\sqrt{c}}{}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b/d*(3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*B*cos(d*x+c)^3*ln(
-cot(d*x+c)+csc(d*x+c)-1)+4*A*sin(d*x+c)*cos(d*x+c)^2+6*C*cos(d*x+c)^2*sin(
d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d
*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.65

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\left[\frac{3\,B\sqrt{b}\cos(dx+c)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\,\mathrm{si}}{\cos(dx+c)^3}\right.}{}\right.$$

$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^4-\left(2\,(2\,A+3\,C)\cos(dx+c)^2+3\,B\cos(dx+c)+2\right.}{6\,b^2d\cos(dx+c)^4}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4), -1/6
*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(co
s(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x
+ c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*co
s(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\, dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(140) = 280$.

Time = 0.52 (sec) , antiderivative size = 1048, normalized size of antiderivative = 6.39

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\, dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x
 + 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + 16*((3*cos(2*d*x + 2*c) + 1)*si
n(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x
+ 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6*
d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*d*
x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c)
 + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*
b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c) + b)*sqrt(b)) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin
```

```
(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x
+ 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin
(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*
(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*
x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*s
in(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4
*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((b*cos(4*d*x + 4*c)^2
+ 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*
d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*c
os(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{5/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(3/2)), x)
```

3.330 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 208

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{B\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*A*sin(d*x+c)/b/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*si
n(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*B*sin(d*x+c)^3/b/d/c
os(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*c
os(d*x+c))^(1/2)+1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*
cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {18, 3100, 2827, 3852, 3853, 3855}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(3A+4C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{8bd\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c +
d*x])^(3/2)),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b*d*Sqrt[b*Cos[c
+ d*x]]) + (A*Sin[c + d*x])/(4*b*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
 + ((3*A + 4*C)*Sin[c + d*x])/(8*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]
]) + (B*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (B*Si
n[c + d*x]^3)/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\sqrt{\cos(c+dx)}\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)\,dx}{4b\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{4b\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{8b\sqrt{b\cos(c+dx)}}\\
&\quad-\frac{\left(B\sqrt{\cos(c+dx)}\right)\text{Subst}\left(\int\left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}\\
&= \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.53

$$\int\frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{3(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\,(6}{24d\cos}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] (3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3
*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c
+ d*x]^2))/(24*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 9.45 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.03

method	result
default	$-\frac{9A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)-9A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+12C(\cos^4(dx+c))\ln(-\cot(dx-}{}$
parts	$\frac{A(-3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+3(\cos^2(dx+c))\sin(dx+c)+2\sin(}{8db\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{i(9A\,e^{6i(dx+c)}+12C\,e^{6i(dx+c)}+33A\,e^{4i(dx+c)}+12C\,e^{4i(dx+c)}-48B\,e^{3i(dx+c)}-33A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-9A-12C-80B\cos(d}{24b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^3 d}$

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(3/2),x ,method=_RETURNVERBOSE)

[Out] -1/24/b/d*(9*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-9*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)+12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-16*B*sin(d*x+c)*cos(d*x+c)^3-9*A*sin(d*x+c)*cos(d*x+c)^2-12*C*cos(d*x+c)^2*sin(d*x+c)-8*B*sin(d*x+c)*cos(d*x+c)-6*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.47

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{3\,(3\,A+4\,C)\sqrt{b}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{}}{\cos(}\right.}{}\right.$$

$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left(16\,B\cos\left(dx+c\right)^3+3\,(3\,A+4\,C)\cos\left(\right.\right.}{24\,b^2d\cos\left(dx+c\right)^5}$$

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^5)]

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))
**(3/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. $2(180) = 360$.

Time = 0.57 (sec) , antiderivative size = 2660, normalized size of antiderivative = 12.79

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
 + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
```

```
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A/((b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*
x + 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*
x + 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*
c) + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) +
 4*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)
*cos(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*si
n(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x
 + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) - 64*((3*cos
(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x
+ 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x
 + 2*c))*B/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x
+ 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) +
 3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*
cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) + 12*(4*(sin(4*d*x + 4*c) + 2*sin
(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(si
n(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x
 + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(
2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/(
```

```
(b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*
d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*c
os(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{7/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
d*x))^(3/2)), x)
```

3.331 $\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 199

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} - \frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/4*C*
cos(d*x+c)^(7/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(
d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(
b*cos(d*x+c))^(1/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {17, 3102, 2827, 2715, 8, 2713}

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} - \frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*
x]^(7/2)*Sin[c + d*x])/(4*b^2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x
]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)(4A+3C+4B\cos(c+dx))\,dx}{4b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8b^2\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+dx)}\right)\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

$$= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} - \frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.48

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(48Ac+36cC+48Adx+36Cdx}{}$$

```
[In] Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d
*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d
*x)]))/(96*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.59

method	result
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(6C\left(\cos^{3}(dx+c)\right)\sin(dx+c)+8B\sin(dx+c)\left(\cos^{2}(dx+c)\right)+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A(\right.}{24b^{2}d\sqrt{\cos(dx+c)b}}$
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)\left(\cos(dx+c)\sin(dx+c)+dx+c\right)}{2d\,b^{2}\sqrt{\cos(dx+c)b}}+\frac{B\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{3d\,b^{2}\sqrt{\cos(dx+c)b}}+\frac{C\left(\sqrt{\cos}(dx+c)\right)\left(2\sin(dx+c)(\cos\right.}{8d\,b}$
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)x(8A+6C)}{16b^{2}\sqrt{\cos(dx+c)b}}+\frac{3B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{4b^{2}d\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(4dx+4c)}{32b^{2}\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(3dx+3c)}{12b^{2}\sqrt{\cos(dx+c)b}\,d}+\frac{(}{}$

```
[In] int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/b^2/d*cos(d*x+c)^(1/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*sin(d*x+c)*cos
(d*x+c)^2+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)
+16*B*sin(d*x+c)+9*C*(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.42

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\left[-\frac{3\,(4\,A+3\,C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(\right.\right.}{}\right.$$

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt
(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(6*C*cos
(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(
b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)), 1/24
*(3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*c
os(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2
 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{5}{2}}} + \frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8}{}}{}$$

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + 3*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/
b^(5/2) + 8*B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))/b^(5/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(b*cos
(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+d\right.\right.}{}$$

```
[In] int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
+ d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*d*
x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x) +
 72*C*d*x*cos(c + d*x)))/(96*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.332 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 155

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/3*C*cos(d*x+
c)^(5/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*x+c)^(1/2)/b^2
/(b*cos(d*x+c))^(1/2)+1/3*(3*A+2*C)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*co
s(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {17, 3102, 2813}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\frac{(3A+2C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]
^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)
*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(3A+2C+3B\cos(c+dx))\,dx}{3b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}} \\
&\quad+\frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.50

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos(c+dx)}(6Bc+6Bdx+3(4A+3C)\sin(c}{12b^2d\sqrt{b\cos}}$$

[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]

[Out] (Sqrt[Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 9.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{(\sqrt{\cos}(dx+c))\left(2C\left(\cos^2(dx+c)\right)\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+6A\sin(dx+c)+3B(dx+c)+4\sin(dx+c)C\right)}{6b^2d\sqrt{\cos(dx+c)b}}$	86
parts	$\frac{A\sin(dx+c)(\sqrt{\cos}(dx+c))}{b^2d\sqrt{\cos(dx+c)b}}+\frac{B(\sqrt{\cos}(dx+c))(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\,b^2\sqrt{\cos(dx+c)b}}+\frac{C\left(2+\cos^2(dx+c)\right)\sin(dx+c)(\sqrt{\cos}(dx+c))}{3d\,b^2\sqrt{\cos(dx+c)b}}$	122
risch	$\frac{Bx(\sqrt{\cos}(dx+c))}{2b^2\sqrt{\cos(dx+c)b}}+\frac{(\sqrt{\cos}(dx+c))(4A+3C)\sin(dx+c)}{4b^2\sqrt{\cos(dx+c)b}\,d}+\frac{(\sqrt{\cos}(dx+c))C\sin(3dx+3c)}{12b^2\sqrt{\cos(dx+c)b}\,d}+\frac{(\sqrt{\cos}(dx+c))B\sin(2dx+2c)}{4b^2\sqrt{\cos(dx+c)b}\,d}$	138

[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/6/b^2/d*cos(d*x+c)^(1/2)*(2*C*cos(d*x+c)^2*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/(cos(d*x+c)*b)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.56

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\left[-\frac{3\,B\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\right.}{}\right.$$

[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c)^

```
2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(b^3*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x +
 c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x
 + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{5}{2}}} + \frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\text{arcta}\right.\right.}{}}{12\,d}$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(5/2) + C*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(5/2) + 12*A*sin(
d*x + c)/b^(5/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos
(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,B\,\sin\left(c+d\,x\right)\right.}{}$$

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*c
 + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*d*
x) + 12*B*d*x*cos(c + d*x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.333 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 135

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)
^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))
^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2717, 2715, 8}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x
]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2
```

```
*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqr
t[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{\left(C\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+4B\sin(c+
}{4b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c
+ d*x)]))/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 10.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c)\right)}{2b^2d\sqrt{\cos(dx+c)b}}$	66
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)x(4A+2C)}{4b^2\sqrt{\cos(dx+c)b}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{b^2d\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(2dx+2c)}{4b^2\sqrt{\cos(dx+c)b}\,d}$	101
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\,b^2\sqrt{\cos(dx+c)b}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{b^2d\sqrt{\cos(dx+c)b}}+\frac{C\left(\sqrt{\cos}(dx+c)\right)(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\,b^2\sqrt{\cos(dx+c)b}}$	110

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^2/d*cos(d*x+c)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x
+c)+C*(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.61

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\left[-\frac{(2\,A+C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx-\right.\right.}{}\right.$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*co
s(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x +
```

```
c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(
d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{5}{2}}} + \frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}} + \frac{4\,B\,\mathrm{s}}{}}{4\,d}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(5/2) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/b^(5/2) + 4*B*sin(d*x + c)/b^(5/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos
(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)\right.}{}$$

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + 4*B*sin(2*c +
2*d*x) + C*sin(3*c + 3*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))
/(4*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.334 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] B*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x
+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b
*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 3102, 2814, 3855}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{A\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*
x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(Bx+\frac{A\text{arctanh}(\sin(c+dx))}{d}+\frac{C\sin(c\text{-}}{d}\right.}{b^2\sqrt{b\cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(B*x + (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d
))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 10.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result
default	$-\frac{(2A\ \text{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c)-\sin(dx+c)C)\left(\sqrt{\cos}(dx+c)\right)}{b^2 d\sqrt{\cos(dx+c)b}}$
parts	$-\frac{2A\left(\sqrt{\cos}(dx+c)\right)\text{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)b}\,b^2}+\frac{B\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\,b^2\sqrt{\cos(dx+c)b}}+\frac{C\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{b^2 d\sqrt{\cos(dx+c)b}}$
risch	$\frac{Bx\left(\sqrt{\cos}(dx+c)\right)}{b^2\sqrt{\cos(dx+c)b}}-\frac{i\left(\sqrt{\cos}(dx+c)\right)C\,\mathrm{e}^{i(dx+c)}}{2b^2\sqrt{\cos(dx+c)b}\,d}+\frac{i\left(\sqrt{\cos}(dx+c)\right)C\,\mathrm{e}^{-i(dx+c)}}{2b^2\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{b^2\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c\right.}{b^2\sqrt{c}}$

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*cos(d*
x+c)^(1/2)/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.03

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \left[-\frac{2A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(\right.}{}\right.$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c) + B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x +
 c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b
) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*
x + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)
*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x
 + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) -
2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+\right.\right.}{b^{\frac{5}{2}}}}{}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/b^(5/2) + 2*C*sin(d*x + c)/b^(5/2))/d
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}$$

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2), x)
```

3.335 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{B\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/
2)/b^2/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b
*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 3100, 2814, 3855}

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (B*ArcTanh[Sin[c + d*
x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b^
2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)} \int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int (B+C\cos(c+dx))\sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(Cdx\cos(c+dx)+B\text{arctanh}(\sin}{d(b\cos(c+dx}$$

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d
*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 10.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

method	result	s
default	$\frac{-2B\cos(dx+c)\,\text{arctanh}(\cot(dx+c)-\csc(dx+c))+C\cos(dx+c)(dx+c)+A\sin(dx+c)}{b^2 d\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	7
parts	$\frac{A\sin(dx+c)}{b^2 d\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}-\frac{2B(\sqrt{\cos}(dx+c))\,\text{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)b}\,b^2}+\frac{C(\sqrt{\cos}(dx+c))(dx+c)}{d\,b^2\sqrt{\cos(dx+c)b}}$	1
risch	$\frac{Cx(\sqrt{\cos}(dx+c))}{b^2\sqrt{\cos(dx+c)b}}+\frac{2i(\sqrt{\cos}(dx+c))A}{b^2\sqrt{\cos(dx+c)b}\,d(e^{2i(dx+c)}+1)}+\frac{(\sqrt{\cos}(dx+c))B\ln(e^{i(dx+c)}+i)}{b^2\sqrt{\cos(dx+c)b}\,d}-\frac{(\sqrt{\cos}(dx+c))B\ln(e^{i(dx+c)}-i)}{b^2\sqrt{\cos(dx+c)b}\,d}$	1

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+C*cos(d*x+c)*(d*x+c
)+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.11

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \left[-\frac{2\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos}{}\right.$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^2 + C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d
```

```
*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
 - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*co
s(d*x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(s
qrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*log(-
(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d
*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt
(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{4\,A\sqrt{b}\sin(2\,dx+2\,c)}{b^3\cos(2\,dx+2\,c)^2+b^3\sin(2\,dx+2\,c)^2+2\,b^3\cos(2\,dx+2\,c)+b^3} + \frac{B}{\ldots}}{\ldots}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + B*(log(cos(d*x + c)^2 + sin(d*x +
 c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d
*x + c) + 1))/b^(5/2) + 4*C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2)
)/d
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2), x)
```

3.336 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 120

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b
^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*arctanh(sin(d*x+c))*
cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {18, 3100, 2827, 3852, 8, 3855}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{(A+2C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c +
d*x])^(5/2)),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c
+ d*x]]) + (A*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]
]) + (B*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{\sqrt{\cos(c+dx)}\int (2B+(A+2C)\cos(c+dx))\sec^2(c+dx)\,dx}{2b^2\sqrt{b\cos(c+dx)}}
\end{aligned}$$

$$= \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left((A+2C)\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

$$= \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}((A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A}{2d(b\cos(c+dx))^{5/2}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Co
s[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A +
2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 10.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

method	result
default	$-\frac{A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)-A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+4C(\cos^2(dx+c))\,\text{arctanh}(\cot(dx}{2b^2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(A\,e^{2i(dx+c)}-A-4B\cos(dx+c))}{2b^2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)d} - \frac{(\sqrt{\cos}(dx+c))(A+2C)\ln(e^{i(dx+c)}-i)}{2b^2\sqrt{\cos(dx+c)b}\,d} + \frac{(\sqrt{\cos}(dx+c))(A+2C)\ln(e^{i(dx+c)}+i}{2b^2\sqrt{\cos(dx+c)b}\,d}$
parts	$\frac{A(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c))}{2d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}} + \frac{B\sin(dx+}{b^2d\sqrt{\cos(dx+c)}\,\sqrt{}}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)-A*cos(d*x+c)^2*ln(-
cot(d*x+c)+csc(d*x+c)+1)+4*C*cos(d*x+c)^2*arctanh(cot(d*x+c)-csc(d*x+c))-2*
B*sin(d*x+c)*cos(d*x+c)-A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.99

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cdots}}{\cos(dx\cdots}\right.}{\cdots}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\left(2\,B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cdots}}{2\,b^3d\cos\left(dx+c\right)^3}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(
sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x
+ c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b^3*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(104) = 208.

Time = 0.53 (sec) , antiderivative size = 820, normalized size of antiderivative = 6.83

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*
c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x
 + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b^2*
cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*
b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*co
s(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*s
qrt(b)) + 2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2))/d
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*s
qrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(5/2)),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(5/2)), x)
```

3.337 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 164

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx &= \frac{B\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} \\ &+ \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} \\ &+ \frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\end{aligned}$$

```
[Out] 1/3*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/2*B*sin(d*x+
c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(d*x+c)/b^2
/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+
c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {18, 3100, 2827, 3853, 3855, 3852, 8}

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx &= \frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} \\ &+ \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}} \\ &+ \frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c +
d*x])^(5/2)),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
 + (A*Sin[c + d*x])/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*
Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3
*C)*Sin[c + d*x])/(3*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}\int\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} \\
&\quad+\frac{\sqrt{\cos(c+dx)}\int(3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)\,dx}{3b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&\quad+\frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{3b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} \\
&\quad+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2b^2\sqrt{b\cos(c+dx)}} \\
&\quad-\frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}} \\
&= \frac{B\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} \\
&\quad+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(3B\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(4A+3}{6d(b\cos(c}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Co
s[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C
+ 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*(b*C
os[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 9.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

method	result
default	$\frac{3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+4A\sin(dx+c)(\cos^2(dx+c))+6C}{6b^2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A(2(\cos^2(dx+c))+1)\sin(dx+c)}{3d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}+\frac{B(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+}{2d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(3B\,e^{4i(dx+c)}-6C\,e^{3i(dx+c)}-3B+(-16A-18C)\cos(dx+c)+i(-8A-6C)\sin(dx+c))}{6b^2\sqrt{\cos(dx+c)b}\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^2d}+\frac{(\sqrt{\cos}(dx+c))B\ln(e^{i(dx+c)}+i)}{2b^2\sqrt{\cos(dx+c)b}\,d}-\frac{(\sqrt{}}{}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b^2/d*(3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)-3*B*cos(d*x+c)^3*l
n(-cot(d*x+c)+csc(d*x+c)-1)+4*A*sin(d*x+c)*cos(d*x+c)^2+6*C*cos(d*x+c)^2*si
n(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos
(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.65

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{3\,B\sqrt{b}\cos(dx+c)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}\right.}{}\right.$$

$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^4-(2\,(2\,A+3\,C)\cos(dx+c)^2+3\,B\cos(dx+c)+}{6\,b^3d\cos(dx+c)^4}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^4), -1/6
*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(co
s(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x
+ c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*co
s(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(140) = 280$.

Time = 0.52 (sec) , antiderivative size = 1098, normalized size of antiderivative = 6.70

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x
 + 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + 16*((3*cos(2*d*x + 2*c) + 1)*si
n(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x
+ 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos(
6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*
sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2
*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) +
 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c
) + b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)) - 3*(4*(sin(4*d*x + 4*
c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
```

```
)) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + c
os(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*l
og(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c
) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*B/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*
x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*
c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(
4*d*x + 4*c))*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*c
os(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{3/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(5/2)), x)
```

3.338 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 208

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{B\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*
sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*B*sin(d*x+c)^3/b
^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b^2/d/cos(d*x+c)^(1
/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)
/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {18, 3100, 2827, 3852, 3853, 3855}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{(3A+4C)\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c +
d*x])^(5/2)),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b^2*d*Sqrt[b*Cos[
c + d*x]]) + (A*Sin[c + d*x])/(4*b^2*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*
x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c
+ d*x]]) + (B*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
 + (B*Sin[c + d*x]^3)/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\cos(c+dx)}\int(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)\,dx}{4b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{4b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{8b^2\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+dx)}\right)\text{Subst}\left(\int(1+x^2)\,dx, x, -\tan(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

$$= \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{B\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.53

$$\int\frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{3(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\,(6A}{24d\cos^{\frac{3}{2}}(}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Co
s[c + d*x])^(5/2)),x]
```

```
[Out] (3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3
*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c
+ d*x]^2))/(24*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 9.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.03

method	result
default	$\frac{-9A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+9A(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-12C(\cos^4(dx+c))\ln(-\cot(dx}{}$
parts	$\frac{A\left(-3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+3(\cos^2(dx+c))\sin(dx+c)+2\sin\right.}{8d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{i\left(9A\,e^{6i(dx+c)}+12C\,e^{6i(dx+c)}+33A\,e^{4i(dx+c)}+12C\,e^{4i(dx+c)}-48B\,e^{3i(dx+c)}-33A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-9A-12C-80B\cos(\right.}{24b^2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)^3 d}$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/b^2/d*(-9*A*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+9*A*cos(d*x+c)^4
*ln(-cot(d*x+c)+csc(d*x+c)+1)-12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1
)+12*C*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)+16*B*sin(d*x+c)*cos(d*x+c)
^3+9*A*sin(d*x+c)*cos(d*x+c)^2+12*C*cos(d*x+c)^2*sin(d*x+c)+8*B*sin(d*x+c)*
cos(d*x+c)+6*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.47

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{3\,(3\,A+4\,C)\sqrt{b}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}}{\cos}\right.}{}\right.$$

$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left(16\,B\cos\left(dx+c\right)^3+3\,(3\,A+4\,C)\cos\right.}{24\,b^3d\cos\left(dx+c\right)^5}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 +
 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(b^3*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos
(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (
16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)
^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2760 vs. $2(180) = 360$.

Time = 0.58 (sec) , antiderivative size = 2760, normalized size of antiderivative = 13.27

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
 + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
```

```
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A/((b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*co
s(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*
b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c
)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b
^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x +
 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x
 + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d
*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(
2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)) - 64*((3*cos(2*d*x + 2*c) + 1)*sin(6*
d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*
c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*B/((b^2*cos(6*d*
x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(
6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3*
b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*(
3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) +
b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)) + 12*(4*(sin(4*d*x + 4*c)
+ 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + co
s(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*s
in(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) +
```

```
2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*C/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x +
 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^
2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d
*x + 4*c))*sqrt(b)))/d
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{5}{2}}\cos\left(dx + c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*c
os(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + B\,\cos\left(c + d\,x\right) + A}{\cos\left(c + d\,x\right)^{5/2}\,\left(b\,\cos\left(c + d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(5/2)), x)
```

3.339 $\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\right.$

Optimal result

Integrand size = 39, antiderivative size = 154

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^2/d-3/88*(11*A+8*C)*(b*cos(d*x+c))
^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c
)^2)^(1/2)-3/11*B*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*
x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{88b^2d\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3B\sin(c+dx)(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)}{11b^3d\sqrt{\sin^2(c+dx)}}$$
$$+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11b^2d}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^2*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c +
 d*x])/(88*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hyper
geometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^3*d*Sqrt[
Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (b\cos(c+dx))^{5/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b}\\
&= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}\\
&\quad+\frac{3\int (b\cos(c+dx))^{5/3}\left(\frac{1}{3}b(11A+8C)+\frac{11}{3}bB\cos(c+dx)\right)\,dx}{11b^2}\\
&= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}+\frac{B\int (b\cos(c+dx))^{8/3}\,dx}{b^2}\\
&\quad+\frac{(11A+8C)\int (b\cos(c+dx))^{5/3}\,dx}{11b}\\
&= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}\\
&\quad-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^2d\sqrt{\sin^2(c+dx)}}\\
&\quad-\frac{3B(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3(b\cos(c+dx))^{5/3}\cot(c+dx)\left(-8C\sin^2(c+dx)+(11A+8C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx\right.\right.}{88bd}$$

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Cot[c + d*x]*(-8*C*Sin[c + d*x]^2 + (11*A + 8*C)
*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 8*
B*Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sqrt[Sin[
c + d*x]^2]))/(88*b*d)
```

Maple [F]

$$\int \cos(dx+c)\left(\cos(dx+c)\,b\right)^{\frac{2}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)*(cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\left(b\cos(dx+c)\right)^{\frac{2}{3}}\cos(dx+c)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
+ c))^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\left(b\cos(dx+c)\right)^{\frac{2}{3}}\cos(dx+c)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int (C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \cos(c+dx)\,(b\,\cos(c+dx))^{2/3}\left(C\cos(c+dx)^2+B\,\cos(c+dx)+A\right)\,dx$$

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2),x)
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2), x)
```

3.340 $\int (b\cos(c+dx))^{2/3} (A + B\cos(c+dx) + C\cos^2(c+d$

Optimal result

Integrand size = 33, antiderivative size = 154

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(5/
3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^
(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*s
in(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

$= \{3102, 2827, 2722\}$

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{40bd\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}} + \frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}$$

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos[c
 + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(40*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeomet
ric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d
*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{3\int (b\cos(c+dx))^{2/3}\left(\frac{1}{3}b(8A+5C)+\frac{8}{3}bB\cos(c+dx)\right)\,dx}{8b}$$

$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{B\int(b\cos(c+dx))^{5/3}\,dx}{b} + \frac{1}{8}(8A+5C)\int(b\cos(c+dx))^{2/3}\,dx$$

$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3(b\cos(c+dx))^{2/3}\left(2(8A+5C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+1\right.}{80}$$

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
[Out] (-3*(b*Cos[c + d*x])^(2/3)*(2*(8*A + 5*C)*Cot[c + d*x]*Hypergeometric2F1[1/
2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 10*B*Cos[c + d*x]*Cot[
c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]
^2] - 5*C*Sin[2*(c + d*x)]))/(80*d)
```

Maple [F]

$$\int(\cos(dx+c)\,b)^{\frac{2}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int((cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int((cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3), x
)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}} \, dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3), x
)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx = \int \left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right) \, dx$$

```
[In] int((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.341 $\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 148

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d}-\frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/3)
*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d
*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int (b\cos(c+dx))^{2/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec(c+dx)\,dx = -\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}} + \frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5d}$$

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) - (3*(5*A + 2*C)*(b*Cos[c +
 d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(10*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F
1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2]
)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d} + \frac{3}{5}\int \frac{\frac{1}{3}b(5A+2C)+\frac{5}{3}bB\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d} + B\int (b\cos(c+dx))^{2/3}\,dx + \frac{1}{5}(b(5A+2C))\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{b\Big(-3(5A+2C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)} - 6B\cos(\ldots}{10d\sqrt[3]{b\ldots}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (b*(-3*(5*A + 2*C)*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*
x]^2]*Sqrt[Sin[c + d*x]^2] - 6*B*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F
1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*C*Sin[2*(c + d*x
)]))/(10*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{\frac{2}{3}} \left(A+B\cos \left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right) \sec \left(dx+c\right) dx$$

```
[In] int((cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
[Out] int((cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x),x)
[Out] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x), x)
```

3.342 $\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 147

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}-\frac{3B(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}+\frac{3(2A-C)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)-3/2*B*(b*cos(d*x+c))^(2/3)*hypergeo
m([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)+3/5*(2*A
-C)*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+
c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

$= \{16, 3100, 2827, 2722\}$

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}} + \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/
3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*Sqrt
[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1
/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
```

```
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b^2 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$$

$$= \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + \frac{3\int \frac{\frac{b^2B}{3} - \frac{1}{3}b^2(2A-C)\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + (bB)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx + (-2A+C)\int (b\cos(c+dx))^{2/3}\,dx$$

$$= \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} - \frac{3B(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

$$\int (b\cos(c+dx))^{2/3}\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$

$$-\frac{3b\big(-10A\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right) + \cot(c+dx)\left(5B\,\text{Hypergeometric2F1}\left(\right.\right.}{10d\sqrt[3]{b\cos(c}}$$

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (-3*b*(-10*A*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]
 + Cot[c + d*x]*(5*B*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*C
*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]))*Sqrt[Sin[
c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int (\cos (dx+c) b)^{\frac{2}{3}} \left(A+B\cos (dx+c)+C\left(\cos^2 (dx+c)\right)\right) \left(\sec^2 (dx+c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
[Out] int((cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos (dx+c)^2 + B\cos (dx+c) + A\right)(b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos (dx+c)^2 + B\cos (dx+c) + A\right)(b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2, x)
```

3.343 $\int (b\cos(c+dx))^{2/3} (A+B\cos(c+dx)+C\cos^2(c+dx)) \sec^3(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 145

$$\int (b\cos(c+dx))^{2/3} (A+B\cos(c+dx)+C\cos^2(c+dx)) \sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3bB\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)+3*b*B*hypergeom([-1/6, 1/2],[5/
6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3/8
*(A+4*C)*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(
d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used

$= \{16, 3100, 2827, 2722\}$

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}} + \frac{3bB\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*b*B*Hypergeometric
2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)
*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric
2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b^3\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx\\
&= \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{3}{4}\int \frac{\frac{4b^2B}{3}+\frac{1}{3}b^2(A+4C)\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx\\
&= \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx+\frac{1}{4}(b(A+4C))\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx\\
&= \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{3bB\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\\
&\quad-\frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2\cos(c+dx)\left(-2B\,\text{Hy}\right.\right.}{}$$

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1
/3, Cos[c + d*x]^2]) + 2*Cos[c + d*x]*(-2*B*Hypergeometric2F1[-1/6, 1/2, 5/
6, Cos[c + d*x]^2] + C*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c
+ d*x]^2]))*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F]

$$\int (\cos(dx+c)\,b)^{\frac{2}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^3(dx+c)\right)dx$$

```
[In] int((cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

```
[In] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3, x)
```

3.344 $\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 152

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3b^2B\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}+\frac{3b(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/4*b^2*B*hypergeom([-2/3, 1/2]
,[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)
+3/7*b*(4*A+7*C)*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*
cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

$= \{16, 3100, 2827, 2722\}$

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3b(4A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}} + \frac{3b^2B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] (3*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*b^2*B*Hypergeometr
ic2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(
4/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2,
5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c +
 d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
```

```
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^4\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx\\
&= \frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{1}{7}(3b)\int \frac{\frac{7b^2B}{3}+\frac{1}{3}b^2(4A+7C)\cos(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx\\
&= \frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}\\
&\quad+\left(b^3B\right)\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx+\frac{1}{7}\left(b^2(4A+7C)\right)\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx\\
&= \frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3b^2B\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}\\
&\quad+\frac{3b(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)+7\cos(c+dx)\right.}{}$$

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1
/6, Cos[c + d*x]^2] + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, C
os[c + d*x]^2] + 4*C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c +
 d*x]^2]))*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(28*d)
```

Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{2}{3}} \left(A+B\cos (dx+c)+C\left(\cos^{2}(dx+c)\right)\right) \left(\sec^{4}(dx+c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
[Out] int((cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\sec^{4}(c+dx)\, dx = \int \left(C\cos(dx+c)^{2}+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^{4}\, dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\sec^{4}(c+dx)\, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\sec^{4}(c+dx)\, dx = \int \left(C\cos(dx+c)^{2}+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^{4}\, dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

```
[In] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4, x)
```

3.345 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\right.$

Optimal result

Integrand size = 39, antiderivative size = 154

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d}-\frac{3(13A+10C)(b\cos(c+dx))^{10/3}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{130b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{13/3}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/13*C*(b*cos(d*x+c))^(10/3)*sin(d*x+c)/b^2/d-3/130*(13*A+10*C)*(b*cos(d*x+
c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d
*x+c)^2)^(1/2)-3/13*B*(b*cos(d*x+c))^(13/3)*hypergeom([1/2, 13/6],[19/6],co
s(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3(13A+10C)\sin(c+dx)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{130b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}} + \frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13b^2d}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^2*d) - (3*(13*A + 10*C)*(b
*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(130*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(13/3)*H
ypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*b^3*d*S
qrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{7/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b}$$

$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d} + \frac{3\int (b\cos(c+dx))^{7/3}\left(\frac{1}{3}b(13A+10C)+\frac{13}{3}bB\cos(c+dx)\right)\,dx}{13b^2}$$

$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d} + \frac{B\int (b\cos(c+dx))^{10/3}\,dx}{b^2} + \frac{(13A+10C)\int (b\cos(c+dx))^{7/3}\,dx}{13b}$$

$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d} - \frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{130b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3(b\cos(c+dx))^{7/3}\cot(c+dx)\left(-10C\sin^2(c+dx)+(13A+10C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+\right.\right.}{130bd}$$

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*(-10*C*Sin[c + d*x]^2 + (13*A + 10*
C)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] +
10*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2]))/(130*b*d)
```

Maple [F]

$$\int \cos \left(dx+c\right) \left(\cos \left(dx+c\right) b\right)^{\frac{4}{3}} \left(A+B\cos \left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)dx$$

```
[In] int(cos(d*x+c)*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
[Out] integral((C*b*cos(d*x + c)^4 + B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*(b*
cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*co
s(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*co
s(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2),x)
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2), x)
```

3.346 $\int (b\cos(c+dx))^{4/3} \left(A + B\cos(c + dx) + C\cos^2(c + d$

Optimal result

Integrand size = 33, antiderivative size = 154

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b/d-3/70*(10*A+7*C)*(b*cos(d*x+c))^(
7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2
)^(1/2)-3/10*B*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^
2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

$= \{3102, 2827, 2722\}$

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{70bd\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}} + \frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}$$

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*Cos
[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(70*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(10/3)*Hypergeo
metric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c
 + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd} + \frac{3\int (b\cos(c+dx))^{4/3}\left(\frac{1}{3}b(10A+7C)+\frac{10}{3}bB\cos(c+dx)\right)\,dx}{10b}$$

$$= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd} + \frac{B\int(b\cos(c+dx))^{7/3}\,dx}{b} + \frac{1}{10}(10A+7C)\int(b\cos(c+dx))^{4/3}\,dx$$

$$= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.74

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\left(-7C\sin^2(c+dx)+(10A+7C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx\right.\right.}{70d}$$

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(-7*C*Sin[c + d*x]^2 + (10*A + 7*C)
*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 7
*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sqrt[Sin[c
 + d*x]^2]))/(70*d)
```

Maple [F]

$$\int (\cos(dx+c)\,b)^{\frac{4}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm ="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co s(d*x + c))^(1/3), x)

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Maxima [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm ="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x
)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int (b\,\cos\left(c+d\,x\right))^{4/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.347 $\int (b\cos(c+dx))^{4/3} (A+B\cos(c+dx)+C\cos^2(c+dx)) \sec(c+dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 148

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec(c+dx)\,dx = \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d} - \frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(4/3)
*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
-3/7*B*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d
*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = -\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{28d\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}} + \frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7d}$$

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*d) - (3*(7*A + 4*C)*(b*Cos[c +
 d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(28*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F
1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]
)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b\int \sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\
&= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d}\\
&\quad+\frac{3}{7}\int \sqrt[3]{b\cos(c+dx)}\left(\frac{1}{3}b(7A+4C)+\frac{7}{3}bB\cos(c+dx)\right)\,dx\\
&= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d}\\
&\quad+B\int (b\cos(c+dx))^{4/3}\,dx+\frac{1}{7}(b(7A+4C))\int \sqrt[3]{b\cos(c+dx)}\,dx\\
&= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d}\\
&\quad-\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28d\sqrt{\sin^2(c+dx)}}\\
&\quad-\frac{3B(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$
\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =
$$

$$
-\frac{3b\sqrt[3]{b\cos(c+dx)}\left((7A+4C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+4B\right.}{28}
$$

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*((7*A + 4*C)*Cot[c + d*x]*Hypergeometric2F1[1/
2, 2/3, 5/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 4*B*Cos[c + d*x]*Cot[c
+ d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^
2] - 2*C*Sin[2*(c + d*x)]))/(28*d)
```

Maple [F]

$$\int (\cos (dx + c) b)^{\frac{4}{3}} \left(A + B\cos (dx + c) + C\left(\cos^2 (dx + c)\right)\right) \sec (dx + c) \, dx$$

```
[In] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
[Out] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x), x)
```

3.348 $\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 145

$$\begin{aligned}
&\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{3bC\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} \\
&-\frac{3b(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}} \\
&-\frac{3B(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

```
[Out] 3/4*b*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d-3/4*b*(4*A+C)*(b*cos(d*x+c))^(1/3
)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2
)-3/4*B*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d
*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = -\frac{3b(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}} + \frac{3bC\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4d}$$

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```

```
[Out] (3*b*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*b*(4*A + C)*(b*Cos[c
 + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x
])/(4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*Hypergeometric2
F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx \\
&= \frac{3bC\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{1}{4}(3b)\int \frac{\frac{1}{3}b(4A+C)+\frac{4}{3}bB\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx \\
&= \frac{3bC\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} + (bB)\int \sqrt[3]{b\cos(c+dx)}\,dx \\
&\quad + \frac{1}{4}\left(b^2(4A+C)\right)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx \\
&= \frac{3bC\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} \\
&\quad - \frac{3b(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{3B(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84

$$
\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{b^2\Big(-6(4A+C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)} - 6B\cos(c+\cdots}{8d(b\cos(c\cdots}
$$

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (b^2*(-6*(4*A + C)*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*
x]^2]*Sqrt[Sin[c + d*x]^2] - 6*B*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F
1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*C*Sin[2*(c + d*x)
]))/(8*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{\frac{4}{3}} \left(A+B\cos \left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right) \left(\sec^{2}\left(dx+c\right)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
[Out] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^{2}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^{2}\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2, x)
```

3.349 $\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 145

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}-\frac{3bB\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}+\frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)-3*b*B*(b*cos(d*x+c))^(1/3)*hype
rgeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)+3/8*
(A-2*C)*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d
*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

$= \{16, 3100, 2827, 2722\}$

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}} - \frac{3bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) - (3*b*B*(b*Cos[c + d*x
])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
Sqrt[Sin[c + d*x]^2]) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2
F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
```

```
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b^3 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3}{2}\int \frac{\frac{2b^2B}{3} - \frac{1}{3}b^2(A-2C)\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx - \frac{1}{2}(b(A-2C))\int \sqrt[3]{b\cos(c+dx)}\,dx$$
$$= \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} - \frac{3bB\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

$$\int (b\cos(c+dx))^{4/3}\left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{3b^2\csc(c+dx)\left(-2A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right) + \cos(c+dx)\left(4B\,\text{Hypergeometric2F1}\right.\right.}{4d(b\cos(c+}$$

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (-3*b^2*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2
] + Cos[c + d*x]*(4*B*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + C*
Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]))*Sqrt[Sin[c
+ d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{4}{3}} \left(A+B\cos (dx+c)+C\left(\cos^{2}(dx+c)\right)\right) \left(\sec^{3}(dx+c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
[Out] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\, dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\, dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\, dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\, dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\, dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3, x)
```

3.350 $\int (b\cos(c+dx))^{4/3} (A + B\cos(c+dx) + C\cos^2(c+dx)) \sec^4(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 152

$$\int (b\cos(c+dx))^{4/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^4(c+dx)\,dx = \frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} + \frac{3b^2B\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3b(2A+5C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)+3/2*b^2*B*hypergeom([-1/3, 1/2]
,[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
-3/5*b*(2*A+5*C)*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)
^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used

$= \{16, 3100, 2827, 2722\}$

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}}-\frac{3b(2A+5C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}}+\frac{3b^2B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] (3*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) + (3*b^2*B*Hypergeometr
ic2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(
2/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*(2*A + 5*C)*(b*Cos[c + d*x])^(1/3)*Hyperg
eometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*Sqrt[Sin[c +
d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
```

```
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b^4 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{8/3}}\,dx$$
$$= \frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} + \frac{1}{5}(3b)\int \frac{\frac{5b^2B}{3} + \frac{1}{3}b^2(2A+5C)\cos(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= \frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} + (b^3B)\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx + \frac{1}{5}(b^2(2A+5C))\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} + \frac{3b^2B\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3b(2A+5C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.82

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$
$$-\frac{3(b\cos(c+dx))^{4/3}\csc(c+dx)\left(-2A\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)+5\cos(c+dx)\left(-B\,\text{Hyp}\right.\right.}{}$$

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-5/6, 1/2,
1/6, Cos[c + d*x]^2] + 5*Cos[c + d*x]*(-(B*Hypergeometric2F1[-1/3, 1/2, 2/3
, Cos[c + d*x]^2]) + 2*C*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[
c + d*x]^2]))*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(10*d)
```

Maple [F]

$$\int (\cos (dx+c) b)^{\frac{4}{3}} \left(A+B\cos (dx+c)+C\left(\cos^2 (dx+c)\right)\right) \left(\sec^4 (dx+c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
[Out] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos (dx+c)^2+B\cos (dx+c)+A\right)(b\cos (dx+c))^{\frac{4}{3}}\sec (dx+c)^4 \,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3)*sec(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos (dx+c)^2+B\cos (dx+c)+A\right)(b\cos (dx+c))^{\frac{4}{3}}\sec (dx+c)^4 \,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4, x)
```

3.351 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 154

$$\begin{aligned}\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d} \\ &\quad - \frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^3d\sqrt{\sin^2(c+dx)}} \\ &\quad - \frac{3B(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^4d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

```
[Out] 3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^3/d-3/88*(11*A+8*C)*(b*cos(d*x+c))
^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c
)^2)^(1/2)-3/11*B*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*
x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= -\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{88b^3d\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3B\sin(c+dx)(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)}{11b^4d\sqrt{\sin^2(c+dx)}}$$
$$+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11b^3d}$$

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^3*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c +
 d*x])/(88*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hyper
geometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^4*d*Sqrt[
Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{5/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^2}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d} + \frac{3\int (b\cos(c+dx))^{5/3}\left(\frac{1}{3}b(11A+8C)+\frac{11}{3}bB\cos(c+dx)\right)\,dx}{11b^3}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d} + \frac{B\int (b\cos(c+dx))^{8/3}\,dx}{b^3} + \frac{(11A+8C)\int (b\cos(c+dx))^{5/3}\,dx}{11b^2}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d} - \frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^3d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^4d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx =$$

$$-\frac{3(b\cos(c+dx))^{2/3}\sin(c+dx)\left((11A+8C)\cot^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sqrt{}\right.}{}$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x]*((11*A + 8*C)*Cot[c + d*x]^2*Hyperg
eometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 8*Cos[c +
 d*x]*(-(C*Cos[c + d*x]) + B*Cot[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17
/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(88*b*d)
```

Maple [F]

$$\int \frac{(\cos^2(dx+c))\,(A+B\cos(dx+c)+C(\cos^2(dx+c)))}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
[Out] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
+ c))^(2/3)/b, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1
/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{{\cos\left(c+d\,x\right)}^2\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/3),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/3), x)
```

3.352 $\int \frac{\cos(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 154

$$
\begin{aligned}
&\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx \\
&= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d} \\
&\quad - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^2d\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{3B(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^3d\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^2/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(
5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)
^2)^(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^
2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= -\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{40b^2d\sqrt{\sin^2(c+dx)}}$$
$$- \frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}$$
$$+ \frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8b^2d}$$

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])
^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^2*d) - (3*(8*A + 5*C)*(b*Cos
[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(40*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hyperge
ometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^3*d*Sqrt[Sin[c
 + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b}$$
$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d} + \frac{3\int (b\cos(c+dx))^{2/3}\left(\frac{1}{3}b(8A+5C)+\frac{8}{3}bB\cos(c+dx)\right)\,dx}{8b^2}$$
$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d} + \frac{B\int (b\cos(c+dx))^{5/3}\,dx}{b^2} + \frac{(8A+5C)\int (b\cos(c+dx))^{2/3}\,dx}{8b}$$
$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx =$$
$$-\frac{3(b\cos(c+dx))^{2/3}\left(2(8A+5C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}\right.}{}$$

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*(2*(8*A + 5*C)*Cot[c + d*x]*Hypergeometric2F1[1/
2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 10*B*Cos[c + d*x]*Cot[
c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]
^2] - 5*C*Sin[2*(c + d*x)]))/(80*b*d)
```

Maple [F]

$$\int \frac{\cos(dx+c)\left(A+B\cos(dx+c)+C(\cos^2(dx+c))\right)}{\left(\cos(dx+c)\,b\right)^{\frac{1}{3}}}dx$$

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
[Out] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos(dx+c)}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b,
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3)
,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos(dx+c)}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(1/3),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(1/3), x)
```

3.353 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 154

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/
3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(
1/2)-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*s
in(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

$= \{3102, 2827, 2722\}$

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= -\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}}$$
$$- \frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}}$$
$$+ \frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*Cos[c
 + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x
])/(10*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometr
ic2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d
*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{3\int \frac{\frac{1}{3}b(5A+2C)+\frac{5}{3}bB\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{5b}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{B\int(b\cos(c+dx))^{2/3}\,dx}{b} + \frac{1}{5}(5A+2C)\int\frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int\frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{-3(5A+2C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)} - 6B\cos(c+dx)\cot}{10d\sqrt[3]{b\cos(c+dx}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(5*A + 2*C)*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^
2]*Sqrt[Sin[c + d*x]^2] - 6*B*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1[1
/2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*C*Sin[2*(c + d*x)])
/(10*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int\frac{A+B\cos(dx+c)+C(\cos^2(dx+c))}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}+A}{(b\cos{(dx+c)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b*
cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}+A}{(b\cos{(dx+c)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(1/3), x
)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}+A}{(b\cos{(dx+c)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(1/3), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3), x)
```

3.354 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 149

$$\begin{aligned}
&\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx\\
&=\frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}\\
&\quad-\frac{3B(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}\\
&\quad+\frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

```
[Out] 3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)-3/2*B*(b*cos(d*x+c))^(2/3)*hypergeom(
[1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)+3/5*(2*A
-C)*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+
c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{16, 3100, 2827, 2722\}$

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx$$
$$= \frac{3(2A - C)\sin(c + dx)(b\cos(c + dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c + dx)\right)}{5b^2d\sqrt{\sin^2(c + dx)}}$$
$$+ \frac{3A\sin(c + dx)}{d\sqrt[3]{b\cos(c + dx)}}$$
$$- \frac{3B\sin(c + dx)(b\cos(c + dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c + dx)\right)}{2bd\sqrt{\sin^2(c + dx)}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])
^(1/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/3)
*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b*d*Sqrt
[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1
/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
```

```
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$$

$$= \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + \frac{3\int \frac{\frac{b^2B}{3}-\frac{1}{3}b^2(2A-C)\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b^2}$$

$$= \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + B\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx - \frac{(2A-C)\int (b\cos(c+dx))^{2/3}\,dx}{b}$$

$$= \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} - \frac{3B(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3\big(10A\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right) - \cot(c+dx)\left(5B\,\text{Hypergeometric2F1}\left(\frac{1}{3},\right.\right.}{10d\sqrt[3]{b\cos(c+}}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c +
 d*x])^(1/3),x]
```

```
[Out] (3*(10*A*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] - C
ot[c + d*x]*(5*B*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*C*Cos
[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]))*Sqrt[Sin[c +
d*x]^2])/(10*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int \frac{(A+B\cos(dx+c)+C(\cos^2(dx+c)))\sec(dx+c)}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(1/3),x)
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x
, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)/(b*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/3)
,x)
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d
*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + B\,\cos\left(c+d\,x\right) + A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(1/3)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(1/3)), x)
```

3.355 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 145

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3B\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)+3*B*hypergeom([-1/6, 1/2],[5/6],c
os(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3/8*(A+
4*C)*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+
c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {16, 3100, 2827, 2722}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8bd\sqrt{\sin^2(c+dx)}} + \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x
])^(1/3),x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*B*Hypergeometric2F1[
-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqr
t[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[
1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b^2 \int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx$$

$$= \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3\int \frac{\frac{4b^2B}{3}+\frac{1}{3}b^2(A+4C)\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx}{4b}$$

$$= \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + (bB)\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx + \frac{1}{4}(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$= \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3B\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.80

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx =$$

$$-\frac{3b\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2\cos(c+dx)\left(-2B\,\text{Hypergeometric}\right.\right.}{4d(b\cos(}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
 + d*x])^(1/3),x]
[Out] (-3*b*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])
+ 2*Cos[c + d*x]*(-2*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
C*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]))*Sqrt[Sin[
c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{(A+B\cos(dx+c)+C(\cos^2(dx+c)))(\sec^2(dx+c))}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(1/3),x)
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3)
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^2/(b*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx = \int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1
/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c
+ d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos(dx + c)^2 + B\cos(dx + c) + A\right)\sec(dx + c)^2}{\left(b\cos(dx + c)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos(dx + c)^2 + B\cos(dx + c) + A\right)\sec(dx + c)^2}{\left(b\cos(dx + c)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{C\cos{(c+d\,x)}^2+B\,\cos{(c+d\,x)}+A}{\cos{(c+d\,x)}^2\,{(b\,\cos{(c+d\,x)})}^{1/3}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(1/3)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(1/3)), x)
```

3.356 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 149

$$\begin{aligned}&\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx\\&=\frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3bB\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}\\&\quad+\frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

```
[Out] 3/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/4*b*B*hypergeom([-2/3, 1/2],[
1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)+3
/7*(4*A+7*C)*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(
d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {16, 3100, 2827, 2722}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3(4A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}} + \frac{3bB\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x
])^(1/3),x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*b*B*Hypergeometric
2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/
3)*Sqrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6,
 Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x
]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b^3\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx\\
&= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3}{7}\int \frac{\frac{7b^2B}{3}+\frac{1}{3}b^2(4A+7C)\cos(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx\\
&= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}\\
&\quad+\left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx+\frac{1}{7}(b(4A+7C))\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx\\
&= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3bB\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}\\
&\quad+\frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\frac{3b^2\csc(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)+7\cos(c+dx)\left(B\,\text{Hypergeometric2F1}\right.\right.}{28d(b\cos(c+}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c
 + d*x])^(1/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]
 + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4*
C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]))*Sqrt[Sin
[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(7/3))
```

Maple [F]

$$\int \frac{(A + B\cos(dx + c) + C(\cos^2(dx + c)))(\sec^3(dx + c))}{(\cos(dx + c)b)^{\frac{1}{3}}}dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(1/3),x)
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}dx = \int \frac{\left(C\cos(dx + c)^2 + B\cos(dx + c) + A\right)\sec(dx + c)^3}{(b\cos(dx + c))^{\frac{1}{3}}}dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3)
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^3/(b*cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1
/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}dx = \int \frac{\left(C\cos(dx + c)^2 + B\cos(dx + c) + A\right)\sec(dx + c)^3}{(b\cos(dx + c))^{\frac{1}{3}}}dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x
 + c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x
 + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^3\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(1/3)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(1/3)), x)
```

3.357 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 154

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^4d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^4d\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^5d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^4/d-3/88*(11*A+8*C)*(b*cos(d*x+c))
^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c
)^2)^(1/2)-3/11*B*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*
x+c)^2)*sin(d*x+c)/b^5/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {16, 3102, 2827, 2722}

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{88b^4d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)}{11b^5d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11b^4d}$$

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(4/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^4*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c +
 d*x])/(88*b^4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hyper
geometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^5*d*Sqrt[
Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{5/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^3}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^4d} + \frac{3\int (b\cos(c+dx))^{5/3}\left(\frac{1}{3}b(11A+8C)+\frac{11}{3}bB\cos(c+dx)\right)\,dx}{11b^4}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^4d} + \frac{B\int (b\cos(c+dx))^{8/3}\,dx}{b^4} + \frac{(11A+8C)\int (b\cos(c+dx))^{5/3}\,dx}{11b^3}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^4d} - \frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^4d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^5d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.79

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\cos^3(c+dx)\cot(c+dx)\left(-8C\sin^2(c+dx)+(11A+8C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sqrt{\ldots}\right.}{88d(b\cos(c+dx)\ldots}$$

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^3*Cot[c + d*x]*(-8*C*Sin[c + d*x]^2 + (11*A + 8*C)*Hyperge
ometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 8*B*Cos[c
+ d*x]*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]
^2]))/(88*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{(\cos^3(dx+c))\left(A+B\cos(dx+c)+C(\cos^2(dx+c))\right)}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos(\ldots}{(b\cos(dx+c))^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
+ c))^(2/3)/b^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4
/3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}$$

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3), x)
```

3.358 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 154

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^3/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(
5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)
^2)^(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^
2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {16, 3102, 2827, 2722}

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{40b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{8b^4d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8b^3d}$$

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(4/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^3*d) - (3*(8*A + 5*C)*(b*Cos
[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(40*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hyperge
ometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^4*d*Sqrt[Sin[c
 + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^2}\\
&= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}+\frac{3\int (b\cos(c+dx))^{2/3}\left(\frac{1}{3}b(8A+5C)+\frac{8}{3}bB\cos(c+dx)\right)\,dx}{8b^3}\\
&= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}+\frac{B\int (b\cos(c+dx))^{5/3}\,dx}{b^3}\\
&\quad+\frac{(8A+5C)\int (b\cos(c+dx))^{2/3}\,dx}{8b^2}
\end{aligned}$$

$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^3d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^4d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\cos^2(c+dx)\cot(c+dx)\left(-5C\sin^2(c+dx)+(8A+5C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\ldots}\right.}{40d(b\cos(c+dx))^{\ldots}}$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*(-5*C*Sin[c + d*x]^2 + (8*A + 5*C)*Hypergeo
metric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 5*B*Cos[c
+ d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2
]))/(40*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{(\cos^2(dx+c))\,(A+B\cos(dx+c)+C(\cos^2(dx+c)))}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos(\ldots}{(b\cos(dx+c))^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b^2
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4
/3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}$$

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3), x)
```

3.359 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 154

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b^2/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(
2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^
2)^(1/2)-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^
2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 3102, 2827, 2722}

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}} + \frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5b^2d}$$

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])
^(4/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d) - (3*(5*A + 2*C)*(b*Cos
[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d
*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeo
metric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c
 + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} + \frac{3\int \frac{\frac{1}{3}b(5A+2C)+\frac{5}{3}bB\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{5b^2}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} + \frac{B\int(b\cos(c+dx))^{2/3}\,dx}{b^2} + \frac{(5A+2C)\int\frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{5b}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

$$\int\frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{-3(5A+2C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3}\right.}{}$$

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(4/3),x]
```

```
[Out] (-3*(5*A + 2*C)*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^
2]*Sqrt[Sin[c + d*x]^2] - 6*B*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1[1
/2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*C*Sin[2*(c + d*x)])
/(10*b*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int\frac{\cos(dx+c)\left(A+B\cos(dx+c)+C(\cos^2(dx+c))\right)}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^
2*cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos(c+d\,x)\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{(b\,\cos(c+d\,x))^{4/3}}\,dx$$

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(4/3),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(4/3), x)
```

3.360 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 152

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} - \frac{3B(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)-3/2*B*(b*cos(d*x+c))^(2/3)*hypergeo
m([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)+3/5*
(2*A-C)*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(
d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3100, 2827, 2722}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{5b^3d\sqrt{\sin^2(c+dx)}} + \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/
3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*
Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2
F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]
^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} + \frac{3\int \frac{\frac{b^2B}{3}-\frac{1}{3}b^2(2A-C)\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b^3} \\
&= \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} + \frac{B\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b} - \frac{(2A-C)\int (b\cos(c+dx))^{2/3}\,dx}{b^2} \\
&= \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} \\
&\quad - \frac{3B(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}} \\
&\quad + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx =$$
$$-\frac{3\cot(c+dx)\left(-10A\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)+\cos(c+dx)\left(5B\,\text{Hypergeometric2F1}\left(\right.\right.\right.}{10d(b\cos(c+\cdots}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(-10*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
 Cos[c + d*x]*(5*B*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*C*C
os[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]))*Sqrt[Sin[c
+ d*x]^2])/(10*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{A+B\cos(dx+c)+C(\cos^2(dx+c))}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^
2*cos(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\, dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\, dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c))^{\frac{4}{3}}}\, dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x
)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\, dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c))^{\frac{4}{3}}}\, dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\, dx = \int \frac{C\cos(c + d\,x)^2 + B\,\cos(c + d\,x) + A}{(b\,\cos(c + d\,x))^{4/3}}\, dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3), x)
```

3.361 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 147

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3B\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)+3*B*hypergeom([-1/6, 1/2],[5/6],cos
(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3/8*(A+
4*C)*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+
c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 3100, 2827, 2722}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}} + \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])
^(4/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*B*Hypergeometric2F1[-1
/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)*Sqr
t[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[
1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx$$

$$= \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3\int \frac{\frac{4b^2B}{3}+\frac{1}{3}b^2(A+4C)\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx}{4b^2}$$

$$= \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + B\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx + \frac{(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{4b}$$

$$= \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3B\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2\cos(c+dx)\left(-2B\,\text{Hypergeometric2F1}\left(\right.\right.\right.}{4d(b\cos(c+\right.}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c +
 d*x])^(4/3),x]
```

```
[Out] (-3*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]) +
2*Cos[c + d*x]*(-2*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] + C*
Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]))*Sqrt[Sin[c
+ d*x]^2])/(4*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{(A+B\cos(dx+c)+C(\cos^2(dx+c)))\sec(dx+c)}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sec(dx+\right.}{(b\cos(dx+c))^{\frac{4}{3}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(4/3)
,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sec(dx}{(b\cos(dx+c))^{\frac{4}{3}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(4/3), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\sec(dx}{(b\cos(dx+c))^{\frac{4}{3}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x
, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A}{\cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{4/3}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(4/3)),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(4/3)), x)
```

3.362 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 149

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3B\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}} + \frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/4*B*hypergeom([-2/3, 1/2],[1/3]
,cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)+3/7*(
4*A+7*C)*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*
x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {16, 3100, 2827, 2722}

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3(4A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6}\right.}{7bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+d}} + \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x
])^(4/3),x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*B*Hypergeometric2F1[
-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*S
qrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos
[c + d*x]^2]*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^
2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b^2\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx$$

$$= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3\int \frac{\frac{7b^2B}{3}+\frac{1}{3}b^2(4A+7C)\cos(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx}{7b}$$

$$= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+(bB)\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx+\frac{1}{7}(4A+7C)\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx$$

$$= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3B\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}} + \frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b^2\cot(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\right.\right.}{}$$

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
 + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]
 + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4*
C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]))*Sqrt[Sin
[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F]

$$\int \frac{(A+B\cos(dx+c)+C(\cos^2(dx+c)))(\sec^2(dx+c))}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(cos(d*x+c)*b)^(4/3),x)
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sec(dx}{(b\cos(dx+c))^{\frac{4}{3}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^2/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(4
/3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(d}{(b\cos(dx + c))^{\frac{4}{3}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(4/3), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(d}{(b\cos(dx + c))^{\frac{4}{3}}}$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos(c + d\,x)^2 + B\,\cos(c + d\,x) + A}{\cos(c + d\,x)^2\,(b\,\cos(c + d\,x))^{4/3}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(4/3)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(4/3)), x)
```

3.363 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$

Optimal result

Integrand size = 41, antiderivative size = 232

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(10+3m)} - \frac{3b(C(7+3m)+A(10+3m))\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+3m)(10+3m)\sqrt{\sin^2(c+dx)}} - \frac{3bB\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(10+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*b*C*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(10+3*m)-3*b*(C*(7
+3*m)+A*(10+3*m))*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6
+1/2*m],[13/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+51*m+70)/(sin(d*x+c)
^2)^(1/2)-3*b*B*cos(d*x+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 5/3+1
/2*m],[8/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

$= \{20, 3102, 2827, 2722\}$

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3b\left(\frac{A}{3m+7}+\frac{C}{3m+10}\right)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+7),\frac{1}{6}(3m+\right.}{d\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+10),\frac{1}{6}(3m+16),\cos^2(c+\right.}{d(3m+10)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{3bC\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)}{d(3m+10)}$$

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (3*b*C*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(10 + 3
*m)) - (3*b*(A/(7 + 3*m) + C/(10 + 3*m))*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*
x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*b*B*Cos[c + d*x]^(3 + m)*(b*Co
s[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c
+ d*x]^2]*Sin[c + d*x])/(d*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{4}{3}+m}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(10+3m)} + \frac{\left(3b\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{4}{3}+m}(c+dx)\left(\frac{1}{3}\left(3C\left(\frac{7}{3}+m\right)+3A\left(\frac{10}{3}+m\right)\right)+\frac{1}{3}B(10+3m)\cos(c+d\right.}{(10+3m)\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(10+3m)} + \frac{\left(bB\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{7}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}} + \frac{\left(b(C(7+3m)+A(10+3m))\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{4}{3}+m}(c+dx)\,dx}{(10+3m)\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(10+3m)} - \frac{3b\left(\frac{A}{7+3m}+\frac{C}{10+3m}\right)\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m\right.}{d\sqrt{\sin^2(c+dx)}} - \frac{3bB\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+dx\right.}{d(10+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.77

$$\int\cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\left(C(7+3m)\sin^2(c+dx)-B(7+3m)\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(C*(7 + 3*m)*Si
n[c + d*x]^2 - B*(7 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8
/3 + m/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - (C*(7 + 3*m) + A*(10 + 3*m
))*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2]))/(d*(7 + 3*m)*(10 + 3*m))
```

Maple [F]

$$\int \left(\cos^{m}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{\frac{4}{3}}\left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)dx$$

```
[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^{m}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*co
s(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*co
s(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int {\cos\left(c+d\,x\right)}^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.364 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+\right.$

Optimal result

Integrand size = 41, antiderivative size = 229

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)}-\frac{3(C(5+3m)+A(8+3m))\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3\right.}{d(5+3m)(8+3m)\sqrt{\sin^2(c+dx)}}-\frac{3B\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{1}{6}(14+3m),\cos^2(c+dx)\right)\sin(c-}{d(8+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d/(8+3*m)-3*(C*(5+3*m)
+A*(8+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 5/6+1/2*m
],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+39*m+40)/(sin(d*x+c)^2)^(1
/2)-3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 4/3+1/2*m],[7
/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(8+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

$= \{20, 3102, 2827, 2722\}$

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\left(\frac{A}{3m+5}+\frac{C}{3m+8}\right)\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+5),\frac{1}{6}(3m+1\right.}{d\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+8),\frac{1}{6}(3m+14),\cos^2(c+d\right.}{d(3m+8)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)}{d(3m+8)}$$

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(8 + 3*m)
) - (3*(A/(5 + 3*m) + C/(8 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2
/3)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*
x])^(2/3)*Hypergeometric2F1[1/2, (8 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*(8 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{2}{3}+m}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$= \frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)} + \frac{\left(3(b\cos(c+dx))^{2/3}\right)\int \cos^{\frac{2}{3}+m}(c+dx)\left(\frac{1}{3}\left(3C\left(\frac{5}{3}+m\right)+3A\left(\frac{8}{3}+m\right)\right)+\frac{1}{3}B(8+3m)\cos(c+d\right.}{(8+3m)\cos^{\frac{2}{3}}(c+dx)}$$

$$= \frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)} + \frac{\left(B(b\cos(c+dx))^{2/3}\right)\int \cos^{\frac{5}{3}+m}(c+dx)\,dx}{\cos^{\frac{2}{3}}(c+dx)} + \frac{\left((C(5+3m)+A(8+3m))(b\cos(c+dx))^{2/3}\right)\int \cos^{\frac{2}{3}+m}(c+dx)\,dx}{(8+3m)\cos^{\frac{2}{3}}(c+dx)}$$

$$= \frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)} - \frac{3\left(\frac{A}{5+3m}+\frac{C}{8+3m}\right)\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3r\right.}{d\sqrt{\sin^2(c+dx)}} - \frac{3B\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{1}{6}(14+3m),\cos^2(c+d\right.}{d(8+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\left(-\left((C(5+3m)+A(8+3m))\,\text{Hyper}\right.\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(-((C*(5 + 3*m)
 + A*(8 + 3*m))*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d
```

```
*x]^2]*Sqrt[Sin[c + d*x]^2]) + (5 + 3*m)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]
*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2]*Sqrt[Sin[c
+ d*x]^2])))/(d*(5 + 3*m)*(8 + 3*m))
```

Maple [F]

$$\int \left(\cos^{m}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{\frac{2}{3}}\left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)dx$$

```
[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^{m}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos
(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int {\cos\left(c+d\,x\right)}^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.365 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx)+C$

Optimal result

Integrand size = 41, antiderivative size = 229

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)}$$
$$-\frac{3(C(4+3m)+A(7+3m))\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3}{d(4+3m)(7+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3B\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(c+}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(7+3*m)-3*(C*(4+3*m)
+A*(7+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 2/3+1/2*m
],[5/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+33*m+28)/(sin(d*x+c)^2)^(1/
2)-3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[13
/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

$= \{20, 3102, 2827, 2722\}$

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3\left(\frac{A}{3m+4}+\frac{C}{3m+7}\right)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+4),\frac{1}{6}(3m+\right.}{d\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+7),\frac{1}{6}(3m+13),\cos^2(c+\right.}{d(3m+7)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)}{d(3m+7)}$$

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*m)
) - (3*(A/(4 + 3*m) + C/(7 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1
/3)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*
x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{b\cos(c+dx)}\int\cos^{\frac{1}{3}+m}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)} + \frac{\left(3\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{1}{3}+m}(c+dx)\left(\frac{1}{3}\left(3C\left(\frac{4}{3}+m\right)+3A\left(\frac{7}{3}+m\right)\right)+\frac{1}{3}B(7+3m)\cos(c+dx)\right)}{(7+3m)\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)} + \frac{\left(B\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{4}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}} + \frac{\left((C(4+3m)+A(7+3m))\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{1}{3}+m}(c+dx)\,dx}{(7+3m)\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)} - \frac{3\left(\frac{A}{4+3m}+\frac{C}{7+3m}\right)\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\right.}{d\sqrt{\sin^2(c+dx)}} - \frac{3B\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76

$$\int\cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(-\left((C(4+3m)+A(7+3m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+\right.\right.\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(-((C*(4 + 3*m)
 + A*(7 + 3*m))*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]
^2]*Sqrt[Sin[c + d*x]^2]) + (4 + 3*m)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hy
pergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c
+ d*x]^2])))/(d*(4 + 3*m)*(7 + 3*m))
```

Maple [F]

$$\int (\cos^m(dx+c))\,(\cos(dx+c)\,b)^{\frac{1}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos
(d*x + c)^m, x)
```

Sympy [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\cos^m(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)*c
os(c + d*x)**m, x)
```

Maxima [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*co
s(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^m\,dx$$

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*co
s(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int {\cos\left(c+d\,x\right)}^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.366 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 229

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3(C(2+3m)+A(5+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx\right.}{d(2+3m)(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(2+3*m)
+A*(5+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x
+c)^2)*sin(d*x+c)/d/(9*m^2+21*m+10)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/
2)-3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x+c)^
2)*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used

$= \{20, 3102, 2827, 2722\}$

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx =$$
$$-\frac{3\left(\frac{A}{3m+2}+\frac{C}{3m+5}\right)\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+2),\frac{1}{6}(3m+8),\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$
$$-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+5),\frac{1}{6}(3m+11),\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$
$$+\frac{3C\sin(c+dx)\cos^{m+1}(c+dx)}{d(3m+5)\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(1/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)
) - (3*(A/(2 + 3*m) + C/(5 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1
/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d
*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric
2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3
*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{\cos(c+dx)}\int\cos^{-\frac{1}{3}+m}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(3\sqrt[3]{\cos(c+dx)}\right)\int\cos^{-\frac{1}{3}+m}(c+dx)\left(\frac{1}{3}\left(3C\left(\frac{2}{3}+m\right)+3A\left(\frac{5}{3}+m\right)\right)+\frac{1}{3}B(5+3m)\cos(c+dx\right.}{(5+3m)\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(B\sqrt[3]{\cos(c+dx)}\right)\int\cos^{\frac{2}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\cos(c+dx)}} + \frac{\left((C(2+3m)+A(5+3m))\sqrt[3]{\cos(c+dx)}\right)\int\cos^{-\frac{1}{3}+m}(c+dx)\,dx}{(5+3m)\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3\left(\frac{A}{2+3m}+\frac{C}{5+3m}\right)\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\text{si}}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76

$$\int\frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$= \frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(-\left((C(2+3m)+A(5+3m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3n\right.\right.\right.}{}$$

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(1/3),x]
[Out] (3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((C*(2 + 3*m) + A*(5 + 3*m))*Hyperge
ometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]
^2]) + (2 + 3*m)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2,
(5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(2 +
3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

Maple [F]

$$\int \frac{(\cos^m (dx + c)) (A + B \cos (dx + c) + C(\cos^2 (dx + c)))}{(\cos (dx + c) b)^{\frac{1}{3}}} dx$$

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx) (A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}} dx = \int \frac{(C\cos(dx+c)^2 + B\cos(dx+c) + A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}} dx$$

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos
(d*x + c)^m/(b*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx) (A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}} dx = \int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}} dx$$

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1
/3),x)
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c
+ d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^m\,\left(C\,\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/3), x)
```

3.367 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 227

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}}$$
$$-\frac{3(C+3Cm+A(4+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+3m)(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)-3*(C+3*C*m+A
*(4+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/6+1/2*m],[7/6+1/2*m],cos(d*x+c
)^2)*sin(d*x+c)/d/(9*m^2+15*m+4)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-
3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 2/3+1/2*m],[5/3+1/2*m],cos(d*x+c)^2)*s
in(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {20, 3102, 2827, 2722}

$$\int \frac{\cos^m(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx =$$
$$-\frac{3(A(3m+4)+3Cm+C)\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+1),\frac{1}{6}(3m+7),\cos^2\right.}{d(3m+1)(3m+4)\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$
$$-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+4),\frac{1}{6}(3m+10),\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$
$$+\frac{3C\sin(c+dx)\cos^{m+1}(c+dx)}{d(3m+4)(b\cos(c+dx))^{2/3}}$$

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(2/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)
) - (3*(C + 3*C*m + A*(4 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2
, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(4 +
 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 +
 m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{-\frac{2}{3}+m}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{(b\cos(c+dx))^{2/3}}\\
&= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}}\\
&\quad+\frac{\left(3\cos^{\frac{2}{3}}(c+dx)\right)\int \cos^{-\frac{2}{3}+m}(c+dx)\left(\frac{1}{3}\left(3C\left(\frac{1}{3}+m\right)+3A\left(\frac{4}{3}+m\right)\right)+\frac{1}{3}B(4+3m)\cos(c+dx)\right)}{(4+3m)(b\cos(c+dx))^{2/3}}\\
&= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}}+\frac{\left(B\cos^{\frac{2}{3}}(c+dx)\right)\int \cos^{\frac{1}{3}+m}(c+dx)\,dx}{(b\cos(c+dx))^{2/3}}\\
&\quad+\frac{\left((C+3Cm+A(4+3m))\cos^{\frac{2}{3}}(c+dx)\right)\int \cos^{-\frac{2}{3}+m}(c+dx)\,dx}{(4+3m)(b\cos(c+dx))^{2/3}}\\
&= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}}\\
&\quad-\frac{3(C+3Cm+A(4+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c\right.}{d(1+3m)(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}\\
&\quad-\frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.76

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(-\left((C+3Cm+A\right.\right.}{}$$

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(2/3),x]
```

```
[Out] (3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((C + 3*C*m + A*(4 + 3*m))*Hypergeom
etric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2
]) + (1 + 3*m)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2, (4
 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(1 + 3*m)*
(4 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

Maple [F]

$$\int \frac{(\cos^m(dx+c))\,(A+B\cos(dx+c)+C(\cos^2(dx+c)))}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)

[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(2/3),x)

Fricas [F]

$$\int \frac{\cos^m(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\cos(}{(b\cos(dx+c))^{\frac{2}{3}}}$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3)
,x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos
(d*x + c)^m/(b*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{\cos^m(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\cos^m}{(b\cos(c+dx))^{\frac{2}{3}}}$$

[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2
/3),x)

[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c
+ d*x))**(2/3), x)

Maxima [F]

$$\int \frac{\cos^m(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)\cos(}{(b\cos(dx+c))^{\frac{2}{3}}}$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3)
,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(2/3), x)

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}$$

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}$$

```
[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(2/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(2/3), x)
```

3.368 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 235

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}}$$
$$-\frac{3(C(1-3m)-A(2+3m))\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(c+dx)\right)s}{bd(1-3m)(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3B\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^m*sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(1-3*m)-A
*(2+3*m))*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m],[5/6+1/2*m],cos(d*x+c)^2
)*sin(d*x+c)/b/d/(-9*m^2-3*m+2)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3
*B*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)*si
n(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {20, 3102, 2827, 2722}

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3\left(\frac{A}{1-3m}-\frac{C}{3m+2}\right)\sin(c+dx)\cos^m(c+dx)\,\text{Hype}}{bd\sqrt{\sin^2(c-}}$$
$$-\frac{3B\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+2),\frac{1}{6}(3m+8),\cos^2(c+dx)\right)}{bd(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$
$$+\frac{3C\sin(c+dx)\cos^m(c+dx)}{bd(3m+2)\sqrt[3]{b\cos(c+dx)}}$$

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(4/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^m*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)) +
(3*(A/(1 - 3*m) - C/(2 + 3*m))*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 +
3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1
/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2
, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + 3*m)*(b
*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{\cos(c+dx)} \int \cos^{-\frac{4}{3}+m}(c+dx) \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(3\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{4}{3}+m}(c+dx)\left(\frac{1}{3}\left(-3C\left(\frac{1}{3}-m\right)+3A\left(\frac{2}{3}+m\right)\right)+\frac{1}{3}B(2+3m)\cos(c+d\right.}{b(2+3m)\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(B\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{1}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\cos(c+dx)}} + \frac{\left((-C(1-3m)+A(2+3m))\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{4}{3}+m}(c+dx)\,dx}{b(2+3m)\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{3\left(\frac{A}{1-3m}-\frac{C}{2+3m}\right)\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(c+dx)\right)\text{si}}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3B\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.74

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx =$$

$$-\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(\left(C(-1+3m)+A(2+3m)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m\right.\right.}{}$$

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*((C*(-1 + 3*m) + A*(2 + 3*m))*Hyperge
ometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x
]^2] - (-1 + 3*m)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2,
 (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(-1 +
 3*m)*(2 + 3*m)*(b*Cos[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{(\cos^m (dx+c)) (A + B \cos (dx+c) + C(\cos^2 (dx+c)))}{(\cos (dx+c) b)^{\frac{4}{3}}} dx$$

[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)

[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(cos(d*x+c)*b)^(4/3),x)

Fricas [F]

$$\int \frac{\cos^m(c+dx) (A + B\cos(c+dx) + C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \int \frac{(C \cos (dx+c)^2 + B \cos (dx+c) + A) \cos (d$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos
(d*x + c)^m/(b^2*cos(d*x + c)^2), x)

Sympy [F]

$$\int \frac{\cos^m(c+dx) (A + B\cos(c+dx) + C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \int \frac{(A + B \cos (c+dx) + C \cos^2 (c+dx)) \cos^m (}{(b \cos (c+dx))^{\frac{4}{3}}}$$

[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4
/3),x)

[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c
+ d*x))**(4/3), x)

Maxima [F]

$$\int \frac{\cos^m(c+dx) (A + B\cos(c+dx) + C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \int \frac{(C \cos (dx+c)^2 + B \cos (dx+c) + A) \cos (d}{(b \cos (dx+c))^{\frac{4}{3}}}$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(4/3), x)

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}$$

```
[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3), x)
```

3.369 $\int (a\cos(c+dx))^m(b\cos(c+dx))^n\,(A+B\cos(c+dx)+$

Optimal result

Integrand size = 41, antiderivative size = 227

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)}$$
$$-\frac{(C(1+m+n)+A(2+m+n))(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n\right.}{ad(1+m+n)(2+m+n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{B(a\cos(c+dx))^{2+m}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2+m+n),\frac{1}{2}(4+m+n),\cos^2(c+dx)\right)}{a^2d(2+m+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*sin(d*x+c)/a/d/(2+m+n)-(C*(1+m+n)+A
*(2+m+n))*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2+1/2*m+1
/2*n],[3/2+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a/d/(1+m+n)/(2+m+n)/(sin(d
*x+c)^2)^(1/2)-B*(a*cos(d*x+c))^(2+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1+1/
2*m+1/2*n],[2+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a^2/d/(2+m+n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

$= \{20, 3102, 2827, 2722\}$

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{B\sin(c+dx)(a\cos(c+dx))^{m+2}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m+n+2),\frac{1}{2}(m+n+4),c\right.}{a^2d(m+n+2)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{(A(m+n+2)+C(m+n+1))\sin(c+dx)(a\cos(c+dx))^{m+1}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{ad(m+n+1)(m+n+2)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{C\sin(c+dx)(a\cos(c+dx))^{m+1}(b\cos(c+dx))^n}{ad(m+n+2)}$$

```
[In] Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (C*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(a*d*(2 + m +
n)) - ((C*(1 + m + n) + A*(2 + m + n))*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c +
d*x])^n*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2
]*Sin[c + d*x])/(a*d*(1 + m + n)*(2 + m + n)*Sqrt[Sin[c + d*x]^2]) - (B*(a*
Cos[c + d*x])^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (2 + m + n)
/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a^2*d*(2 + m + n)*Sqrt[Si
n[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left((a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int (a\cos(c+dx))^{m+n}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)} + \frac{\left((a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int (a\cos(c+dx))^{m+n}(a(C(1+m+n)+A(2+m+n))+aB}{a(2+m+n)}$$

$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)} + \frac{\left(B(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int (a\cos(c+dx))^{1+m+n}\,dx}{a} + \frac{\left((C(1+m+n)+A(2+m+n))(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int (a\cos(c+dx))^{m+n}\,dx}{2+m+n}$$

$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)} - \frac{(C(1+m+n)+A(2+m+n))(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1}{ad(1+m+n)(2+m+n)\sqrt{\sin^2(c+dx)}} - \frac{B(a\cos(c+dx))^{2+m}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2+m+n),\frac{1}{2}(4+m+n),\cos^2(c}{a^2d(2+m+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.69

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$= \frac{(a\cos(c+dx))^m(b\cos(c+dx))^n\cot(c+dx)\left(C\sin^2(c+dx)-\frac{(C(1+m+n)+A(2+m+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m}{1+m+n}\right.}{}$$

```
[In] Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] ((a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Cot[c + d*x]*(C*Sin[c + d*x]^2 - ((C
*(1 + m + n) + A*(2 + m + n))*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m
```

```
+ n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(1 + m + n) - B*Cos[c + d*x]*
Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2]))/(d*(2 + m + n))
```

Maple [F]

$$\int (\cos (dx+c)\, a)^{m} (\cos (dx+c)\, b)^{n} \left(A+B\cos (dx+c)+C\left(\cos^{2}(dx+c)\right)\right) dx$$

```
[In] int((cos(d*x+c)*a)^m*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((cos(d*x+c)*a)^m*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a\cos(c+dx))^m (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\, dx$$
$$= \int \left(C\cos (dx+c)^{2}+B\cos (dx+c)+A\right)(a\cos (dx+c))^{m}\, (b\cos (dx+c))^{n}\, dx$$

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*cos(
d*x + c))^n, x)
```

Sympy [F]

$$\int (a\cos(c+dx))^m (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\, dx$$
$$= \int (a\cos (c+dx))^{m}\, (b\cos (c+dx))^{n} \left(A+B\cos (c+dx)+C\cos^{2}(c+dx)\right)\, dx$$

```
[In] integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
```

```
[Out] Integral((a*cos(c + d*x))**m*(b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*co
s(c + d*x)**2), x)
```

Maxima [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(a\cos\left(dx+c\right))^m\left(b\cos\left(dx+c\right)\right)^n dx$$

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*cos
(d*x + c))^n, x)
```

Giac [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(a\cos\left(dx+c\right))^m\left(b\cos\left(dx+c\right)\right)^n dx$$

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*cos
(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int (a\cos\left(c+d\,x\right))^m\,\left(b\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\cos\left(c+d\,x\right)+A\right) dx$$

```
[In] int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.370 $\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\right.$

Optimal result

Integrand size = 39, antiderivative size = 187

$$\begin{aligned}
&\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\
&=\frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)}\\
&\quad-\frac{(C(3+n)+A(4+n))(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)(4+n)\sqrt{\sin^2(c+dx)}}\\
&\quad-\frac{B(b\cos(c+dx))^{4+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

```
[Out] C*(b*cos(d*x+c))^(3+n)*sin(d*x+c)/b^3/d/(4+n)-(C*(3+n)+A*(4+n))*(b*cos(d*x+
c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b
^3/d/(3+n)/(4+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(4+n)*hypergeom([1/2
, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(4+n)/(sin(d*x+c)^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{(A(n+4)+C(n+3))\sin(c+dx)(b\cos(c+dx))^{n+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+5}{2},\cos^2(c+dx)\right)}{b^3d(n+3)(n+4)\sqrt{\sin^2(c+dx)}} - \frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+4}{2},\frac{n+6}{2},\cos^2(c+dx)\right)}{b^4d(n+4)\sqrt{\sin^2(c+dx)}} + \frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{b^3d(n+4)}$$

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(3 + n)*Sin[c + d*x])/(b^3*d*(4 + n)) - ((C*(3 + n) + A
*(4 + n))*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*(4 + n)*Sqrt[Sin[c + d*x]
^2]) - (B*(b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{2+n}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b^2}$$

$$= \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)} + \frac{\int (b\cos(c+dx))^{2+n}(b(C(3+n)+A(4+n))+bB(4+n)\cos(c+dx))\,dx}{b^3(4+n)}$$

$$= \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)} + \frac{B\int (b\cos(c+dx))^{3+n}\,dx}{b^3} + \frac{(C(3+n)+A(4+n))\int (b\cos(c+dx))^{2+n}\,dx}{b^2(4+n)}$$

$$= \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)} - \frac{(C(3+n)+A(4+n))(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c\cdots}{b^3d(3+n)(4+n)\sqrt{\sin^2(c+dx)}} - \frac{B(b\cos(c+dx))^{4+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.82

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{\cos^2(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left((C(3+n)+A(4+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^{\cdots}\right.\right.}{\cdots}$$

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*((C*(3 + n) + A*(4 + n))*
Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d
*x]^2] - (3 + n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2,
(4 + n)/2, (6 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(3 + n)*(4
 + n)))
```

Maple [F]

$$\int \left(\cos^2(dx+c)\right)\left(\cos(dx+c)\,b\right)^n\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int(cos(d*x+c)^2*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n\cos(dx+c)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*
x + c))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n\cos(dx+c)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^2 \, dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= \int \cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right) \, dx$$

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.371 $\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 37, antiderivative size = 187

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)} - \frac{(C(2+n)+A(3+n))(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)(3+n)\sqrt{\sin^2(c+dx)}} - \frac{B(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(2+n)*sin(d*x+c)/b^2/d/(3+n)-(C*(2+n)+A*(3+n))*(b*cos(d*x+
c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d
/(2+n)/(3+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(3+n)*hypergeom([1/2, 3/
2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{(A(n+3)+C(n+2))\sin(c+dx)(b\cos(c+dx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{b^2d(n+2)(n+3)\sqrt{\sin^2(c+dx)}} - \frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+5}{2},\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}} + \frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{b^2d(n+3)}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
,x]
```

```
[Out] (C*(b*Cos[c + d*x])^(2 + n)*Sin[c + d*x])/(b^2*d*(3 + n)) - ((C*(2 + n) + A
*(3 + n))*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*(3 + n)*Sqrt[Sin[c + d*x]
^2]) - (B*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int (b\cos(c+dx))^{1+n}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx}{b}$$

$$= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)} + \frac{\int (b\cos(c+dx))^{1+n}(b(C(2+n)+A(3+n))+bB(3+n)\cos(c+dx))\,dx}{b^2(3+n)}$$

$$= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)} + \frac{B\int (b\cos(c+dx))^{2+n}\,dx}{b^2} + \frac{(C(2+n)+A(3+n))\int (b\cos(c+dx))^{1+n}\,dx}{b(3+n)}$$

$$= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)} - \frac{(C(2+n)+A(3+n))(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+}{b^2d(2+n)(3+n)\sqrt{\sin^2(c+dx)}} - \frac{B(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

$$\int \cos(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{\cos(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left((C(2+n)+A(3+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c\right.\right.}{}$$

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*((C*(2 + n) + A*(3 + n))*Hy
pergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x
]^2] - (2 + n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2, (3
 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(2 + n)*(3 +
 n)))
```

Maple [F]

$$\int \cos \left(d x+c\right) \left(\cos \left(d x+c\right) b\right)^{n} \left(A+B \cos \left(d x+c\right)+C \left(\cos^{2}\left(d x+c\right)\right)\right)d x$$

```
[In] int(cos(d*x+c)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="fricas")
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
+ c))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2)
,x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2)
, x)
```

3.372 $\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$

Optimal result

Integrand size = 31, antiderivative size = 187

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} - \frac{(C(1+n)+A(2+n))(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)(2+n)\sqrt{\sin^2(c+dx)}} - \frac{B(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(1+n)*sin(d*x+c)/b/d/(2+n)-(C*(1+n)+A*(2+n))*(b*cos(d*x+c)
)^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b/d
/(1+n)/(2+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(2+n)*hypergeom([1/2, 1+
1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3102, 2827, 2722}

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{(A(n+2)+C(n+1))\sin(c+dx)(b\cos(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{bd(n+1)(n+2)\sqrt{\sin^2(c+dx)}} - \frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}} + \frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(2 + n)) - ((C*(1 + n) + A*(
2 + n))*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*(2 + n)*Sqrt[Sin[c + d*x]^2])
 - (B*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2,
 Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} \\ &\quad + \frac{\int (b\cos(c+dx))^n(b(C(1+n)+A(2+n))+bB(2+n)\cos(c+dx))\,dx}{b(2+n)} \\ &= \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} + \frac{B\int (b\cos(c+dx))^{1+n}\,dx}{b} \\ &\quad + \left(A+\frac{C(1+n)}{2+n}\right)\int (b\cos(c+dx))^n\,dx\end{aligned}$$

$$= \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} - \frac{\left(A+\frac{C(1+n)}{2+n}\right)(b\cos(c+dx))^{1+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}} - \frac{B(b\cos(c+dx))^{2+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^n\cot(c+dx)\left(-\left((C(1+n)+A(2+n))\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sqrt{\right.\right.}{}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^n*Cot[c + d*x]*(-((C*(1 + n) + A*(2 + n))*Hypergeometric2
F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]) + (1 +
n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4
+ n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(1 + n)*(2 + n))
```

Maple [F]

$$\int (\cos (dx+c)\, b)^n \left(A+B\cos (dx+c)+C\left(\cos^2 (dx+c)\right)\right) dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (b\cos{(c+dx)})^n \left(A+B\cos{(c+dx)}+C\cos^2{(c+dx)}\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A\right)(b\cos{(dx+c)})^n\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A\right)(b\cos{(dx+c)})^n\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.373 $\int (b\cos(c+dx))^n (A+B\cos(c+dx)+C\cos^2(c+dx)) \sec(c+dx)\, dx$

Optimal result

Integrand size = 37, antiderivative size = 170

$$\begin{aligned}
&\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx \\
&= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)} \\
&\quad - \frac{(A+An+Cn)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn(1+n)\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{B(b\cos(c+dx))^{1+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

```
[Out] C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+n)-(A*n+C*n+A)*(b*cos(d*x+c))^n*hypergeo
m([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(1+n)/(sin(d*x+c)^2)^
(1/2)-B*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x
+c)^2)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$
$$-\frac{(An+A+Cn)\sin(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(c+dx)\right)}{dn(n+1)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{C\sin(c+dx)(b\cos(c+dx))^n}{d(n+1)}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
,x]
```

```
[Out] (C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)) - ((A + A*n + C*n)*(b*Cos[c
 + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d
*x])/(d*n*(1 + n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^(1 + n)*Hyper
geometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*
(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\int(b\cos(c+dx))^{-1+n}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\
&= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)}\\
&\quad+\frac{\int(b\cos(c+dx))^{-1+n}(b(A+An+Cn)+bB(1+n)\cos(c+dx))\,dx}{1+n}\\
&= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)}+B\int(b\cos(c+dx))^n\,dx\\
&\quad+\frac{(b(A+An+Cn))\int(b\cos(c+dx))^{-1+n}\,dx}{1+n}\\
&= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)}\\
&\quad-\frac{(A+An+Cn)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn(1+n)\sqrt{\sin^2(c+dx)}}\\
&\quad-\frac{B(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

$$\int(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\left(-\left((A+An+Cn)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(\right.\right.}{dn}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x],x]
```

```
[Out] (b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(-((A + A*n + C*n)*Hypergeometric
2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]) + n*(C*Sin[c
 + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos
[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*n*(1 + n))
```

Maple [F]

$$\int (\cos (dx+c)\, b)^{n} \left(A+B\cos (dx+c)+C\left(\cos^{2}(dx+c)\right)\right)\sec (dx+c)\, dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{n} \left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\sec(c+dx)\, dx$$
$$= \int \left(C\cos (dx+c)^{2}+B\cos (dx+c)+A\right)(b\cos (dx+c))^{n}\sec (dx+c)\ dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x
 + c), x)
```

Sympy [F]

$$\int (b\cos(c+dx))^{n} \left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\sec(c+dx)\, dx$$
$$= \int (b\cos (c+dx))^{n} \left(A+B\cos (c+dx)+C\cos^{2}(c+dx)\right)\sec (c+dx)\, dx$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c
 + d*x), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^{n} \left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\sec(c+dx)\, dx$$
$$= \int \left(C\cos (dx+c)^{2}+B\cos (dx+c)+A\right)(b\cos (dx+c))^{n}\sec (dx+c)\ dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x), x)
```

3.374 $\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 173

$$\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn}$$
$$- \frac{b(C(1-n)-An)(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)n\sqrt{\sin^2(c+dx)}}$$
$$- \frac{B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}$$

```
[Out] b*C*(b*cos(d*x+c))^(-1+n)*sin(d*x+c)/d/n-b*(C*(1-n)-A*n)*(b*cos(d*x+c))^(-1
+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n
)/n/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2*n],[1+1/2*n
],cos(d*x+c)^2)*sin(d*x+c)/d/n/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = -\frac{b(C(1-n)-An)\sin(c+dx)(b\cos(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\cos^2(c+dx)\right)}{d(1-n)n\sqrt{\sin^2(c+dx)}} - \frac{B\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}} + \frac{bC\sin(c+dx)(b\cos(c+dx))^{n-1}}{dn}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^2,x]
```

```
[Out] (b*C*(b*Cos[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*n) - (b*(C*(1 - n) - A*n)*(
b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*n*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c
+ d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*
x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^2 \int (b\cos(c+dx))^{-2+n} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\, dx \\
&= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn} \\
&\quad + \frac{b\int (b\cos(c+dx))^{-2+n}(-b(C(1-n)-An) + bBn\cos(c+dx))\, dx}{n} \\
&= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn} + (bB)\int (b\cos(c+dx))^{-1+n}\, dx \\
&\quad - \frac{(b^2(C(1-n)-An))\int (b\cos(c+dx))^{-2+n}\, dx}{n} \\
&= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn} \\
&\quad - \frac{b(C(1-n)-An)(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1+n), \frac{1+n}{2}, \cos^2(c+dx)\right)\sin}{d(1-n)n\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.75

$$
\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^2(c+dx)\, dx =
$$

$$
-\frac{(b\cos(c+dx))^n \left((C(-1+n)+An)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1+n), \frac{1+n}{2}, \cos^2(c+dx)\right) + (-1+n\right.}{d(-1+n)n\sqrt{\sin}}
$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^2,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*((C*(-1 + n) + A*n)*Hypergeometric2F1[1/2, (-1 + n)/2
, (1 + n)/2, Cos[c + d*x]^2] + (-1 + n)*(B*Cos[c + d*x]*Hypergeometric2F1[1
/2, n/2, (2 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))*Tan[c + d*x]
)/(d*(-1 + n)*n*Sqrt[Sin[c + d*x]^2]))
```

Maple [F]

$$\int (\cos(dx+c)\,b)^n \left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^2(dx+c)\right)dx$$

[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n\sec(dx+c)^2\,dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\text{Timed out}$$

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2, x)

[Out] Timed out

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n\sec(dx+c)^2\,dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \frac{\left(b\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^2}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^2, x)
```

3.375 $\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 194

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=-\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)}$$
$$+\frac{b^2(A(1-n)+C(2-n))(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{bB(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b^2*C*(b*cos(d*x+c))^(-2+n)*sin(d*x+c)/d/(1-n)+b^2*(A*(1-n)+C*(2-n))*(b*co
s(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+c)
/d/(n^2-3*n+2)/(sin(d*x+c)^2)^(1/2)+b*B*(b*cos(d*x+c))^(-1+n)*hypergeom([1/
2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \frac{b^2(A(1-n)+C(2-n))\sin(c+dx)(b\cos(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\cos^2(c+dx)\right)}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}}$$
$$- \frac{b^2C\sin(c+dx)(b\cos(c+dx))^{n-2}}{d(1-n)}$$
$$+ \frac{bB\sin(c+dx)(b\cos(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^3,x]
```

```
[Out] -((b^2*C*(b*Cos[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(1 - n))) + (b^2*(A*(1
- n) + C*(2 - n))*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)
/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*(2 - n)*Sqrt[Sin[c + d*x]
^2]) + (b*B*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1
 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = b^3 \int (b\cos(c+dx))^{-3+n} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx$$
$$= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)} - \frac{b^2 \int (b\cos(c+dx))^{-3+n}(-b(A(1-n)+C(2-n)) - bB(1-n)\cos(c+dx))\,dx}{1-n}$$
$$= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)} + \left(b^2B\right)\int (b\cos(c+dx))^{-2+n}\,dx + \frac{\left(b^3(A(1-n)+C(2-n))\right)\int (b\cos(c+dx))^{-3+n}\,dx}{1-n}$$
$$= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)} + \frac{b^2(A(1-n)+C(2-n))(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}} + \frac{bB(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{(b\cos(c+dx))^n\csc(c+dx)\sec^2(c+dx)\left(-\left((C(-2+n)+A(-1+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n\right.\right.\right.}{}$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^3,x]
```

```
[Out] ((b*Cos[c + d*x])^n*Csc[c + d*x]*Sec[c + d*x]^2*(-((C*(-2 + n) + A*(-1 + n)
)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]
^2]) + (-2 + n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2, (
-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(-2 + n)*(
-1 + n))
```

Maple [F]

$$\int (\cos (dx + c) b)^n \left(A + B \cos (dx + c) + C\left(\cos^2 (dx + c)\right)\right) \left(\sec^3 (dx + c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos (dx + c)^2 + B\cos (dx + c) + A\right)(b\cos (dx + c))^n \sec (dx + c)^3 \, dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x
 + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,
x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos (dx + c)^2 + B\cos (dx + c) + A\right)(b\cos (dx + c))^n \sec (dx + c)^3 \, dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^3, x)
```

3.376 $\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$ $dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 196

$$
\begin{aligned}
&\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx \\
&= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)} \\
&\quad + \frac{b^3(A(2-n)+C(3-n))(b\cos(c+dx))^{-3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+d\right.}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}} \\
&\quad + \frac{b^2B(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

```
[Out] -b^3*C*(b*cos(d*x+c))^(-3+n)*sin(d*x+c)/d/(2-n)+b^3*(A*(2-n)+C*(3-n))*(b*co
s(d*x+c))^(-3+n)*hypergeom([1/2, -3/2+1/2*n],[-1/2+1/2*n],cos(d*x+c)^2)*sin
(d*x+c)/d/(n^2-5*n+6)/(sin(d*x+c)^2)^(1/2)+b^2*B*(b*cos(d*x+c))^(-2+n)*hype
rgeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{16, 3102, 2827, 2722\}$

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \frac{b^3(A(2-n)+C(3-n))\sin(c+dx)(b\cos(c+dx))^{n-3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-3}{2},\frac{n-1}{2},\cos^2(c+dx)\right)}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{b^3C\sin(c+dx)(b\cos(c+dx))^{n-3}}{d(2-n)}$$
$$+\frac{b^2B\sin(c+dx)(b\cos(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^4,x]
```

```
[Out] -((b^3*C*(b*Cos[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(2 - n))) + (b^3*(A*(2
- n) + C*(3 - n))*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)
/2, (-1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*(3 - n)*Sqrt[Sin[c
 + d*x]^2]) + (b^2*B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 +
 n)/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^4 \int (b\cos(c+dx))^{-4+n} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx \\
&= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)} \\
&\quad -\frac{b^3\int(b\cos(c+dx))^{-4+n}(-b(A(2-n)+C(3-n))-bB(2-n)\cos(c+dx))\,dx}{2-n} \\
&= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)} + \left(b^3B\right)\int(b\cos(c+dx))^{-3+n}\,dx \\
&\quad +\frac{\left(b^4(A(2-n)+C(3-n))\right)\int(b\cos(c+dx))^{-4+n}\,dx}{2-n} \\
&= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)} \\
&\quad +\frac{b^3(A(2-n)+C(3-n))(b\cos(c+dx))^{-3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos\right.}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}} \\
&\quad +\frac{b^2B(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$
\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx
$$
$$
= \frac{(b\cos(c+dx))^n\csc(c+dx)\sec^3(c+dx)\left(-\left((C(-3+n)+A(-2+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+\right.\right.\right.}{}
$$

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^4,x]
```

```
[Out] ((b*Cos[c + d*x])^n*Csc[c + d*x]*Sec[c + d*x]^3*(-((C*(-3 + n) + A*(-2 + n)
)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c
 + d*x]^2]) + (-3 + n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1
[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(-3 + n)*
(-2 + n))
```

Maple [F]

$$\int (\cos (dx+c) b)^n \left(A+B\cos (dx+c)+C\left(\cos^2 (dx+c)\right)\right) \left(\sec^4 (dx+c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n \sec(dx+c)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x
 + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,
x)
[Out] Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n \sec(dx+c)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^4, x)
```

3.377 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\right.$

Optimal result

Integrand size = 41, antiderivative size = 223

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)}$$
$$- \frac{2(C(5+2n)+A(7+2n))\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos\right.}{d(5+2n)(7+2n)\sqrt{\sin^2(c+dx)}}$$
$$- \frac{2B\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*C*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(7+2*n)-2*(C*(5+2*n)+A*(
7+2*n))*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1
/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2+24*n+35)/(sin(d*x+c)^2)^(1/2)-2*B*c
os(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2*n],co
s(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used

$= \{20, 3102, 2827, 2722\}$

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx =$$

$$-\frac{2\left(\frac{A}{2n+5}+\frac{C}{2n+7}\right)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),\right.}{d\sqrt{\sin^2(c+dx)}}$$
$$-\frac{2B\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+7),\frac{1}{4}(2n+11),\cos^2(c+dx\right.}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n}{d(2n+7)}$$

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) - (2
*(A/(5 + 2*n) + C/(7 + 2*n))*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*S
qrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeom
etric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
7 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)} + \frac{\left(2\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\left(\frac{1}{2}\left(2C\left(\frac{5}{2}+n\right)+2A\left(\frac{7}{2}+n\right)\right)+\frac{1}{2}B(7+2n)\cos\right.}{7+2n}$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)} + \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{5}{2}+n}(c+dx)\,dx + \frac{\left((C(5+2n)+A(7+2n))\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\,dx}{7+2n}$$

$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)} - \frac{2\left(\frac{A}{5+2n}+\frac{C}{7+2n}\right)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c\right.}{d\sqrt{\sin^2(c+dx)}} - \frac{2B\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left((C(5+2n)+A(7+2n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\right.\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*((C*(5 + 2*n) + A*(7
 + 2*n))*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*S
```

```
qrt[Sin[c + d*x]^2] - (5 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeo
metric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]
^2])))/(d*(5 + 2*n)*(7 + 2*n))
```

Maple [F]

$$\int \left(\cos^{\frac{3}{2}}\left(dx+c\right)\right) \left(\cos\left(dx+c\right) b\right)^{n} \left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right) dx$$

```
[In] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{n}\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
+ c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{n}\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{3}{2}} dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{3}{2}} dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \cos\left(c+d\,x\right)^{3/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right) dx$$

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.378 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\left(A+B\cos(c+dx)+\right.$

Optimal result

Integrand size = 41, antiderivative size = 223

$$
\begin{aligned}
&\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx \\
&= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)} \\
&\quad - \frac{2(C(3+2n)+A(5+2n))\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\text{c}\right.}{d(3+2n)(5+2n)\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{2B\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(c+dx}{d(5+2n)\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

```
[Out] 2*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(5+2*n)-2*(C*(3+2*n)+A*(
5+2*n))*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1
/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2+16*n+15)/(sin(d*x+c)^2)^(1/2)-2*B*c
os(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*n],cos
(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used

$= \{20, 3102, 2827, 2722\}$

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = -\frac{2\left(\frac{A}{2n+3}+\frac{C}{2n+5}\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\text{co}\right.}{d\sqrt{\sin^2(c+dx)}} - \frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}} + \frac{2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n}{d(2n+5)}$$

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) - (2
*(A/(3 + 2*n) + C/(5 + 2*n))*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*S
qrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeom
etric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5
 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)} + \frac{(2\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{1}{2}+n}(c+dx)\left(\frac{1}{2}\left(2C\left(\frac{3}{2}+n\right)+2A\left(\frac{5}{2}+n\right)\right)+\frac{1}{2}B(5+2n)}{5+2n}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)} + \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\,dx + \frac{((C(3+2n)+A(5+2n))\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{1}{2}+n}(c+dx)\,dx}{5+2n}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)} - \frac{2\left(\frac{A}{3+2n}+\frac{C}{5+2n}\right)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2\right.}{d\sqrt{\sin^2(c+dx)}} - \frac{2B\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left((C(3+2n)+A(5+2n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n\right.\right.}{}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*((C*(3 + 2*n) + A*(5
 + 2*n))*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*S
```

```
qrt[Sin[c + d*x]^2] - (3 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeo
metric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^
2])))/(d*(3 + 2*n)*(5 + 2*n))
```

Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{n} \left(A+B\cos \left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right) \left(\sqrt{\cos}(dx+c)\right) dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{n} \left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right) dx$$
$$=\int \left(C\cos \left(dx+c\right)^{2}+B\cos \left(dx+c\right)+A\right)\left(b\cos \left(dx+c\right)\right)^{n} \sqrt{\cos \left(dx+c\right)}\, dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(co
s(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{n} \left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right) dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(c
os(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(c
os(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt{\cos(c+d\,x)}\,(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.379 $\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 221

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)}$$
$$- \frac{2(C+2Cn+A(3+2n))\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos\right.}{d(1+2n)(3+2n)\sqrt{\sin^2(c+dx)}}$$
$$- \frac{2B\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*C*(b*cos(d*x+c))^n*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(3+2*n)-2*B*cos(d*x+c)^(
3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*(C+2*C*n+A*(3+2*n))*(b*cos(d*x+
c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*cos(d
*x+c)^(1/2)/d/(4*n^2+8*n+3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

= {20, 3102, 2827, 2722}

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx =$$
$$-\frac{2(A(2n+3)+2Cn+C)\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+1),\right.}{d(2n+1)(2n+3)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\cos^2(c+dx)\right.}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n}{d(2n+3)}$$

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c
 + d*x]],x]
```

```
[Out] (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2
*(C + 2*C*n + A*(3 + 2*n))*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeome
tric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1
+ 2*n)*(3 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(3/2)*(b*Cos[c +
 d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*S
in[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$= \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)} + \frac{(2\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{1}{2}+n}(c+dx)\left(\frac{1}{2}\left(2C\left(\frac{1}{2}+n\right)+2A\left(\frac{3}{2}+n\right)\right)+\frac{1}{2}B(3+2n)\right.}{3+2n}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)} + \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+dx)\,dx + \frac{((C+2Cn+A(3+2n))\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{1}{2}+n}(c+dx)\,dx}{3+2n}$$

$$= \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)} - \frac{2(C+2Cn+A(3+2n))\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+\right.}{d(1+2n)(3+2n)\sqrt{\sin^2(c+dx)}} - \frac{2B\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.77

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\csc(c+dx)\left(-\left((C+2Cn+A(3+2n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n)\right.\right.\right.}{}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(-((C + 2*C*n + A*(3
+ 2*n))*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sq
```

```
rt[Sin[c + d*x]^2]) + (1 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeo
metric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^
2])))/(d*(1 + 2*n)*(3 + 2*n))
```

Maple [F]

$$\int \frac{(\cos (dx + c) b)^n (A + B \cos (dx + c) + C(\cos^2 (dx + c)))}{\sqrt{\cos (dx + c)}} dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n (A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx = \int \frac{(C\cos(dx+c)^2 + B\cos(dx+c) + A)(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(co
s(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n (A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx = \int \frac{(b\cos(c+dx))^n (A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1
/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/sqrt(
cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n (A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(c
os(d*x + c)), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n (A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(c
os(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n (A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(1/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(1/2), x)
```

3.380 $\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 217

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \frac{2(A-C(1-2n)+2An)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)}{d\left(1-4n^2\right)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{2B\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+2*n)/cos(d*x+c)^(1/2)+2*(A-C*(1-2*n)+2
*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(-4*n^2+1)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)-2*B*(b*cos(
d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*c
os(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used

$= \{20, 3102, 2827, 2722\}$

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2(2An+A-C(1-2n))\sin(c+dx)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\cos^2(c\right.}{d\left(1-4n^2\right)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}$$
$$-\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(2n+1)\sqrt{\cos(c+dx)}}$$

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(3/2),x]
```

```
[Out] (2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Cos[c + d*x]]) + (2
*(A - C*(1 - 2*n) + 2*A*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 +
2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 4*n^2)*Sqrt[Cos[
c + d*x]]*Sqrt[Sin[c + d*x]^2]) - (2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^
n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c +
d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \frac{(2\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{3}{2}+n}(c+dx)\left(\frac{1}{2}\left(-2C\left(\frac{1}{2}-n\right)+2A\left(\frac{1}{2}+n\right)\right)+\frac{1}{2}B(1+}{1+2n}$$

$$= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)\,dx + \frac{((A-C(1-2n)+2An)\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{3}{2}+n}(c+dx)\,dx}{1+2n}$$

$$= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \frac{2(A-C(1-2n)+2An)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(}{d\left(1-4n^2\right)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{2B\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\text{si}}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.76

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(b\cos(c+dx))^n\csc(c+dx)\left(-\left((A+2An+C(-1+2n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2\right.\right.\right.}{}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(3/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(-((A + 2*A*n + C*(-1 + 2*n))*Hypergeome
tric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2
]) + (-1 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2, (
1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(-1 + 4
*n^2)*Sqrt[Cos[c + d*x]])
```

Maple [F]

$$\int \frac{(\cos (dx+c)\, b)^n \left(A+B\cos (dx+c)+C(\cos^2 (dx+c))\right)}{\cos (dx+c)^{\frac{3}{2}}}dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos (dx+c)^2+B\cos (dx+c)+A\right)(b\cos (dx+c))^n}{\cos (dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x
 + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos (c+dx))^n \left(A+B\cos (c+dx)+C\cos^2 (c+dx)\right)}{\cos^{\frac{3}{2}} (c+dx)}\,dx$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3
/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/cos(c
 + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(3/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(3/2), x)
```

3.381 $\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 221

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}$$
$$+\frac{2(A+C(3-2n)-2An)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\text{s}}{d(1-2n)(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{2B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(3/2)+2*(A+C*(3-2*n)-
2*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/d/(4*n^2-8*n+3)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*
cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*sin(d*x
+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

$= \{20, 3102, 2827, 2722\}$

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2(-2An+A+C(3-2n))\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\cos^2\right.}{d(1-2n)(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$
$$+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}$$
$$-\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(5/2),x]
```

```
[Out] (-2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Cos[c + d*x]^(3/2)) + (
2*(A + C*(3 - 2*n) - 2*A*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 +
 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*(3 - 2*n)*
Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(2\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\left(\frac{1}{2}\left(-2A\left(\frac{1}{2}-n\right)-2C\left(\frac{3}{2}-n\right)\right)-\frac{1}{2}B(1-2}{1-2n}$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)} + \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\,dx - \frac{\left(\left(-2A\left(\frac{1}{2}-n\right)-2C\left(\frac{3}{2}-n\right)\right)\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\,dx}{1-2n}$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(\frac{C}{1-2n}+\frac{A}{3-2n}\right)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}} + \frac{2B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(b\cos(c+dx))^n\csc(c+dx)\left(-\left(\left(C(-3+2n)+A(-1+2n)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+\right.\right.\right.}{}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(5/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(-((C*(-3 + 2*n) + A*(-1 + 2*n))*Hyperge
ometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x
]^2]) + (-3 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2
, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(-3
 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2))
```

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n\,(A+B\cos(dx+c)+C(\cos^2(dx+c)))}{\cos(dx+c)^{\frac{5}{2}}}dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x
 + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5
/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(5/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(5/2), x)
```

3.382 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 223

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)}$$
$$+\frac{2(A(3-2n)+C(5-2n))(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+d\right.}{d(3-2n)(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{2B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(5/2)+2*(A*(3-2*n)+C*
(5-2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+
c)^2)*sin(d*x+c)/d/(4*n^2-16*n+15)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)+2*
B*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*si
n(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used

$= \{20, 3102, 2827, 2722\}$

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(\frac{A}{5-2n}+\frac{C}{3-2n}\right)\sin(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-5),\frac{1}{4}(2n-1),\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)}$$
$$+\frac{2B\sin(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$
$$-\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)}$$

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(7/2),x]
```

```
[Out] (-2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(5/2)) + (
2*(C/(3 - 2*n) + A/(5 - 2*n))*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5
 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2
)*Sqrt[Sin[c + d*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-
3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c +
 d*x]^(3/2)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} - \frac{(2\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{7}{2}+n}(c+dx)\left(\frac{1}{2}\left(-2A\left(\frac{3}{2}-n\right)-2C\left(\frac{5}{2}-n\right)\right)-\frac{1}{2}B(3-}{3-2n}$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} + \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\,dx - \frac{\left(\left(-2A\left(\frac{3}{2}-n\right)-2C\left(\frac{5}{2}-n\right)\right)\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx)\,dx}{3-2n}$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(\frac{C}{3-2n}+\frac{A}{5-2n}\right)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+dx)\right)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}} + \frac{2B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(b\cos(c+dx))^n\csc(c+dx)\left(-\left((C(-5+2n)+A(-3+2n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-\right.\right.\right.}{}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(7/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(-((C*(-5 + 2*n) + A*(-3 + 2*n))*Hyperge
ometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*
x]^2]) + (-5 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/
2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(-
5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2))
```

Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n\,(A+B\cos(dx+c)+C(\cos^2(dx+c)))}{\cos(dx+c)^{\frac{7}{2}}}dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x
 + c)^(7/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7
/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\int \frac{(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A)(b\cos{(dx+c)})^n}{\cos{(dx+c)}^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\int \frac{(b\,\cos{(c+d\,x)})^n\,\left(C\cos{(c+d\,x)}^2+B\,\cos{(c+d\,x)}+A\right)}{\cos{(c+d\,x)}^{7/2}}\,dx$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(7/2), x)
```

3.383 $\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+\right.$

Optimal result

Integrand size = 33, antiderivative size = 183

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)}$$
$$+\frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}$$
$$+\frac{2^{\frac{1}{2}+m}(Bm(2+m)+C(1+m+m^2)+A(2+3m+m^2))\,(1+\cos(e+fx))^{-\frac{1}{2}-m}(a+a\cos(e+fx))^m\,\mathrm{H}}{f(1+m)(2+m)}$$

```
[Out] -(C-B*(2+m))*(a+a*cos(f*x+e))^m*sin(f*x+e)/f/(1+m)/(2+m)+C*(a+a*cos(f*x+e))
^(1+m)*sin(f*x+e)/a/f/(2+m)+2^(1/2+m)*(B*m*(2+m)+C*(m^2+m+1)+A*(m^2+3*m+2))
*(1+cos(f*x+e))^(-1/2-m)*(a+a*cos(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2],1/
2-1/2*cos(f*x+e))*sin(f*x+e)/f/(m^2+3*m+2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used

$= \{3102, 2830, 2731, 2730\}$

$$\int (a + a\cos(e + fx))^m \left(A + B\cos(e + fx) + C\cos^2(e + fx)\right)\,dx$$
$$= \frac{2^{m+\frac{1}{2}}(A(m^2 + 3m + 2) + Bm(m + 2) + C(m^2 + m + 1))\sin(e + fx)(\cos(e + fx) + 1)^{-m-\frac{1}{2}}(a\cos(e + f\ldots}{f(m + 1)(m + 2)}$$
$$- \frac{(C - B(m + 2))\sin(e + fx)(a\cos(e + fx) + a)^m}{f(m + 1)(m + 2)}$$
$$+ \frac{C\sin(e + fx)(a\cos(e + fx) + a)^{m+1}}{af(m + 2)}$$

```
[In] Int[(a + a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] -((((C - B*(2 + m))*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m)*(2 + m))
) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 +
 m)*(B*m*(2 + m) + C*(1 + m + m^2) + A*(2 + 3*m + m^2))*(1 + Cos[e + f*x])^
(-1/2 - m)*(a + a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 -
 Cos[e + f*x])/2]*Sin[e + f*x])/(f*(1 + m)*(2 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

integral

$$= \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{\int (a+a\cos(e+fx))^m(a(C(1+m)+A(2+m))-a(C-B(2+m))\cos(e+fx))\,dx}{a(2+m)}$$

$$= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)} + \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{(Bm(2+m)+C(1+m+m^2)+A(2+3m+m^2))\int (a+a\cos(e+fx))^m\,dx}{(1+m)(2+m)}$$

$$= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)} + \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{((Bm(2+m)+C(1+m+m^2)+A(2+3m+m^2))\,(1+\cos(e+fx))^{-m}(a+a\cos(e+fx))^m)\int (1+c}{(1+m)(2+m)}$$

$$= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)} + \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{2^{\frac{1}{2}+m}(Bm(2+m)+C(1+m+m^2)+A(2+3m+m^2))\,(1+\cos(e+fx))^{-\frac{1}{2}-m}(a+a\cos(e+fx))^m\,\mathrm{H}}{f(1+m)(2+m)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.05

$$\int (a+a\cos(e+fx))^m\left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= \frac{i4^{-1-m}e^{ifmx}\left(1+e^{i(e+fx)}\right)^{-2m}\left(e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\right)^{2m}\cos^{-2m}\left(\frac{1}{2}(e+fx)\right)(a(1+\cos(e+fx)))^m\left(\frac{Ce^{-}}{}\right.}{}$$

```
[In] Integrate[(a + a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (I*4^(-1 - m)*E^(I*f*m*x)*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m)
*(a*(1 + Cos[e + f*x]))^m*((C*Hypergeometric2F1[-2 - m, -2*m, -1 - m, -E^(I
*(e + f*x))])/(E^(I*(2*e + f*(2 + m)*x))*(2 + m)) + (2*B*Hypergeometric2F1[
-1 - m, -2*m, -m, -E^(I*(e + f*x))])/(E^(I*(e + f*x))*(1 + m)*x))*(1 + m)) + (2
```

```
*B*E^(I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -2*m, 2 - m, -E^(I*(e
+ f*x))])/(-1 + m) + (C*E^((2*I)*e - I*f*(-2 + m)*x)*Hypergeometric2F1[2 -
m, -2*m, 3 - m, -E^(I*(e + f*x))])/(-2 + m) + (4*A*Hypergeometric2F1[-2*m,
-m, 1 - m, -E^(I*(e + f*x))])/(E^(I*f*m*x)*m) + (2*C*Hypergeometric2F1[-2*m
, -m, 1 - m, -E^(I*(e + f*x))])/(E^(I*f*m*x)*m)))/((1 + E^(I*(e + f*x)))^(2
*m)*f*Cos[(e + f*x)/2]^(2*m))
```

Maple [F]

$$\int (a+\cos (fx+e)\, a)^{m} \left(A+\cos (fx+e)\, B+C\left(\cos^{2}(fx+e)\right)\right) dx$$

```
[In] int((a+cos(f*x+e)*a)^m*(A+cos(f*x+e)*B+C*cos(f*x+e)^2),x)
[Out] int((a+cos(f*x+e)*a)^m*(A+cos(f*x+e)*B+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$=\int \left(C\cos\left(fx+e\right)^{2}+B\cos\left(fx+e\right)+A\right)\left(a\cos\left(fx+e\right)+a\right)^{m} dx$$

```
[In] integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
fricas")
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$=\int \left(a(\cos\left(e+fx\right)+1)\right)^{m}\left(A+B\cos\left(e+fx\right)+C\cos^{2}\left(e+fx\right)\right) dx$$

```
[In] integrate((a+a*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)
[Out] Integral((a*(cos(e + f*x) + 1))**m*(A + B*cos(e + f*x) + C*cos(e + f*x)**2)
, x)
```

Maxima [F]

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)+A\right)\left(a\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
maxima")
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m, x
)
```

Giac [F]

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)+A\right)\left(a\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
giac")
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m, x
)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(a+a\,\cos\left(e+f\,x\right)\right)^m\,\left(C\cos\left(e+f\,x\right)^2+B\,\cos\left(e+f\,x\right)+A\right)\,dx$$

```
[In] int((a + a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2),x)
[Out] int((a + a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

3.384 $\int (a+a\cos(c+dx))^{2/3} (A+B\cos(c+dx)+C\cos^2(c$

Optimal result

Integrand size = 35, antiderivative size = 144

$$\int (a+a\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{3(8B-3C)(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d} + \frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} + \frac{(40A+16B+19C)(a+a\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{10\ 2^{5/6}d(1+\cos(c+dx))^{7/6}}$$

```
[Out] 3/40*(8*B-3*C)*(a+a*cos(d*x+c))^(2/3)*sin(d*x+c)/d+3/8*C*(a+a*cos(d*x+c))^(
5/3)*sin(d*x+c)/a/d+1/20*(40*A+16*B+19*C)*(a+a*cos(d*x+c))^(2/3)*hypergeom(
[-1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(
7/6)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3102, 2830, 2731, 2730}

$$\int (a+a\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{(40A+16B+19C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\right.}{10\ 2^{5/6}d(\cos(c+dx)+1)^{7/6}} + \frac{3(8B-3C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{40d} + \frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

```
[In] Int[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*(8*B - 3*C)*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(40*d) + (3*C*(a +
a*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*a*d) + ((40*A + 16*B + 19*C)*(a + a*
Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]
*Sin[c + d*x])/(10*2^(5/6)*d*(1 + Cos[c + d*x])^(7/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} + \frac{3\int(a+a\cos(c+dx))^{2/3}\left(\frac{1}{3}a(8A+5C)+\frac{1}{3}a(8B-3C)\cos(c+dx)\right)\,dx}{8a}$$

$$
\begin{aligned}
&= \frac{3(8B-3C)(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d} \\
&\quad + \frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} \\
&\quad + \frac{1}{40}(40A+16B+19C)\int (a+a\cos(c+dx))^{2/3}\,dx \\
&= \frac{3(8B-3C)(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d} \\
&\quad + \frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} + \frac{\left((40A+16B+19C)(a+a\cos(c+dx))^{2/3}\right)\int (1+\cos(}{40(1+\cos(c+dx))^{2/3}} \\
&= \frac{3(8B-3C)(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d} + \frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} \\
&\quad + \frac{(40A+16B+19C)(a+a\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\text{si}}{10\,2^{5/6}d(1+\cos(c+dx))^{7/6}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$
\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3(a(1+\cos(c+dx)))^{2/3}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-2i(40A+16B+19C)\,\text{Hypergeometric2}\right.}{}
$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] (3*(a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*((-2*I)*(40*A + 16*B + 1
9*C)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^(I*(c + d*x))]*(1 + Cos[c + d*x] +
 I*Sin[c + d*x])^(2/3) + 2*(40*A + 32*B + 28*C + 2*(8*B + 7*C)*Cos[c + d*x]
 + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(320*d)
```

Maple [F]

$$
\int (a+\cos(dx+c)\,a)^{\frac{2}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx
$$

```
[In] int((a+cos(d*x+c)*a)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+cos(d*x+c)*a)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a + a\cos(c + dx))^{2/3} (A + B\cos(c + dx) + C\cos^2(c+dx)) \, dx = \int (C\cos(dx + c)^2 + B\cos(dx + c) + A)(a\cos(dx + c) + a)^{\frac{2}{3}} \, dx$$

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{2/3} (A + B\cos(c + dx) + C\cos^2(c + dx)) \, dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a\cos(c + dx))^{2/3} (A + B\cos(c + dx) + C\cos^2(c+dx)) \, dx = \int (C\cos(dx + c)^2 + B\cos(dx + c) + A)(a\cos(dx + c) + a)^{\frac{2}{3}} \, dx$$

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3
), x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3
), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int (a+a\,\cos\left(c+d\,x\right))^{2/3}\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

```
[In] int((a + a*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int((a + a*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.385 $\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c$

Optimal result

Integrand size = 35, antiderivative size = 144

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3(7B-3C)\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{28d}+\frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad}$$
$$+\frac{(28A+7B+13C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{14\sqrt[6]{2}d(1+\cos(c+dx))^{5/6}}$$

```
[Out] 3/28*(7*B-3*C)*(a+a*cos(d*x+c))^(1/3)*sin(d*x+c)/d+3/7*C*(a+a*cos(d*x+c))^(
4/3)*sin(d*x+c)/a/d+1/28*(28*A+7*B+13*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([
1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/d/(1+cos(d*x+c))^(5/
6)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3102, 2830, 2731, 2730}

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{(28A+7B+13C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{14\sqrt[6]{2}d(\cos(c+dx)+1)^{5/6}}$$
$$+\frac{3(7B-3C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{28d}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

```
[In] Int[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*(7*B - 3*C)*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(28*d) + (3*C*(a +
a*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*a*d) + ((28*A + 7*B + 13*C)*(a + a*C
os[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*S
in[c + d*x])/(14*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad} \\
&\quad + \frac{3\int \sqrt[3]{a+a\cos(c+dx)}\left(\frac{1}{3}a(7A+4C)+\frac{1}{3}a(7B-3C)\cos(c+dx)\right)\,dx}{7a} \\
&= \frac{3(7B-3C)\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{28d} + \frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad} \\
&\quad + \frac{1}{28}(28A+7B+13C)\int \sqrt[3]{a+a\cos(c+dx)}\,dx
\end{aligned}
$$

$$= \frac{3(7B-3C)\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{28d} + \frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad} + \frac{\left((28A+7B+13C)\sqrt[3]{a+a\cos(c+dx)}\right)\int\sqrt[3]{1+\cos(c+dx)}\,dx}{28\sqrt[3]{1+\cos(c+dx)}}$$

$$= \frac{3(7B-3C)\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{28d} + \frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad} + \frac{(28A+7B+13C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{14\sqrt[6]{2}d(1+\cos(c+dx))^{5/6}}$$

Mathematica [F]

$$\int\sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int\sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
[Out] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
), x]
```

Maple [F]

$$\int\left(a+\cos(dx+c)\,a\right)^{\frac{1}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int((a+cos(d*x+c)*a)^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int((a+cos(d*x+c)*a)^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int\sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\left(a\cos(dx+c)+a\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3)
, x)
```

Sympy [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
[Out] Integral((a*(cos(c + d*x) + 1))**(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)
**2), x)
```

Maxima [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3
), x)
```

Giac [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (a+a\,\cos(c+d\,x))^{1/3}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

```
[In] int((a + a*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int((a + a*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.386 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 144

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}}+\frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad}+\frac{(10A-5B+7C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{5\ 2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}$$

```
[Out] 3/10*(5*B-3*C)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/3)+3/5*C*(a+a*cos(d*x+c))^(
2/3)*sin(d*x+c)/a/d+1/10*(10*A-5*B+7*C)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*
cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(1/6)/(a+a*cos(d*x+c))^(1/3
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3102, 2830, 2731, 2730}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{(10A-5B+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{5\ 2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\sqrt[3]{a\cos(c+dx)+a}}+\frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a\cos(c+dx)+a}}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*(5*B - 3*C)*Sin[c + d*x])/(10*d*(a + a*Cos[c + d*x])^(1/3)) + (3*C*(a +
a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*a*d) + ((10*A - 5*B + 7*C)*Hypergeom
etric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(5*2^(5/6)*d*(1
 + Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad}+\frac{3\int\frac{\frac{1}{3}a(5A+2C)+\frac{1}{3}a(5B-3C)\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx}{5a}\\
&= \frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}}+\frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad}\\
&\quad+\frac{1}{10}(10A-5B+7C)\int\frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx
\end{aligned}
$$

$$= \frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}} + \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{\left((10A-5B+7C)\sqrt[3]{1+\cos(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\cos(c+dx)}}\,dx}{10\sqrt[3]{a+a\cos(c+dx)}}$$

$$= \frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}} + \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{(10A-5B+7C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{5\ 2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.73

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{-3i(10A-5B+7C)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-e^{i(c+dx)}\right)(1+\cos(c+dx)+i\sin(c+dx))^{2/3}+3(5B}{10d\sqrt[3]{a(1+\cos(c+dx))}}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3
),x]
```

```
[Out] ((-3*I)*(10*A - 5*B + 7*C)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^(I*(c + d*x)
)]*(1 + Cos[c + d*x] + I*Sin[c + d*x])^(2/3) + 3*(5*B - C + 2*C*Cos[c + d*x
])*Sin[c + d*x])/(10*d*(a*(1 + Cos[c + d*x]))^(1/3))
```

Maple [F]

$$\int \frac{A+B\cos(dx+c)+C(\cos^2(dx+c))}{(a+\cos(dx+c)\,a)^{\frac{1}{3}}}dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorit
hm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3)
, x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \int \frac{A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)}{\sqrt[3]{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/3),x)
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**(
1/3), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorit
hm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3
), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorit
hm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3), x)
```

3.387 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 144

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx=\frac{3(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}}+\frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad}-\frac{(4A-8B+7C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2\sqrt[6]{2}ad(1+\cos(c+dx))^{5/6}}$$

```
[Out] 3*(A-B+C)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(2/3)+3/4*C*(a+a*cos(d*x+c))^(1/3)*
sin(d*x+c)/a/d-1/4*(4*A-8*B+7*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2
],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/a/d/(1+cos(d*x+c))^(5/6)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3102, 2829, 2731, 2730}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx=-\frac{(4A-8B+7C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}ad(\cos(c+dx)+1)^{5/6}}+\frac{3(A-B+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*(A - B + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) + (3*C*(a + a*C
os[c + d*x])^(1/3)*Sin[c + d*x])/(4*a*d) - ((4*A - 8*B + 7*C)*(a + a*Cos[c
+ d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c
+ d*x])/(2*2^(1/6)*a*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad} + \frac{3\int \frac{\frac{1}{3}a(4A+C)+\frac{1}{3}a(4B-3C)\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx}{4a}$$

$$= \frac{3(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad} - \frac{(4A-8B+7C)\int \sqrt[3]{a+a\cos(c+dx)}\,dx}{4a}$$

$$= \frac{3(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad} - \frac{\left((4A-8B+7C)\sqrt[3]{a+a\cos(c+dx)}\right)\int\sqrt[3]{1+\cos(c+dx)}\,dx}{4a\sqrt[3]{1+\cos(c+dx)}}$$

$$= \frac{3(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad} - \frac{(4A-8B+7C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx}{2\sqrt[6]{2}ad(1+\cos(c+dx))^{5/6}}$$

Mathematica [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3
),x]
```

```
[Out] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3
), x]
```

Maple [F]

$$\int \frac{A+B\cos(dx+c)+C(\cos^2(dx+c))}{(a+\cos(dx+c)\,a)^{\frac{2}{3}}}dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3)
, x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx=\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a(\cos(c+dx)+1))^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(2/3),x)
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**(
2/3), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorit
hm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3
), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorit
hm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A}{(a+a\,\cos(c+d\,x))^{2/3}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3), x)
```

3.388 $\int (a+b\cos(c+dx))^{2/3} \left(A + B\cos(c + dx) + C\cos^2(c -\right.$

Optimal result

Integrand size = 35, antiderivative size = 290

$$\int (a+b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx) + C\cos^2(c+dx)\right) dx = \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}$$

$$+ \frac{(a+b)(8bB-3aC)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+\ldots}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

$$+ \frac{(8Ab^2-8abB+3a^2C+5b^2C)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))\ldots}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] 3/8*C*(a+b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d+1/8*(a+b)*(8*B*b-3*C*a)*AppellF
1(1/2,-5/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+
c))^(2/3)*sin(d*x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*
x+c))^(1/2)+1/8*(8*A*b^2-8*B*a*b+3*C*a^2+5*C*b^2)*AppellF1(1/2,-2/3,1/2,3/2
,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+
c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3102, 2835, 2744, 144, 143}

$$\int (a+b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\sin(c+dx)\left(3a^2C-8abB+8Ab^2+5b^2C\right)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\ldots\right)}{4\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{(a+b)(8bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+\ldots}{a+b}\right)}{4\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

```
[In] Int[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((a + b)*(8*b*B - 3
*a*C)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + ((8*A*b^2 - 8*
a*b*B + 3*a^2*C + 5*b^2*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])
/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x]
)/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2
/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} \\
&\quad + \frac{3\int(a+b\cos(c+dx))^{2/3}\left(\frac{1}{3}b(8A+5C)+\frac{1}{3}(8bB-3aC)\cos(c+dx)\right)\,dx}{8b} \\
&= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{(8bB-3aC)\int(a+b\cos(c+dx))^{5/3}\,dx}{8b^2} \\
&\quad + \frac{\left(3\left(\frac{1}{3}b^2(8A+5C)-\frac{1}{3}a(8bB-3aC)\right)\right)\int(a+b\cos(c+dx))^{2/3}\,dx}{8b^2} \\
&= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} \\
&\quad - \frac{((8bB-3aC)\sin(c+dx))\text{Subst}\left(\int\frac{(a+bx)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{8b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} \\
&\quad - \frac{\left(3\left(\frac{1}{3}b^2(8A+5C)-\frac{1}{3}a(8bB-3aC)\right)\sin(c+dx)\right)\text{Subst}\left(\int\frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{8b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{((-a-b)(8bB-3aC)(a+b\cos(c+dx))^{2/3}\sin(c+dx))\,\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c\right.}{8b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}} - \frac{\left(3\left(\frac{1}{3}b^2(8A+5C)-\frac{1}{3}a(8bB-3aC)\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right.}{\sqrt{1-x}\sqrt{1+x}}\right.}{8b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}}$$

$$= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{(a+b)(8bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx)}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{(8Ab^2-8abB+3a^2C+5b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.02

$$\int (a+b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(20(-a^2+b^2)(8bB-3aC)\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+\right.\right.}{a+b}}{}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(20*(-a^2 + b^2)*(8*b*B - 3*a*C
)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
 d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos
[c + d*x]))/(a - b))] + 4*(40*A*b^2 + 16*a*b*B - 6*a^2*C + 25*b^2*C)*Appell
F1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(
a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x])
)/(-a + b)]*(a + b*Cos[c + d*x]) - 20*b^2*(8*b*B + 2*a*C + 5*b*C*Cos[c + d*
x])*Sin[c + d*x]^2))/(800*b^3*d)
```

Maple [F]

$$\int (a + \cos (dx + c) b)^{\frac{2}{3}} \left(A + B \cos (dx + c) + C \left(\cos^2 (dx + c)\right)\right) dx$$

```
[In] int((a+cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int((a+cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a + b\cos(c + dx))^{2/3} \left(A + B\cos(c + dx) + C\cos^2(c+dx)\right) dx = \int \left(C\cos (dx + c)^2 + B\cos (dx + c) + A\right)(b\cos (dx + c) + a)^{\frac{2}{3}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx))^{2/3} \left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a + b\cos(c + dx))^{2/3} \left(A + B\cos(c + dx) + C\cos^2(c+dx)\right) dx = \int \left(C\cos (dx + c)^2 + B\cos (dx + c) + A\right)(b\cos (dx + c) + a)^{\frac{2}{3}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3
), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)^{\frac{2}{3}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3
), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \int (a+b\cos(c+dx))^{2/3} \left(C\cos(c+dx)^2+B\cos(c+dx)+A\right) dx$$

```
[In] int((a + b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int((a + b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.389 $\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c$

Optimal result

Integrand size = 35, antiderivative size = 290

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd}$$
$$+\frac{\sqrt{2}(a+b)(7bB-3aC)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{\sqrt{2}(7Ab^2-7abB+3a^2C+4b^2C)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(}}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 3/7*C*(a+b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d+1/7*(a+b)*(7*B*b-3*C*a)*AppellF
1(1/2,-4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+
c))^(1/3)*sin(d*x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*
x+c))^(1/2)+1/7*(7*A*b^2-7*B*a*b+3*C*a^2+4*C*b^2)*AppellF1(1/2,-1/3,1/2,3/2
,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+
c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3102, 2835, 2744, 144, 143}

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{2}\sin(c+dx)\left(3a^2C-7abB+7Ab^2+4b^2C\right)\sqrt[3]{a+b\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+d\right.}{7b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{\sqrt{2}(a+b)(7bB-3aC)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\text{cc}}{\text{c}}\right.}{7b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

```
[In] Int[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + (Sqrt[2]*(a + b)*(7
*b*B - 3*a*C)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - C
os[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(7*A*
b^2 - 7*a*b*B + 3*a^2*C + 4*b^2*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c
 + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[
c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1
/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} \\
&\quad + \frac{3\int \sqrt[3]{a+b\cos(c+dx)}\left(\frac{1}{3}b(7A+4C)+\frac{1}{3}(7bB-3aC)\cos(c+dx)\right)\,dx}{7b} \\
&= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{(7bB-3aC)\int (a+b\cos(c+dx))^{4/3}\,dx}{7b^2} \\
&\quad + \frac{\left(3\left(\frac{1}{3}b^2(7A+4C)-\frac{1}{3}a(7bB-3aC)\right)\right)\int \sqrt[3]{a+b\cos(c+dx)}\,dx}{7b^2} \\
&= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} \\
&\quad - \frac{((7bB-3aC)\sin(c+dx))\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{7b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} \\
&\quad - \frac{\left(3\left(\frac{1}{3}b^2(7A+4C)-\frac{1}{3}a(7bB-3aC)\right)\sin(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{7b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}
\end{aligned}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{\left((-a-b)(7bB-3aC)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+\right.}{7b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}} - \frac{\left(3\left(\frac{1}{3}b^2(7A+4C)-\frac{1}{3}a(7bB-3aC)\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\operatorname{Subst}\left(\int \frac{\sqrt[3]{-\frac{a}{-a-b}-}}{\sqrt{1-x}\sqrt{1}}\right.}{7b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{\sqrt{2}(a+b)(7bB-3aC)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c-}}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{2}(7Ab^2-7abB+3a^2C+4b^2C)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a}}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] (verified)

Time = 3.07 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.01

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(-a^2+b^2)(7bB-3aC)\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+}{a+b}\right.\right.}{}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*(7*b*B - 3*a*C)
*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[
c + d*x]))/(a - b))] + (28*A*b^2 + 7*a*b*B - 3*a^2*C + 16*b^2*C)*AppellF1[4
/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a +
b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-
a + b)]*(a + b*Cos[c + d*x]) - 4*b^2*(7*b*B + a*C + 4*b*C*Cos[c + d*x])*Sin
[c + d*x]^2))/(112*b^3*d)
```

Maple [F]

$$\int (a+\cos(dx+c)\,b)^{\frac{1}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

```
[In] int((a+cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int((a+cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3)
, x)
```

Sympy [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
[Out] Integral((a + b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)**
2), x)
```

Maxima [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3
), x)
```

Giac [F]

$$\int \sqrt[3]{a + b\cos(c + dx)}\left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) dx$$
$$= \int \left(C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A\right)\left(b\cos\left(dx + c\right) + a\right)^{\frac{1}{3}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a + b\cos(c + dx)}\left(A + B\cos(c + dx) + C\cos^2(c + dx)\right) dx$$
$$= \int \left(a + b\,\cos\left(c + d\,x\right)\right)^{1/3}\left(C\cos\left(c + d\,x\right)^2 + B\,\cos\left(c + d\,x\right) + A\right) dx$$

```
[In] int((a + b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int((a + b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.390 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 287

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}$$
$$+\frac{\sqrt{2}(5bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+d}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$
$$+\frac{\sqrt{2}(5Ab^2-5abB+3a^2C+2b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c-}{a+b}}}{5b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

```
[Out] 3/5*C*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d+1/5*(5*B*b-3*C*a)*AppellF1(1/2,
-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2
/3)*sin(d*x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^
(1/2)+1/5*(5*A*b^2-5*B*a*b+3*C*a^2+2*C*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-c
os(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x
+c)*2^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3102, 2835, 2744, 144, 143}

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\sin(c+dx)\left(3a^2C-5abB+5Ab^2+2b^2C\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx\right.}{5b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}$$
$$+\frac{\sqrt{2}(5bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx\right.}{5b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$
$$+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + (Sqrt[2]*(5*b*B - 3
*a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(5*A*b^2 - 5*
a*b*B + 3*a^2*C + 2*b^2*C)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/
2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin
[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}+\frac{3\int\frac{\frac{1}{3}b(5A+2C)+\frac{1}{3}(5bB-3aC)\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx}{5b}\\
&= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}+\frac{(5bB-3aC)\int(a+b\cos(c+dx))^{2/3}\,dx}{5b^2}\\
&\quad+\frac{\left(3\left(\frac{1}{3}b^2(5A+2C)-\frac{1}{3}a(5bB-3aC)\right)\right)\int\frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx}{5b^2}\\
&= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}\\
&\quad-\frac{((5bB-3aC)\sin(c+dx))\text{Subst}\left(\int\frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{5b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\\
&\quad-\frac{\left(3\left(\frac{1}{3}b^2(5A+2C)-\frac{1}{3}a(5bB-3aC)\right)\sin(c+dx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+bx}}\,dx,x,\cos(c+dx)\right)}{5b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}
\end{aligned}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{((5bB-3aC)(a+b\cos(c+dx))^{2/3}\sin(c+dx))\,\text{Subst}\left(\int\frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{5b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}} - \frac{\left(3\left(\frac{1}{3}b^2(5A+2C)-\frac{1}{3}a(5bB-3aC)\right)\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}\sin(c+dx)\right)\text{Subst}\left(\int\frac{}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{\cdots}}\right.}{5b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+\cdots}}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{\sqrt{2}(5bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{\sqrt{2}(5Ab^2-5abB+3a^2C+2b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a-\cdots}{\cdots}}}{5b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.93

$$\int\frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = -\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(5Ab^2-5abB+3a^2C+2b^2C)\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\right.\right.}{}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3
),x]

[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(5*A*b^2 - 5*a*b*B + 3*a^2*C
 + 2*b^2*C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a +
 b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*
(1 + Cos[c + d*x]))/(-a + b)] + 2*(5*b*B - 3*a*C)*AppellF1[5/3, 1/2, 1/2, 8
/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(
-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*
Cos[c + d*x]) - 10*b^2*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Maple [F]

$$\int \frac{A+B\cos(dx+c)+C(\cos^2(dx+c))}{(a+\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3)
, x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(1/
3), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3
), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3), x)
```

3.391 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 286

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}$$
$$+\frac{(4bB-3aC)\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{(4Ab^2-4abB+3a^2C+b^2C)\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\sin(c}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

```
[Out] 3/4*C*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d+1/4*(4*B*b-3*C*a)*AppellF1(1/2,
-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1
/3)*sin(d*x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)*2^(1/2)/(1+cos(d*x+c))^
(1/2)+1/4*(4*A*b^2-4*B*a*b+3*C*a^2+C*b^2)*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos
(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c
)/b^2/d/(a+b*cos(d*x+c))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{3102, 2835, 2744, 144, 143\}$

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{\sin(c+dx)\left(3a^2C - 4abB + 4Ab^2 + b^2C\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\text{Ap}}{2\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}(a -}$$
$$+ \frac{(4bB-3aC)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{2\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+ \frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((4*b*B - 3*a*C)*Ap
pellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a
 + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + ((4*A*b^2 - 4*a*b*B +
3*a^2*C + b^2*C)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 -
 Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])
/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
```

```
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}+\frac{3\int\frac{\frac{1}{3}b(4A+C)+\frac{1}{3}(4bB-3aC)\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx}{4b}\\
&= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}+\frac{(4bB-3aC)\int\sqrt[3]{a+b\cos(c+dx)}\,dx}{4b^2}\\
&\quad+\frac{1}{4}\left(4A+C-\frac{a(4bB-3aC)}{b^2}\right)\int\frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx\\
&= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}\\
&\quad-\frac{((4bB-3aC)\sin(c+dx))\text{Subst}\left(\int\frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{4b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\\
&\quad+\frac{\left(\left(-4A-C+\frac{a(4bB-3aC)}{b^2}\right)\sin(c+dx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{2/3}}\,dx,x,\cos(c+dx)\right)}{4d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}
\end{aligned}
$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}$$

$$-\frac{\left((4bB-3aC)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\mathrm{Subst}\left(\int\frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}}dx,x,\cos(c+\right.}{4b^2d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}}$$

$$+\frac{\left(\left(-4A-C+\frac{a(4bB-3aC)}{b^2}\right)\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}\sin(c+dx)\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}\right.}{4d\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}$$

$$+\frac{(4bB-3aC)\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$+\frac{\left(4A+C-\frac{a(4bB-3aC)}{b^2}\right)\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/}}{2\sqrt{2}d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

Mathematica [A] (warning: unable to verify)

Time = 2.34 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.93

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx =$$

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(4Ab^2-4abB+3a^2C+b^2C)\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos}{a+}\right.\right.}{}$$

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3
),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(4*A*b^2 - 4*a*b*B + 3*a^2*C
 + b^2*C)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b
*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1
 + Cos[c + d*x]))/(-a + b)] + (4*b*B - 3*a*C)*AppellF1[4/3, 1/2, 1/2, 7/3,
(a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 +
 Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[
c + d*x]) - 4*b^2*C*Sin[c + d*x]^2))/(16*b^3*d)
```

Maple [F]

$$\int \frac{A + B\cos(dx + c) + C(\cos^2(dx + c))}{(a + \cos(dx + c)\, b)^{\frac{2}{3}}} dx$$

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(2/3),x)
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*b)^(2/3),x)
```

Fricas [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{2/3}} dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c) + a)^{\frac{2}{3}}} dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorit
hm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3)
, x)
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{2/3}} dx = \int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{\frac{2}{3}}} dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3),x)
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(2/
3), x)
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{2/3}} dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c) + a)^{\frac{2}{3}}} dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorit
hm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3
), x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{2/3}}\,dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c) + a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorit
hm="giac")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(a + b\cos(c + dx))^{2/3}}\,dx = \int \frac{C\cos(c + d\,x)^2 + B\,\cos(c + d\,x) + A}{(a + b\,\cos(c + d\,x))^{2/3}}\,dx$$

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3), x)
```

3.392 $\int (a+b\cos(e+fx))^m (A+(A+C)\cos(e+fx)+C\,\mathrm{co}$

Optimal result

Integrand size = 35, antiderivative size = 215

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right) dx$$

$$= \frac{4\sqrt{2}C\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\mathrm{si}}{f\sqrt{1+\cos(e+fx)}}$$

$$+\frac{2\sqrt{2}(A-C)\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+}{a+b}\right.}{f\sqrt{1+\cos(e+fx)}}$$

```
[Out] 4*C*AppellF1(1/2,-m,-3/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+
b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/f/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*
x+e))^(1/2)+2*(A-C)*AppellF1(1/2,-m,-1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2
*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/f/(((a+b*cos(f*x+e))/(a+
b))^m)/(1+cos(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3096, 2834, 144, 143, 2863}

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\frac{2\sqrt{2}(A-C)\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+f$$
$$f\sqrt{\cos(e+fx)+1}$$
$$+\frac{4\sqrt{2}C\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),$$
$$f\sqrt{\cos(e+fx)+1}$$

```
[In] Int[(a + b*Cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (4*Sqrt[2]*C*AppellF1[1/2, -3/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos
[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e
 + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (2*Sqrt[2]*(A - C)*AppellF1[1/
2, -1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a
+ b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e +
 f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
```

```
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 3096

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[A - C, I
nt[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Dist[C, Int[(a + b*S
in[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, B, C,
m}, x] && EqQ[A - B + C, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (A-C)\int (1+\cos(e+fx))(a+b\cos(e+fx))^m\,dx \\ &\quad + C\int (1+\cos(e+fx))^2(a+b\cos(e+fx))^m\,dx \\ &= -\frac{(C\sin(e+fx))\text{Subst}\left(\int \frac{(1+x)^{3/2}(a+bx)^m}{\sqrt{1-x}}\,dx, x, \cos(e+fx)\right)}{f\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} \\ &\quad + \frac{((-A+C)\sin(e+fx))\text{Subst}\left(\int \frac{\sqrt{1+x}(a+bx)^m}{\sqrt{1-x}}\,dx, x, \cos(e+fx)\right)}{f\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} \\ &= \\ &\quad -\frac{\left(C(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{3/2}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^m}{\sqrt{1-x}}\,dx, x,\right.}{f\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} \\ &\quad + \frac{\left((-A+C)(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^m}{\sqrt{1-x}}\right.}{f\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}}\end{aligned}$$

$$= \frac{4\sqrt{2}C \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{3}{2}, -m, \frac{3}{2}, \frac{1}{2}(1-\cos(e+fx)), \frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+}{a+b}\right.}{f\sqrt{1+\cos(e+fx)}}$$
$$+ \frac{2\sqrt{2}(A-C) \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2}(1-\cos(e+fx)), \frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m \left(\right.}{f\sqrt{1+\cos(e+fx)}}$$

Mathematica [F]

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right) dx$$

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*Cos[e + f*x]
^2),x]
[Out] Integrate[(a + b*Cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*Cos[e + f*x]
^2), x]
```

Maple [F]

$$\int (a+b\cos(fx+e))^m \left(A+(A+C)\cos(fx+e)+C\left(\cos^2(fx+e)\right)\right) dx$$

```
[In] int((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x)
[Out] int((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos(fx+e)^2+(A+C)\cos(fx+e)+A\right)(b\cos(fx+e)+a)^m dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algorit
hm="fricas")
[Out] integral((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) + a)
^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(f*x+e))**m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= \int \left(C\cos(fx+e)^2+(A+C)\cos(fx+e)+A\right)(b\cos(fx+e)+a)^m\,dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) + a
)^m, x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= \int \left(C\cos(fx+e)^2+(A+C)\cos(fx+e)+A\right)(b\cos(fx+e)+a)^m\,dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) + a
)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (a+b\,\cos\left(e+f\,x\right))^m\,\left(C\cos\left(e+f\,x\right)^2+(A+C)\,\cos\left(e+f\,x\right)+A\right)\,dx$$

```
[In] int((a + b*cos(e + f*x))^m*(A + C*cos(e + f*x)^2 + cos(e + f*x)*(A + C)),x)
[Out] int((a + b*cos(e + f*x))^m*(A + C*cos(e + f*x)^2 + cos(e + f*x)*(A + C)), x
)
```

3.393 $\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+\right.$

Optimal result

Integrand size = 33, antiderivative size = 303

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$
$$-\frac{\sqrt{2}(a+b)(aC-bB(2+m))\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e-}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$
$$+\frac{\sqrt{2}(a^2C+b^2C(1+m)+Ab^2(2+m)-abB(2+m))\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(}{a+}\right.}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$

```
[Out] C*(a+b*cos(f*x+e))^(1+m)*sin(f*x+e)/b/f/(2+m)-(a+b)*(a*C-b*B*(2+m))*AppellF
1(1/2,-1-m,1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+
e))^m*sin(f*x+e)*2^(1/2)/b^2/f/(2+m)/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*
x+e))^(1/2)+(a^2*C+b^2*C*(1+m)+A*b^2*(2+m)-a*b*B*(2+m))*AppellF1(1/2,-m,1/2
,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+
e)*2^(1/2)/b^2/f/(2+m)/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used

$= \{3102, 2835, 2744, 144, 143\}$

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= \frac{\sqrt{2}\sin(e+fx)\left(a^2C-abB(m+2)+Ab^2(m+2)+b^2C(m+1)\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}}{b^2f(m+2)\sqrt{\cos(e+fx)+1}}$$
$$-\frac{\sqrt{2}(a+b)\sin(e+fx)(aC-bB(m+2))(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m-\right.}{b^2f(m+2)\sqrt{\cos(e+fx)+1}}$$
$$+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

```
[In] Int[(a + b*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*(a +
 b)*(a*C - b*B*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/
2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^
2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqr
t[2]*(a^2*C + b^2*C*(1 + m) + A*b^2*(2 + m) - a*b*B*(2 + m))*AppellF1[1/2,
1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*
Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b
*Cos[e + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

integral

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} + \frac{\int (a+b\cos(e+fx))^m(b(C(1+m)+A(2+m))-(aC-bB(2+m))\cos(e+fx))\,dx}{b(2+m)}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} + \frac{(-aC+bB(2+m))\int (a+b\cos(e+fx))^{1+m}\,dx}{b^2(2+m)} + \frac{(b^2(C(1+m)+A(2+m))-a(-aC+bB(2+m)))\int (a+b\cos(e+fx))^m\,dx}{b^2(2+m)}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \frac{((-aC+bB(2+m))\sin(e+fx))\text{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(e+fx)\right)}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}} - \frac{((b^2(C(1+m)+A(2+m))-a(-aC+bB(2+m)))\sin(e+fx))\,\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(e+fx)\right)}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$
$$+ \frac{\left((-a-b)(-aC+bB(2+m))(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}\right.}{\sqrt{1-}}\right.}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}}$$
$$- \frac{\left((b^2(C(1+m)+A(2+m))-a(-aC+bB(2+m)))\,(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+\right.}{b^2f(2+m)\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}}$$
$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$
$$- \frac{\sqrt{2}(a+b)(aC-bB(2+m))\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$
$$+ \frac{\sqrt{2}(a^2C+b^2C(1+m)+Ab^2(2+m)-abB(2+m))\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\mathrm{co}}{c}\right.}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 16142 vs. $2(303) = 606$.

Time = 26.98 (sec) , antiderivative size = 16142, normalized size of antiderivative = 53.27

$$\int (a+b\cos(e+fx))^m\left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
[Out] Result too large to show
```

Maple [F]

$$\int (a+b\cos(fx+e))^m\left(A+\cos(fx+e)\,B+C\left(\cos^2(fx+e)\right)\right)dx$$

```
[In] int((a+b*cos(f*x+e))^m*(A+cos(f*x+e)*B+C*cos(f*x+e)^2),x)
[Out] int((a+b*cos(f*x+e))^m*(A+cos(f*x+e)*B+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
fricas")
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
maxima")
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x
)
```

Giac [F]

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^m\,dx$$

```
[In] integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x
)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (a+b\,\cos\left(e+f\,x\right))^m\,\left(C\cos\left(e+f\,x\right)^2+B\,\cos\left(e+f\,x\right)+A\right)\,dx$$

```
[In] int((a + b*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2),x)
```

```
[Out] int((a + b*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
          'sin','cos','tan','cot','sec','csc',
          'arcsin','arccos','arctan','arccot','arcsec','arccsc',
          'sinh','cosh','tanh','coth','sech','csch',
          'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```